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Disturbance observer-based multirate control for rejecting periodic disturbances to the Nyquist frequency and beyond ☆

Weili Yan [a,b], Chee Khiang Pang [a], Chunling Du [b]

[a] Department of Electrical and Computer Engineering, National University of Singapore, Singapore

[b] Data Storage Institute, A*STAR, 2 Fusionopolis Way, #08-01 Innovis, Singapore

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ABSTRACT

Intersample behavior cannot be ignored when a sampled-data control system is subject to periodic disturbances beyond the Nyquist frequency. In this paper, a disturbance observer-based multirate control scheme is proposed to deal with such a periodic disturbance problem. First, using discrete-time Fourier series, the effect of the periodic disturbance on steady-state response of the plant output including intersample information is derived. Next, based on the steady-state output response, a sufficient condition is provided for minimization of the disturbance effect on the output. It turns out that solving the sufficient condition is a problem of quadratic optimization with several equality constraints. The proposed approach is applied on vibration control of mechanical resonant modes beyond the Nyquist frequency in a commercial hard disk drive. Perfect disturbance elimination and disturbance deduction of 72% in fast-rate discrete-time response are observed in the simulation and experiment, respectively.

1. Introduction

Various control systems are subject to periodic disturbances (Longman, Akogyeram, Juang, & Hutton, 2000), such as rotating mechanical systems (Reinig & Desrochers, 1986), vibration suppression in helicopters (Chen, Li, Teo, & Tan, 2017; Pigg & Bodson, 2010), and disk drive servo systems with repeatable runout disturbance (Amara, Kabamba, & Ulsoy, 1999). In current literature, many approaches can be found for rejecting periodic disturbances, such as the repetitive control technique (Hara, Yamamoto, Omata, & Nakano, 1988), the phase-locked loop technique-based methods (Bodson, 2001; Bodson & Douglas, 1997; Bodson, Sacks, & Khosla, 1994; Wu & Bodson, 2003), the internal model principle-based feedback control (Brown & Zhang, 2004; Kim, Kim, Chung, & Tomizuka, 2011), and the adaptive control methods (Jafari, Ioannou, Fitzpatrick, & Wang, 2015; Kim, Shim, & Jo, 2014).

Nowadays, many applications of control systems are implemented digitally (Chen & Francis, 1995; Franklin, Powell, & Workman, 1998), and a fully assembled digital control system usually has a limited output sampling rate due to hardware constrains in ADC/DAC or manufacturing costs (Fujimoto & Hori, 2002; Yamaguchi, Hirata & Pang, 2013; Yan, Du, & Pang, 2015). When this control system is subject to disturbances at frequencies beyond the Nyquist frequency of the output sampling rate, unobservable oscillations or ripples of the output occurring between samples can degrade the performance or even destabilize the closed-loop control system (Atsumi & Messner, 2012; Pang, Yan, & Du, 2016). To the best of our knowledge, there are few works for rejecting disturbances beyond the Nyquist frequency. By employing both frequency responses of the digital controller and the plant (Atsumi, Okuyama, & Nakagawa, 2008, 2010), a design method was presented to suppress disturbances beyond the Nyquist frequency for hard disk drive servo systems. Recently, an add-on multirate adaptive control scheme (Yan, Du, & Pang, 2016) is proposed for compensating of uncertain mechanical resonances beyond the Nyquist frequency in high-performance mechatronic systems. Using models of the fast-rate discrete-time plant and the disturbance, a discrete-time regulation scheme is proposed for disturbance rejection beyond the Nyquist frequency (Chen & Xiao, 2016).

In this paper, we design a disturbance observer-based multirate control scheme to reject periodic disturbances beyond the Nyquist frequency of the measured output sampling rate. The essential

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E-mail addresses: weiliyan@u.nus.edu (W. Yan), justinpang@nus.edu.sg (C.K. Pang), Du_Chunling@dsi.a-star.edu.sg (C. Du).

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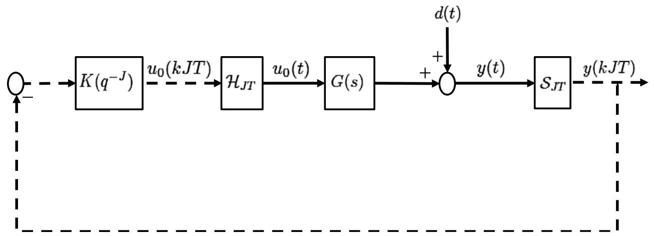

Fig. 1. Blocked diagram of single-rate sampled-data control system. Solid line: continuous-time signals. Dashed line: discrete-time signals with sampling period $JT, J \in \mathbb{Z}^+ \setminus \{1\}$.

contribution of this paper is threefold. First, the effect of the periodic disturbance on the fast-rate plant output is established by a linear difference equation where coefficients of the disturbance are periodic time-varying. Second, the discrete-time Fourier series is employed to derive steady-state response of the fast-rate plant output including intersample information. Third, minimization of steady-state output response under the disturbance is formulated as a problem of quadratic optimization with several equality constraints. The disturbance observer can be obtained after solving the quadratic optimization problem. The proposed approach is applied to suppress mechanical resonances beyond the Nyquist frequency for a commercial hard disk drive. Perfect disturbance elimination and disturbance deduction of 72% in fast-rate discrete-time response are observed in the simulation and experiment, respectively.

The remaining parts of the paper are organized as follows. Section 2 formulates the disturbance rejection problem. Our main results are presented in Section 3. Section 4 gives an application example to illustrate the effectiveness of the proposed approach. Section 5 summarizes our conclusions.

2. Problem formulation

Consider a single-rate sampled-data control system as shown in Fig. 1, where $G(s)$ is a continuous-time linear time-invariant (LTI) plant to be regulated, $\mathcal{H}_{JT}$ is a zero-order hold (ZOH) with period JT, $\mathcal{S}_{JT}$ is a sampler with period JT, and $J \in \mathbb{Z}^+ \setminus \{1\}$. It is worth noting that the output sampling period JT is fixed, and $T \to 0$ if $J \to \infty$. Based on the sampled signal $\{y(kJT), k = 0, 1, 2, \ldots\}$, the discrete-time feedback LTI controller $K(q^{-J})$ is designed to regulate $y(t)$ when the output of $G(s)$ is subject to the external disturbance $d(t)$, where q^{-1} is a backward shift operator with sampling period T, i.e.,

$$q^{-1}x(kT) = x(kT - T). \tag{1}$$

For such a single-rate sampled-data control system, the controller $K(q^{-J})$ is sufficient to regulate $y(t)$ when $d(t)$ has no significant frequency components at the frequency above $\frac{1}{2JT}$, which is the Nyquist frequency of the measured output $y(kJT)$. However, it is challenging for $K(q^{-J})$ to regulate $y(t)$ when $d(t)$ contains significant frequency components above $\frac{1}{2JT}$.

In this paper, we shall deal with disturbance rejection problem when $d(t)$ is periodic and its frequency can be up to $\frac{1}{2T}$. As such, we need to at least analyze the fictitious fast-rate output $y(kT)$. It is worth noting that a T-periodic discrete-time signal $\{x(kT), k = 0, 1, \ldots\}$ can be denoted by $\{x(kJT + mT), m = 0, 1, \ldots, J - 1\}$ involving the slow-rate signal $x(kJT)$ and intersample signal $\{x(kJT + mT), m = 1, 2, \ldots, J - 1\}$.

We propose the disturbance observer-based multirate control scheme as shown in the red dash-dotted box of Fig. 2, where the output $y(kJT + mT)$ consists of the measured data $y(kJT)$ and unmeasured intersample data $\{y(kJT + mT), m = 1, 2, \ldots, J - 1\}$. $G(q^{-1})$ is the ZOH-equivalent discrete-time model of $G(s)$ with period T and $\hat{G}(q^{-1})$ is a precise estimation for $G(q^{-1})$. This paper aims to design the finite impulse response (FIR) filter $Q(q^{-1})$ for minimizing the effect of $d(kT)$ on $y(kT)$.

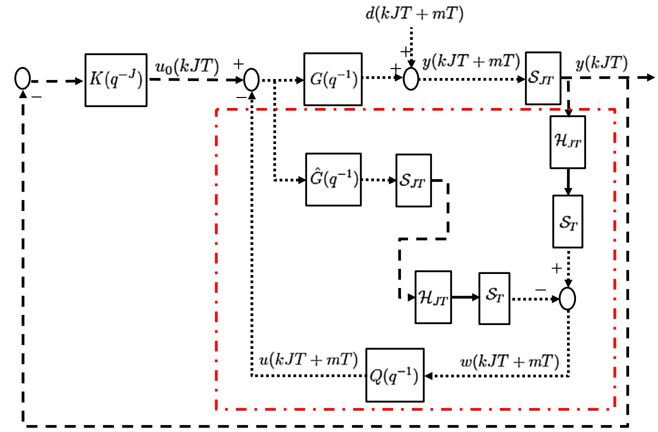

Fig. 2. Disturbance observer-based multirate control. Solid line: continuous-time signals. Dashed line: discrete-time signals with sampling period $JT, J \in \mathbb{Z}^+ \setminus \{1\}$. Dotted line: discrete-time signals with sampling period T.

Remark 1. As mentioned earlier, the fictitious fast-rate output $y(kT)$ or $y(kJT + mT)$ is used for mathematical analysis. Only the measured slow-rate output $y(kJT)$ is fed back to the controller $K(q^{-J})$, which can be seen both from Figs. 1 and 2.

Remark 2. For $G(s)$ embedded in Fig. 1, we can apply the identification methods (Yan et al., 2015) or (Pang et al., 2016) to precisely estimate $\hat{G}(q^{-1})$ by only using the slow-rate output $y(kJT)$.

Remark 3. It should be pointed out that the following results depend on the precise model for the plant. Simulation studies in Section 4 are also carried out to show the robustness of the proposed method when the plant shifts in a certain range.

As a periodic discrete-time signal can be decomposed into several sinusoids, we consider $d(kT)$ without loss of generality as a single sinusoid, i.e.,

$$d(kT) = e^{j2\pi \frac{M}{N} kT}, \quad 0 \leqslant \frac{M}{N} < \frac{1}{2}, \tag{2}$$

where $j^2 = -1, M \in \mathbb{Z}^+, N \in \mathbb{Z}^+$, and $\frac{N}{J} \in \mathbb{Z}^+$.

Without loss of generality, let $G(q^{-1})$ take the form as

$$G(q^{-1}) = \frac{B(q^{-1})}{A(q^{-1})} = \frac{\sum_{i=0}^{n_g J} b_i q^{-i}}{\sum_{i=0}^{n_g J} a_i q^{-i}}, \tag{3}$$

where $n_g \in \mathbb{Z}^+$ and $a_0 = 1$. The coefficient b_0 can possibly be zero.

Let the FIR filter $Q(q^{-1})$ take the form as

$$Q(q^{-1}) = \sum_{i=0}^{nJ} c_i q^{-i}, \quad n \in \mathbb{Z}^+. \tag{4}$$

Our main results are presented in the following section. Hereafter, $T = 1$ is used for simplicity of notation.

3. Main results

3.1. Effect of $d(k)$ on $y(k)$

In this subsection, the effect of $d(k)$ on $y(k)$ is derived from the parameters in $G(q^{-1})$ and $Q(q^{-1})$.