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Extended information filter on matrix Lie groups ☆

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ABSTRACT

In this paper we propose a new state estimation algorithm called the extended information filter on Lie groups. The proposed filter is inspired by the extended Kalman filter on Lie groups and exhibits the advantages of the information filter with regard to multisensor update and decentralization, while keeping the accuracy of stochastic inference on Lie groups. We present the theoretical development and demonstrate its performance on multisensor rigid body attitude tracking by forming the state space on the $\mathrm{SO}(3) \times \mathbb{R}^3$ group, where the first and second component represent the orientation and angular rates, respectively. The performance of the filter is compared with respect to the accuracy of attitude tracking with parametrization based on Euler angles and with respect to execution time of the extended Kalman filter formulation on Lie groups. The results show that the filter achieves higher performance consistency and smaller error by tracking the state directly on the Lie group and that it keeps smaller computational complexity of the information form with respect to high number of measurements.

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1. Introduction

The information filter (IF) is the dual of the Kalman filter (KF) relying on the state representation by a Gaussian distribution (Maybeck, 1979), and hence is the subject of the same assumptions underlying the KF. Whereas the KF family of algorithms is represented by the first two moments involving the mean and covariance, the IF relies on the canonical parametrization consisting of an information matrix and information vector (Grocholsky, Makarenko, & Durrant-Whyte, 2003). Both the KF and IF operate cyclically in two steps: the prediction and update step. The advantages of the IF lie in the update step, especially when the number of measurements is significantly larger than the size of the state space, since this step is additive for the IF. For the KF, the opposite applies; it is the prediction step which is additive and computationally less complex. What is computationally complex in one parametrization turns out to be simple in the other (and vice-versa) (Thrun, Burgard, & Fox, 2006). Given this duality, the IF has proven its mettle in a number of applications facing large number of measurements, features or demanding a decentralized filter form. For example, if the system is linear and the state is modeled as Gaussian, then multisensor fusion can be performed with the decentralized KF proposed in Rao, Durrant-Whyte, and Sheen (1993), which enables fusion of not only the measurements, but also of the local independent KFs. Therein, the inverse covariance form is utilized, thus resulting in additive fusion equations, which can further be elegantly translated to the IF form as shown in Nettleton, Durrant-Whyte, and Sukkarieh (2003). In Zhang, Chai Soh, and Chen (2005) an IF is presented for robust decentralized estimation based on the robustness property of the H_∞ filter with respect to noise statistics, whereas in Battistelli and Chisci (2016) stability of consensus extended Kalman filter for distributed state estimation was investigated. In Onel, Ersoy, and Delic (2009) collaborative target tracking is developed for wireless sensor networks and a mutual-information-based sensor selection is adopted for participation in the IF form fusion process. In Fu, Ling, and Tian (2012) the IF form is used in multitarget tracking sensor allocation based on solving a constrained optimization problem. In Vercauteren and Wang (2005) a sigma-point IF was used for decentralized target tracking, in Campbell and Whitacre (2007) a square root form of the same filter was used for cooperative tracking with unmanned aerial vehicles, and in Liu, Wörgötter, and Markelić (2012); Wang, Feng, and Tse (2014) square-root information filtering was further explored with respect to numerical stability. The unscented IF was presented in Lee (2008) for tracking of a re-entry vehicle entering into an atmosphere from space, and in Pakki, Chandra, and Postlethwaite (2013) the square root cubature IF was proposed and

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demonstrated on the example of speed and rotor position estimation of a two phase permanent magnet synchronous motor.

Another important aspect of estimation is the state space geometry, hence many works have been dedicated to dealing with uncertainty and estimation techniques accounting for it. For example, Lie groups are natural ambient (state) spaces for description of the dynamics of rigid body mechanical systems (Murray, Li, & Sastry, 1994; Selig, 1996). Furthermore, error propagation on the $\mathrm{SE}(3)$ group with applications to manipulator kinematics was presented in Wang and Chirikjian (2006a) by developing closed-form solutions for the convolution of the concentrated Gaussian distributions on $\mathrm{SE}(3)$. Furthermore, in Wolfe, Mashner, and Chirikjian (2011) the authors propose a solution to Bayesian fusion on Lie groups by assuming conditional independence of observations on the group, thus setting the fusion result as a product of concentrated Gaussian distributions, and finding the single concentrated Gaussian distribution parameters which are closest to the starting product. Uncertainty association, propagation and fusion on $\mathrm{SE}(3)$ was investigated in Barfoot and Furgale (2014) along with sigma point method for uncertainty propagation through a nonlinear camera model. In Forster, Carlone, Dellaert, and Scaramuzza (2015) the authors preintegrated a large number of inertial measurement unit measurements for visual-inertial navigation into a single relative motion constraint by respecting the structure of the $\mathrm{SO}(3)$ group and defining the uncertainty thereof in the pertaining tangent space. A state estimation method based on an observer and a predictor cascade for invariant systems on Lie groups with delayed measurements was proposed in Khosravian, Trumpf, Mahony, and Hamel (2015). Recently, some works have also addressed the uncertainty on the $\mathrm{SE}(2)$ group proposing new distributions (Gilitschenski, Kurz, Julier, & Hanebeck, 2014; Kurz, Gilitschenski, & Hanebeck, 2014); however, these approaches do not yet provide a closed-form Bayesian recursion framework (involving both the prediction and update) that can include higher order motion and non-linear models. A least squares optimization and nonlinear KF on manifolds in the vein of the unscented KF was proposed in Hertzberg, Wagner, Frese, and Schröder (2013) along with an accompanying software library. Therein the authors demonstrate the filter on a synthetic dataset addressing the problem of trajectory estimation by posing the system state to reside on the manifold $\mathbb{R}^3 \times \mathrm{SO}(3) \times \mathbb{R}^3$, i.e., the position, orientation and velocity. In the end, the authors also demonstrate the approach on real-world simultaneous localization and mapping (SLAM) data and perform pose relation graph optimization. In the vein of the extended Kalman filter (EKF) a nonlinear continuous–discrete extended Kalman filter on Lie groups (LG-EKF) was proposed in Bourmaud, Mégret, Arnaudon, and Giremus (2015). Therein, the prediction step is presented in the continuous domain, while the update step is discrete. The authors have demonstrated the efficiency of the filter on a synthetic camera pose filtering problem by forming the system state to reside on the $\mathrm{SO}(3) \times \mathbb{R}^9$ group, i.e. the camera orientation, position, angular and radial velocities. In an earlier publication (Bourmaud, Mégret, Giremus, & Berthoumieu, 2013), the authors have presented a discrete version of the LG-EKF, which servers as the inspiration for the filter proposed in the present paper. In Ćesić, Marković, Cvišić, and Petrović (2016) we have explored modeling of the pose of tracked objects on the $\mathrm{SE}(2)$ group within the LG-EKF framework, and applied it on the problem of multitarget tracking by fusing a radar sensor and stereo vision. Given the advantages of the IF and filtering on Lie groups, a natural question arises; Can LG-EKF be cast in the information form and will the corresponding information filter on Lie groups keep the additivity and computational advantages of the update step?

A quite prominent example of an application where the need arises for computational benefits of the IF and the geometric accuracy of Lie groups is SLAM. SLAM is of great practical importance in many robotic and autonomous system applications and the earliest solutions were based on the EKF. However, EKF in practice can handle maps that contain a few hundred features, while in many applications maps are orders of magnitude larger (Thrun et al., 2004). Therefore, the extended information filter (EIF) is often employed and widely accepted for SLAM (Bailey, Upcroft, & Durrant-Whyte, 2006), and has reached its zenith with sparsification approaches resulting with sparse EIF (SEIF) (Thrun et al., 2004) and exactly sparse delayed-state filter (ESDF) (Eustice, Singh, & Leonard, 2006). However, the localization component of SLAM conforms the pose estimation problem as arising on Lie groups, i.e., describing the pose in the special euclidean group $\mathrm{SE}(3)$ (Barfoot & Furgale, 2014). Furthermore, the mapping part of SLAM consists of landmarks whose position, as well, arises on $\mathrm{SE}(3)$. Therefore, some recent SLAM solutions approached the problem by respecting the geometry of the state space (Kümmerle et al., 2011; Ros, Guerrero, Sappa, Ponsa, & Lopez, 2013), since significant cause of error in such application was determined to stem from the state space geometry approximations. However, these SLAM solutions, although able to account for the geometry of the state space, exclusively rely on graph optimization (Engel, Sch, & Cremers, 2014; Mur-Artal, Montiel, & Tardos, 2015), but not on filtering approaches. By using the herein proposed algorithm, one can extend the SLAM filtering approaches, such as SEIF or ESDF, and at the same time respect the geometry of the state space via formulation on Lie groups.

The main contribution of this paper is a new state estimation algorithm called the extended information filter on Lie groups (LG-EIF), which exhibits the advantages of the IF with regard to multisensor update and decentralization, while keeping the accuracy of the LG-EKF for stochastic inference on Lie groups. We present the theoretical development of the LG-EIF recursion equations and the applicability of the proposed approach is demonstrated on a rigid body attitude tracking problem with multiple sensors. In the experiments we define the state space to reside on the Cartesian product of the special orthogonal group $\mathrm{SO}(3)$ and $\mathbb{R}^3$, with the first component representing the attitude of the rigid body and the second component representing the pertaining angular rates. Given that, the model of the system is then set as a constant angular rate model acting on the state space $\mathrm{SO}(3) \times \mathbb{R}^3$. Note that, just like the LG-EKF, the proposed filter can be applied on any matrix Lie group or combination thereof. In the end, we compare the proposed LG-EIF to an EIF based on Euler angles, and we analyze the computational complexity of the LG-EIF multisensor update with respect to the LG-EKF. The results show that the proposed filter achieves higher performance consistency and smaller error by tracking the state directly on the Lie group and that it keeps smaller computational complexity of the information form with respect to large number of measurements.

The rest of the paper is organized as follows. In Section 2 we present the theoretical preliminaries addressing Lie groups and uncertainty definition in the form of the concentrated Gaussian distribution. In Section 3 we derive the proposed LG-EIF, while in Section 4 we present the experimental results. In the end, Section 5 concludes the paper.

2. Preliminaries

2.1. Lie groups and Lie algebras

Generally, a Lie group is a group which has also the structure of a differentiable manifold and the group operations (product and inversion) are differentiable. In this paper we restrict our attention to a special class of Lie groups, the matrix groups over the field of reals, where the group operations are matrix multiplication and inversion, with the identity matrix I^d being the identity element

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