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Stabilization and robustness analysis for time-varying systems with time-varying delays using a sequential subpredictors approach[☆]

Frédéric Mazenc[a], Michael Malisoff[b]

[a] EPI DISCO Inria-Saclay, Laboratoire des Signaux et Systèmes, CNRS, CentraleSupélec, Université Paris-Sud, 3 rue Joliot Curie, 91192, Gif-sur-Yvette, France
[b] Department of Mathematics, Louisiana State University, Baton Rouge, LA 70803-4918, USA

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ABSTRACT

We provide a new sequential predictors approach for the exponential stabilization of linear time-varying systems. Our method circumvents the problem of constructing and estimating distributed terms in the control laws, and allows arbitrarily large input delay bounds, pointwise time-varying input delays, and uncertainties. Instead of using distributed terms, our approach to handling longer delays is to increase the number of predictors. We obtain explicit formulas to find lower bounds for the number of required predictors. The formulas involve bounds on the delays and on the derivatives of the delays. We illustrate our method in three examples.

1. Introduction

This work continues our search (which we began in Mazenc & Malisoff, in press) for predictive control methods for time-varying systems that can be applied without computing Lie derivatives and without computing distributed terms, and which can compensate for arbitrarily long input delays. Our work is motivated by the ubiquity of input delays across engineering, coupled with the challenges that one may encounter when building delay tolerant feedback controls, if one applies traditional emulation or prediction methods that can involve distributed terms. See, e.g., Bekiaris-Liberis and Krstic (2013), Michiels and Niculescu (2007), and Richard (2003) for overviews on delay compensating control, Sharma, Gregory, and Dixon (2011) for constant electromechanical input delays in muscle response in neuromuscular electrical stimulation (or NMES), and Kamalapurkar, Fischer, Obuz, and Dixon (2016) and Merad, Downey, Obuz, and Dixon (2016) for extensions to NMES under time-varying input delays.

For constant coefficient linear systems, it often suffices to use linear matrix inequalities (or LMIs) to build delay tolerant controls, but many important linear systems are time-varying. For instance, when tracking reference trajectories and linearizing around the reference trajectories, we obtain time-varying linear systems, even if the original system is time invariant. See Germani, Manes, and Pepe (2002), and Mazenc, Malisoff, and Niculescu (2014) for systems with delayed outputs, which lead to input delayed systems when the output for one system is an input for another system.

Traditional input delay compensation methods can roughly be grouped into three approaches. One approach is to solve a stabilization problem with the input delay set equal to 0, and to then look for upper bounds on the input delay that the resulting closed loop system can tolerate, without sacrificing the desired stability properties. Two advantages of this so-called emulation approach are that (a) it makes it possible to use relatively simple controls for undelayed systems (such as Lie derivative feedbacks and other approaches from Khalil, 2002) and (b) the strict Lyapunov functions that one obtains from solving the feedback design problem for the corresponding undelayed system can often be transformed into Lyapunov–Krasovskii functionals, which can in turn be used to compute bounds on the input delays that the closed loop system can tolerate. See Fridman and Niculescu (2008) for background on Lyapunov–Krasovskii functionals, and Mazenc, Malisoff, and Lin (2008) for ways to transform strict Lyapunov functions for undelayed systems into Lyapunov–Krasovskii functionals for the corresponding input delayed systems.

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E-mail addresses: frederic.mazenc@l2s.centralesupelec.fr (F. Mazenc), malisoff@lsu.edu (M. Malisoff).

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Another approach is the reduction model method, where the control is expressed implicitly as a solution of an integral equation, which can lead to challenging problems of numerically computing controls, especially when the system has uncertainty or the delay is time-varying. Prediction is another useful method, where the states in the controls are replaced by numerical predictions of the states. Standard prediction or reduction model methods can compensate for arbitrarily long input delays, and so may have an advantage over emulation for communication networks or multi-agent problems that are prone to long input delays, but the distributed terms in their controls (which use all values of the input or of the state on some interval of times) may make them harder to implement; see Artstein (1982), Bekiaris-Liberis and Krstic (2012), Mazenc, Niculescu, and Krstic (2012), and Witrant, Canudas de Wit, Georges, and Alamir (2007).

This paper provides a new sequential predictors approach to exponential stabilization of time-varying linear systems with time-varying input delays, and therefore builds on recent notable works such as Najafi, Hosseinnia, Sheikholeslam, and Karimadin (2013) (which used LMI methods for time invariant linear systems to build sequential predictors) and Léchappé (2015) (which extended Najafi et al., 2013 by studying constant coefficient linear systems with time-varying delays, which is a smaller class of systems than the time-varying systems that we consider here). It also extends Mazenc and Malisoff (in press), which was confined to constant delays. Since we do not use any distributed terms or Lie derivatives in our control, our work is also very different from the classical reduction model or the more recent prediction approaches that have been used by M. Krstic and others (as in Bresch-Pietri & Petit, 2014; Karafyllis & Krstic, 2013, and Krstic, 2009).

Our approach uses several dynamic extensions. Each dynamic extension has the same dimension as the original system. This contrasts with the reduction model approach, where the integral equation that produces the control has the same dimension as the control. Since we do not use distributed terms, our work differs from Ahmed-Ali, Karafyllis, Krstic, and Lamnabhi-Lagarrigue (2016); Ahmed-Ali, Karafyllis, and Lamnabhi-Lagarrigue (2013), and other works that use several dynamic extensions and distributed terms. We obtain closed form control formulas and ways to compute lower bounds on the number of required extensions. Our work is mainly a theoretical and methodological development. However, we illustrate our work in three examples, including a pendulum dynamics that we studied in Mazenc et al. (2014), where we used a reduction model approach and distributed controls but did not cover time-varying delays.

There are several other notable works that use prediction without using distributed terms, but which do not cover the problems that we solve in the present work. The works Ahmed-Ali, Cherrier, and Lamnabhi-Lagarrigue (2012), Cacace, Conte, and Germani (2016), Cacace, Conte, Germani, and Palombo (2016), and Zhou, Lin, and Duan (2012) focused on time-invariant linear systems $\dot{x} = Ax + Bu$, and they use eigenvalue conditions on A and bounds on the delays or controllability conditions, without guaranteeing robustness under uncertainty; strict feedback systems were covered in Cacace, Conte, Germani, and Pepe (2016) by adding conditions on the coefficient matrices of a new system that is obtained by using a diffeomorphic transformation, but we do not require such conditions here; Cacace, Germani, and Manes (2014b) proved asymptotic stability results with prescribed decay rates for linear time-invariant systems by using partial spectrum assignment; Cacace, Germani, and Manes (2014c) was also limited to time-invariant linear systems; Germani et al. (2002) used a globally drift-observability condition (which we also do not require here) to cover nonlinear systems; and Zhou (2014a,b) gave sufficient conditions for stabilizability for time-varying linear systems under pseudo-predictor feedback with an integral delay system that is also not needed in the present work. Our work is also reminiscent of Cacace, Germani, and Manes (2014a), which is devoted to chain observers for time invariant nonlinear systems with time-varying measurement delays, and so does not cover the uniform global exponential stabilization results that we present here.

We use standard notation and definitions. Throughout the sequel, the dimensions are arbitrary, unless otherwise noted. We omit arguments of functions when they are clear, and we assume that the initial times t_0 for our solutions of our systems are $t_0 = 0$, but we can write analogs for general choices of $t_0 \geq 0$. We use $|\cdot|$ to denote the usual Euclidean norm and the induced matrix norm, $|\phi|_\infty$ (resp., $|\phi|_{\mathcal{I}}$) is the essential supremum (resp., supremum over any interval $\mathcal{I}$) for any bounded measurable function ϕ, and I_n is the $n \times n$ identity matrix.

Our preliminary version (Mazenc & Malisoff, 2016) of this work only provides a sketch of the proof of its main result, while here we provide a complete and new proof, and a new example from identification theory that was not in Mazenc and Malisoff (2016). Our new proof includes a new Lyapunov–Krasovskii functional approach that can allow smaller values for the number of required sequential subpredictors than what was required in Mazenc and Malisoff (2016); see Remark 4.

2. Main result

We study systems of the form

$$\dot{x}(t) = A(t)x(t) + B(t)u(t - h(t)) + \delta(t), \tag{1}$$

where the state x and the control u are valued in $\mathbb{R}^n$ and $\mathbb{R}^\ell$, respectively, $h : \mathbb{R} \to [0, \infty)$ is a known time-varying delay, and $\delta : [0, \infty) \to \mathbb{R}^n$ is an unknown measurable essentially bounded function representing unmodeled features or actuator errors. We make two assumptions:

Assumption 1. The function h is C^1 and bounded from above by a constant $c_h > 0$, $\dot{h}$ has a finite lower bound $\underline{h} \in \mathbb{R}$, $\dot{h}$ is bounded from above by a constant $l_h \in (0, 1)$, and $\dot{h}$ has a global Lipschitz constant $n_h > 0$. □

Assumption 2. The functions A and B are bounded and continuous, and there is a known bounded continuous function $K : [0, \infty) \to \mathbb{R}^{\ell \times n}$ such that

$$\dot{x}(t) = [A(t) + B(t)K(t)]x(t) \tag{2}$$

is uniformly globally exponentially stable on $\mathbb{R}^n$ to 0. □

Assumption 1 can model many delays, e.g., by using a standard denseness argument to arbitrarily closely approximate many non-C^1 delays, including discontinuous delays; see Remark 2. In terms of an integer $m > 1$ that we specify later, we use the functions

$$\Omega_i(t) = t - \frac{i}{m}h(t) \quad \text{and} \quad \theta_j(t) = \Omega_{m-j+1}^{-1}(\Omega_{m-j}(t)) \tag{3}$$

for all $i \in \{0, \dots, m\}$ and $j \in \{1, \dots, m\}$, and define

$$R_1 = \dot{\theta}_1 \quad \text{and} \quad R_i(t) = \dot{\theta}_i(t)R_{i-1}(\theta_i(t)) \quad \text{for } i > 1. \tag{4}$$

The preceding functions are used to define the coefficients in our subpredictors and to produce the required exponential decay estimate of our transformed error vector in our theorem. Such functions exist because our upper bounds c_h and $l_h \in (0, 1)$ from Assumption 1 imply that the Ω_i's have the range $\mathbb{R}$ and are strictly increasing. Hence, the θ_i's are also strictly increasing and C^1. The inverses in (3) can be computed numerically using standard programs, e.g., the command

$$\texttt{g = InverseFunction[Function[t, t - h[t]/m]]} \tag{5}$$