Automatica 81 (2017) 429–437

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Stabilization by using artificial delays: An LMI approach[☆]

Emilia Fridman, Leonid Shaikhet

School of Electrical Engineering, Tel-Aviv University, Tel-Aviv 69978, Israel

ARTICLE INFO

Article history:
Received 21 September 2016
Received in revised form
27 December 2016
Accepted 7 March 2017

Keywords:
Time-delay
Stabilization by using artificial delay
Lyapunov–Krasovskii method
LMIs

ABSTRACT

Static output-feedback stabilization for the nth order vector differential equations by using artificial multiple delays is considered. Under assumption of the stabilizability of the system by a static feedback that depends on the output and its derivatives up to the order $n-1$, a delayed static output-feedback is found that stabilizes the system. The conditions for the stability analysis of the resulting closed-loop system are given in terms of simple LMIs. It is shown that the LMIs are always feasible for appropriately chosen gains and small enough delays. Robust stability analysis in the presence of uncertain time-varying delays and stochastic perturbation of the system coefficients is provided. Numerical examples including chains of three and four integrators that are stabilized by static output-feedbacks with multiple delays illustrate the efficiency of the method.

1. Introduction

It is well-known that some classes of systems (e.g. chains of integrators or oscillators, inverted pendulums) that cannot be stabilized by memoryless static output-feedbacks, can be stabilized by using static output-feedbacks with delays (French, Ilchmann, & Mueller, 2009; Karafyllis, 2008; Kharitonov, Niculescu, Moreno, & Michiels, 2005; Michiels & Niculescu, 2014; Niculescu & Michiels, 2004). The idea of feedback design in this case is usually based on the employing of a stabilizing feedback that depends on the output derivatives, and further approximation of the output derivatives (e.g. by finite differences). In the existing works it is proved that the resulting delayed static output-feedback stabilizes the system for small enough delays. However, efficient and simple conditions for the design and robustness analysis are missing.

The objective of the present paper is to fill this gap for systems that are governed by nth order vector differential equation and that can be stabilized by a static feedback that depends on the output and its derivatives up to the order $n-1$. Some first results for $n = 2$ were obtained recently in Fridman and Shaikhet (2016), where simple LMIs for robust stability analysis of the closed-loop delayed systems were derived. Comparatively to more general LMIs for stability analysis of time-delay systems provided e.g. in Gu, Kharitonov, and Chen (2003) and Seuret and Gouaisbaut (2013) (that may also be applicable to delay-induced stability), the conditions of Fridman and Shaikhet (2016) are essentially simpler leading in numerical examples to slightly more conservative results. Moreover, differently from Gu et al. (2003) and Seuret and Gouaisbaut (2013), the feasibility of LMIs was justified in Fridman and Shaikhet (2016) for small enough delays.

In the present paper, we suggest a new idea to represent the delayed outputs in the form of Taylor expansion with the *integral (Lagrange) form of the remainder*. This leads to novel controller design and robust stability analysis via a novel simple Lyapunov functional. For $n = 2$, the suggested Lyapunov functional is different from the one of Fridman and Shaikhet (2016) and leads to less restrictive conditions. However, as in Fridman and Shaikhet (2016), this method employs a Lyapunov functional depending on the state derivative that seems to be not applicable to the stochastic case.

For the stochastic case, we develop the model transformation-based analysis initiated in Borne, Kolmanovskii, and Shaikhet (2000) and Shaikhet (2013) and applied in Fridman and Shaikhet (2016). The feasibility of the resulting LMIs is justified for appropriately chosen gains and small enough delays. Extension to time-varying delays and stochastic perturbations is considered. Numerical examples including chains of three and four integrators illustrate the efficiency of the results.

[☆] This work was supported by Israel Science Foundation (grant no. 1128/14) and by the Israeli Ministry of Absorption. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Akira Kojima under the direction of Editor Ian R. Petersen.

E-mail addresses: emilia@eng.tau.ac.il (E. Fridman), leonid.shaikhet@usa.net (L. Shaikhet).

http://dx.doi.org/10.1016/j.automatica.2017.04.015
0005-1098/

2. Problem formulation and preliminaries

Consider the nth order vector system

$$y^{(n)}(t) = \sum_{i=0}^{n-1} A_i y^{(i)}(t) + Bu(t), \tag{2.1}$$

where $y(t) = y^{(0)}(t) \in \mathbb{R}^k$ is the measurement, $y^{(i)}(t)$ is the ith derivative of $y(t)$, $u(t) \in \mathbb{R}^m$ is the control input, $A_i \in \mathbb{R}^{k\times k}$ and $B \in \mathbb{R}^{k\times m}$ are constant matrices. Assume that the open-loop system is unstable, and we are looking for a simple static output-feedback that will stabilize the system. It may happen that (2.1) is not stabilizable by $u(t) = K_0 y(t)$, but may be stabilizable by using artificial multiple delays (Karafyllis, 2008; Niculescu & Michiels, 2004).

Denote

$$\begin{aligned} x(t) &= col\{y^{(0)}(t), \dots, y^{(n-1)}(t)\} \\ &= col\{x_0(t), \dots, x_{n-1}(t)\}, \\ \bar{B} &= col\{0, \dots, 0, B\} \in \mathbb{R}^{nk\times m}, \\ A &= \begin{bmatrix} 0 & I_k & 0 & \cdots & 0 \\ 0 & 0 & I_k & \cdots & 0 \\ \dots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & I_k \\ A_0 & A_1 & A_2 & \cdots & A_{n-1} \end{bmatrix} \in \mathbb{R}^{nk\times nk}. \end{aligned} \tag{2.2}$$

Then (2.1) can be presented as

$$\dot{x}(t) = Ax(t) + \bar{B}u(t). \tag{2.3}$$

Assume that the pair $(A, \bar{B})$ is stabilizable, i.e. there exists a matrix $\bar{K} = [\bar{K}_0 \ \dots \bar{K}_{n-1}] \in \mathbb{R}^{m\times nk}$ such that the matrix $D = A + \bar{B}\bar{K}$ is Hurwitz. The corresponding state-feedback has a form

$$u(t) = \sum_{j=0}^{n-1} \bar{K}_j x_j(t), \quad \bar{K}_j \in \mathbb{R}^{m\times k}.$$

Since the derivatives $y^{(j)}(t) = x_j(t)$, $j = 1, \dots, n-1$ are not available, we approximate them by using the delayed measurements $x_0(t - h_j)(j = 1, \dots, n-1)$, where

$$0 < h_1 < \dots < h_{n-1}. \tag{2.4}$$

Differently from Fridman and Shaikhet (2016), we employ in this paper the Taylor expansion with the integral form of the remainder:

$$x_0(t - h_i) = \sum_{j=0}^{n-1} \frac{1}{j!}(-h_i)^j x_j(t) + W_i(x_{nt}), \quad i = 1, \dots, n-1, \tag{2.5}$$

where

$$W_i(x_{nt}) = \frac{(-1)^n}{(n-1)!}\int_{t-h_i}^{t} (s - t + h_i)^{n-1} x_n(s)ds \tag{2.6}$$

and where $x_n(s) = \dot{x}_{n-1}(s)$. Note that $W_i(x_{nt}) = O(h_i^n)$. In Fridman and Shaikhet (2016) the delayed state was presented as $x_0(t - h_1) = x_0(t) - hx_1(t) + \delta(t)$ with $\delta(t) = O(h^2)$, whereas a particular form of the remainder δ was not exploited. In such a way it was not clear how to extend the results of Fridman and Shaikhet (2016) to $n > 2$.

Remark 2.1. For $n = 1$ representation (2.5) coincides with the basic relation

$$x_0(t - h_1) = x_0(t) - \int_{t-h_1}^{t} \dot{x}_0(s)ds$$

for delay-dependent stability conditions (see e.g. Fridman, 2014; Kolmanovskii & Myshkis, 1999). In this sense the Lyapunov-based analysis of Section 3 naturally extends simple delay-dependent conditions from the 1st order to the nth order systems.

Denoting $h_0 = 0$, we will find a delayed stabilizing static output-feedback

$$u(t) = \sum_{i=0}^{n-1} K_i x_0(t - h_i), \quad K_i \in \mathbb{R}^{r\times k}. \tag{2.7}$$

Substituting (2.7) into (2.3), we obtain the following closed-loop system with delays

$$\dot{x}(t) = Ax(t) + \sum_{i=0}^{n-1} \bar{B}K_i x_0(t - h_i). \tag{2.8}$$

From (2.5) we have

$$\sum_{i=0}^{n-1} K_i x_0(t - h_i) = \sum_{j=0}^{n-1} \bar{K}_j x_j(t) + \sum_{i=1}^{n-1} K_i W_i(x_{nt}), \tag{2.9}$$

where

$$\begin{aligned} \bar{K}_0 &= \sum_{i=0}^{n-1} K_i \quad \text{and} \\ \bar{K}_j &= \frac{(-1)^j}{j!}\sum_{i=1}^{n-1} h_i^j K_i, \quad j = 1, \dots, n-1. \end{aligned} \tag{2.10}$$

From (2.10) for $K = [K_0 \ \dots K_{n-1}]$ we obtain

$$\begin{aligned} &\bar{K} = KM, \\ &M = \begin{bmatrix} I_k & 0 & 0 & \cdots & 0 \\ I_k & -h_1 I_k & \frac{h_1^2}{2} I_k & \cdots & \frac{(-h_1)^{n-1}}{(n-1)!} I_k \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ I_k & -h_{n-1} I_k & \frac{h_{n-1}^2}{2} I_k & \cdots & \frac{(-h_{n-1})^{n-1}}{(n-1)!} I_k \end{bmatrix}. \end{aligned} \tag{2.11}$$

Since all the delays are different, the Vandermonde-type matrix M is invertible. Moreover, the following holds:

Lemma 2.1. *Let $h_i = ih (i = 0, \dots, n-1)$ for some $h > 0$ and M be given by (2.11). Then $M^{-1} = O(h^{-n+1})$, i.e. the absolute values of the entries of M^{-1} are bounded from above by Ch^{-n+1} with a positive constant $C = C(n)$.*

Proof. The matrix M can be regarded as a matrix consisting of k equal Vandermonde-type blocks M_b of size n (each block is Vandermonde up to division of columns by corresponding factorials). In particular, the determinant of each block is given by

$$\begin{aligned} \det M_b &= C_1 \prod_{0\le i<j\le n-1} (h_i - h_j) \\ &= C_1(h_0 - h_1)\cdots(h_0 - h_{n-1})(h_1 - h_2)\cdots(h_1 - h_{n-1}) \\ &\quad \times \cdots (h_{n-2} - h_{n-1}) = C_2 h^{n(n-1)/2} \end{aligned}$$

with C_1, C_2 being functions of n.

Similarly to M, the inverse M^{-1} consists of k inverse matrices $(M_b)^{-1}$. We can write $(M_b)^{-1} = \frac{1}{\det M_b}\,\mathrm{Adj}(M_b)$, where the entries of $\mathrm{Adj}(M_b)$ are $(n-1)\times(n-1)$ minors of M_b with some signs. Any $(n-1)\times(n-1)$ minor of M_b, regarded as the sum of products of elements taken one from each column, appears to be proportional to $h^{n(n-1)/2-s+1}$, where s is the number of the removed column of M_b. Thus, the minimal order of the $(n-1)\times(n-1)$ minors of M_b corresponds to the last removed column with $s = n$. Hence, the minimal order of an entry of M^{-1} is $(h^{\frac{n(n-1)}{2}-n+1})/h^{\frac{n(n-1)}{2}} = h^{-n+1}$. □