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Brief paper

# Frequency-domain tools for stability analysis of reset control systems☆

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ABSTRACT

The potential of reset controllers to improve the transient performance of linear (motion) systems has been extensively demonstrated in the literature. The design and stability analysis of these reset controllers generally rely on the availability of parametric models and on the numerical solution of linear matrix inequalities. Both these aspects may hamper the application of reset control in industrial settings. To remove these hurdles and stimulate broader application of reset control techniques in practice, we present new sufficient conditions, based on measured frequency response data of the system to be controlled, to guarantee the stability of closed-loop reset control systems. The effectiveness of these conditions is demonstrated through experiments on an industrial piezo-actuated motion system.



## 1. Introduction

A reset controller is a linear time-invariant (LTI) control system of which the state, or a part of the state is reset to a certain value (usually zero) whenever appropriate algebraic conditions on its input and output are satisfied. Reset controllers were proposed in 1958, see Clegg (1958), in order to overcome the inherent performance limitations of linear feedback controllers imposed by Bode's gain–phase relationship. Especially in the last two decades, reset control has regained attention from the control community in both theoretically oriented research, see e.g., Aangenent, Witvoet, Heemels, van de Molengraft, and Steinbuch (2010), Baños and Barreiro (2012), Beker, Hollot, and Chait (2001); Beker, Hollot, Chait, and Han (2004), Nešić, Teel, and Zaccarian (2011); Nešić, Zaccarian, and Teel (2008), Prieur, Tarbouriech, and Zaccarian (2013) and Zhao and Wang (2016), as well as in applications (Baños & Barreiro, 2012; Heertjes, Gruntjens, van Loon, Kontaras, & Heemels, 2015; Panni, Waschl, Alberer, & Zaccarian, 2014; Zheng, Chait, Hollot, Steinbuch, & Norg, 2000). However, despite the potential of a reset controller to improve the transient performance of linear systems, reset controllers are often not so easily embraced by (motion) control engineers in industry. To a large extent, this is caused by the fact that the vast majority of existing tools for the stability analysis and the design of reset controllers rely on parametric models and on solving linear matrix inequalities using those models. As such, they do not interface well with the current industrial (motion) control design practice, in which typically frequency-domain tools and non-parametric models are exploited, see, e.g., Butler (2011). Therefore, an important open problem is to obtain easy-to-use, 'industry-friendly' design tools for reset control systems using frequency-domain techniques as a basis.

In this paper, we contribute to solving this important open problem and focus, in particular, on deriving stability conditions that are graphically verifiable on the basis of *measured* frequency response data concerning the system dynamics. These conditions apply, amongst others, to the reset condition employed in Aangenent et al. (2010), Forni, Nešić, and Zaccarian (2011), Nešić et al. (2008) and Zaccarian, Nešić, and Teel (2011), and have some connections to recent developments in variable gain control (VGC), see, e.g., Heertjes and Steinbuch (2004), Hunnekens, van de Wouw, Heertjes, and Nijmeijer (2015) and van de Wouw, Pastink, Heertjes,

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Pavlov, and Nijmeijer (2008). In VGC, the use of the circle criterion, see, e.g., Khalil (2000), is central in obtaining stability conditions based on frequency-domain system models. A key step in the approach for VGC is to write the closed-loop system as a so-called Lur'e-type system, i.e., a feedback interconnection of an LTI dynamical system and a static memoryless nonlinearity, see Khalil (2000). Unfortunately, such an approach does not transfer easily to reset controllers as the closed-loop system would be an interconnection of an LTI dynamical system and a reset controller. This is not a (true) Lur'e-type system as the reset controller (as opposed to the VGC element) consists of a *dynamical* system that exhibits discontinuities (jumps) in the state variables rather than a static memoryless element. As such, applying Lur'e-type stability arguments calls for a new perspective on reset control systems, which we will provide in this paper by abstracting away from the internal dynamics of the reset controller and focusing instead on its input/output behavior, that can be confined to a certain sector bound, see Khalil (2000). This sector bound can subsequently be employed in a circle criterion-like condition. We will formally prove that this will yield sufficient conditions to assess input-to-state stability (ISS), see Cai and Teel (2009) and Sontag and Wang (1995), of reset control systems (including the internal dynamics) by evaluating (measured) frequency response data. In addition, this new perspective on reset control can also be directly used for the design of such controllers.

The results presented in this paper are not the first stability conditions for reset control systems that are graphically verifiable on the basis of *measured* frequency response data. In Beker, Hollot, Chen, and Chait (1999), see also Beker et al. (2004) containing an overview of the work on reset control until the mid 2000s, the $H_\beta$-condition was developed involving a strictly positive real condition to guarantee closed-loop stability of a class of reset control systems. However, the result still required a parametric model for the search of both a positive definite matrix and a vector (both of size equal to the dimension of the states of the controller that are reset) defining the output of the transfer matrix that has to be strictly positive real. In this paper, we aim for frequency-domain conditions for the analysis and design of reset control systems, i.e., employing measured data instead of using parametric models, with the additional advantage that the linear part of the controller design and analysis can be performed by shaping the frequency response of the open-loop and/or closed-loop transfer functions, see Steinbuch and Norg (1998). In Carrasco, Baños, and van der Schaft (2010) and Forni et al. (2011), the concept of passivity has been used to analyze stability of reset systems. Key in the work of Carrasco et al. (2010) is that a (full) reset system retains the passivity properties of its underlying base system, i.e., the system without the reset part. As a result, $\mathcal{L}_2$-stability conditions can be verified in the frequency domain. In addition, the results in Forni et al. (2011) can be seen as a generalization of the results in Carrasco et al. (2010). The novelty in our stability results compared to Carrasco et al. (2010) and Forni et al. (2011) is the link to the circle criterion, resulting in less strict conditions on the underlying base system. The relaxation lies in the fact that the underlying linear system does not need to be strictly positive real (as in Carrasco et al., 2010; Forni et al., 2011) but should satisfy less stringent (circle-criterion) conditions. This fact significantly widens the applicability scope of the results. An important class of systems for which such relaxation is essential for the application of reset control, is the class of motion control systems as studied as a central application in this paper.

The outline of this paper is as follows. In Section 2, we present the control architecture. In Section 3, we present our main results. In Section 4, we discuss an industrial case study and demonstrate the applicability of the presented results in practice. Finally in Section 5, we provide the conclusions.

### 1.1. Nomenclature

The following notational conventions will be used. Let $\mathbb{N}$, $\mathbb{R}$, $\mathbb{R}_{\geq 0}$, $\mathbb{C}$ denote the set of non-negative integers, real numbers, nonnegative real numbers and complex numbers, respectively. The Laplace transform of a signal $x : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ is denoted by $\mathfrak{L}\{x\}$ and $s \in \mathbb{C}$ denotes the Laplace variable. Some further hybrid system notations from Goebel, Sanfelice, and Teel (2012) can be found in the Appendix.

## 2. System description and problem formulation

In this section, we will formally introduce the reset control system as considered in this paper and derive a closed-loop hybrid model. In addition, we pose a problem formulation.

### 2.1. Hybrid closed-loop model

We will mainly focus on the single-input–single-output (SISO) control architecture as depicted in Fig. 1, although our results are applicable to other configurations as well, see Remark 8. The closed-loop system in Fig. 1 consists of a linear time-invariant (LTI) plant given by the transfer function $\mathcal{P}(s)$, $s \in \mathbb{C}$, a nominal LTI controller with transfer function $\mathcal{C}(s)$, reference $r \in \mathbb{R}$, output $y_p \in \mathbb{R}$, tracking error $e := r - y_p \in \mathbb{R}$ and an external disturbance $d \in \mathbb{R}$. In this figure, $\mathcal{R}$ denotes a reset controller, which is modeled in terms of the hybrid system formalism of Goebel et al. (2012) as

$$\mathcal{R} : \begin{cases} \dot{x}_r &= A_r x_r + B_r e & \text{if } (e, -u) \in \mathcal{F} \\ x_r^+ &= 0 & \text{if } (e, -u) \in \mathcal{J} \\ u &= -C_r x_r \end{cases} \tag{1}$$

with state $x_r \in \mathbb{R}^{n_r}$, controller output $u \in \mathbb{R}$, and $A_r$, $B_r$, $C_r$ are constant real matrices of appropriate dimensions. In (1), flow of the reset controller state $x_r$ occurs when the input/output pair $(e, -u)$ is in the flow set $\mathcal{F}$ given by

$$\mathcal{F} := \left\{(e, -u) \in \mathbb{R}^2 \mid eu \leq -\tfrac{1}{\alpha}u^2\right\} \tag{2a}$$

with $\alpha \in (0, \infty)$, and state resets occur when the input/output pair $(e, -u)$ is in the jump set $\mathcal{J}$ given by

$$\mathcal{J} := \left\{(e, -u) \in \mathbb{R}^2 \mid eu \geq -\tfrac{1}{\alpha}u^2\right\}. \tag{2b}$$

A schematic representation of the flow set $\mathcal{F}$ and the jump set $\mathcal{J}$ can be found in Fig. 2(a). Later, the concept of hybrid time domains and solutions (solution pairs) of hybrid systems of the form (1), (2) will be used, which are defined for a general class of hybrid systems with inputs in the Appendix for convenience of the reader. For more details on this hybrid modeling framework we refer the reader to Cai and Teel (2009) and Goebel et al. (2012).

**Remark 1.** The general class of reset controllers in (1), (2) encompasses two of the most well-known reset controllers in the literature, i.e., the Clegg integrator (Clegg, 1958) and the First-Order-Reset-Element (FORE) (Horowitz & Rosenbaum, 1975). Indeed, these can be modeled as in (1) using

$$\text{Clegg integrator} : (A_r, B_r, C_r) = (0, \omega_i, 1), \tag{3}$$

$$\text{FORE} : (A_r, B_r, C_r) = (\beta, \omega_i, 1), \tag{4}$$

in which $n_r = 1$, $\omega_i \in \mathbb{R}_{\geq 0}$ represents the integrator gain, and $\beta \in \mathbb{R}$ denotes the single pole of the FORE, see, e.g., Zaccarian, Nešić, and Teel (2005) and the references therein.

Let us adopt the following assumption on the reset controller (1), (2).

**Assumption 2.** The pair $(A_r, C_r)$ is detectable.