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Brief paper

Coherent-classical estimation for linear quantum systems[☆]

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ABSTRACT

We study a coherent-classical estimation scheme for a class of linear quantum systems, where the estimator is a mixed quantum–classical system that may or may not involve coherent feedback. We show that when the quantum plant or the quantum part of the estimator (coherent controller) is an annihilation operator only system, coherent-classical estimation without coherent feedback can provide no improvement over purely-classical estimation. Otherwise, coherent-classical estimation without feedback can be better than classical-only estimation for certain homodyne detector angles, although the former is inferior to the latter for the best choice of homodyne detector angle. Moreover, we show that coherent-classical estimation with coherent feedback is no better than classical-only estimation, when both the plant and the coherent controller are annihilation operator only systems. Otherwise, coherent-classical estimation with coherent feedback can be superior to purely-classical estimation, and in this case, the former is better than the latter for the optimal choice of homodyne detector angle.

1. Introduction

Estimation and control problems for quantum systems are of significant interest (Gough, Gohm, & Yanagisawa, 2008; Gough, James, & Nurdin, 2010; James, Nurdin, & Petersen, 2008; Maalouf & Petersen, 2011a,b; Nurdin, James, & Petersen, 2009; Petersen, 2010; Wiseman & Milburn, 2010; Yamamoto, 2006; Yanagisawa & Kimura, 2003a,b). An important class is linear quantum systems (Gardiner & Zoller, 2000; Gough et al., 2008, 2010; James et al., 2008; Nurdin, James, & Doherty, 2009; Nurdin, James, & Petersen, 2009; Petersen, 2013b; Roy & Petersen, 2016; Wiseman & Doherty, 2005; Wiseman & Milburn, 2010; Yamamoto, 2006), that describe quantum optical devices such as optical cavities (Bachor & Ralph, 2004; Walls & Milburn, 1994), linear quantum amplifiers (Gardiner & Zoller, 2000), and finite bandwidth squeezers (Gardiner & Zoller, 2000). Coherent feedback control for linear quantum systems has been studied, where the feedback controller is also a quantum system (James et al., 2008; Lloyd, 2000; Maalouf & Petersen, 2011a,b; Nurdin, James, & Petersen, 2009; Wiseman & Milburn, 1994). A related coherent-classical estimation scheme was introduced by the authors in Petersen (2013a) and Roy et al. (2014), where the estimator has a classical part, which yields the desired final estimate, and a quantum part, which may involve coherent feedback. This is different from the quantum observer studied in Miao and James (2012). A quantum observer is a purely quantum system, that gives a quantum estimate of a variable for a quantum plant. By contrast, a coherent-classical estimator is a mixed quantum–classical system, that yields a classical estimate of a variable for a quantum plant.

In this paper, we elaborate and build on the results of the conference papers Petersen (2013a) and Roy et al. (2014) to present two key theorems, propose three relevant conjectures, and illustrate our findings with several examples. We show that a coherent-classical estimator without feedback, where either of the plant and the coherent controller is a physically realizable annihilation operator only system, it is not possible to get better estimates than the corresponding purely-classical estimator. Otherwise it is possible to get better estimates in certain cases. But we observe in examples that for the optimal choice of the homodyne angle, classical-only estimation is always superior. Moreover, we demonstrate that a coherent-classical estimator with coherent feedback can provide with higher estimation precision than classical-only estimation.

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This is possible only if either of the plant and the controller cannot be defined purely using annihilation operators. Furthermore, if there is any improvement with the coherent-classical estimator (with feedback) over purely-classical estimation, we see in examples that the latter is always inferior for the optimal choice of the homodyne angle.

The paper is structured as follows. Section 2 introduces the class of linear quantum systems considered here and discusses physical realizability for such systems. Section 3 formulates the problem of optimal purely-classical estimation. In Section 4, we formulate the optimal coherent-classical estimation problem without coherent feedback, and present our first theorem and two conjectures supported by examples. Section 5 discusses the coherent-classical estimation scheme involving coherent feedback and lays down our second theorem and another conjecture with pertinent examples. Finally, Section 6 concludes the paper with relevant summarizing remarks.

2. Linear quantum systems

The class of linear quantum systems we consider here is described by the quantum stochastic differential equations (QSDEs) (Gough et al., 2010; James et al., 2008; Petersen, 2010, 2013a; Shaiju & Petersen, 2012):

$$\begin{bmatrix} da(t) \\ da(t)^{\#} \end{bmatrix} = F \begin{bmatrix} a(t) \\ a(t)^{\#} \end{bmatrix} dt + G \begin{bmatrix} d\mathcal{A}(t) \\ d\mathcal{A}(t)^{\#} \end{bmatrix}; \\ \begin{bmatrix} d\mathcal{A}^{out}(t) \\ d\mathcal{A}^{out}(t)^{\#} \end{bmatrix} = H \begin{bmatrix} a(t) \\ a(t)^{\#} \end{bmatrix} dt + K \begin{bmatrix} d\mathcal{A}(t) \\ d\mathcal{A}(t)^{\#} \end{bmatrix}, \tag{1}$$

where

$$F = \Delta(F_1, F_2), \qquad G = \Delta(G_1, G_2), \\ H = \Delta(H_1, H_2), \qquad K = \Delta(K_1, K_2). \tag{2}$$

Here, $a(t) = [a_1(t) \dots a_n(t)]^T$ is a vector of annihilation operators. The adjoint a_i^* of the operator a_i is called a creation operator. The notation $\Delta(F_1, F_2)$ denotes the matrix $\begin{bmatrix} F_1 & F_2 \\ F_2^{\#} & F_1^{\#} \end{bmatrix}$. Also, $F_1, F_2 \in \mathbb{C}^{n\times n}$, $G_1, G_2 \in \mathbb{C}^{n\times m}$, $H_1, H_2 \in \mathbb{C}^{m\times n}$, and $K_1, K_2 \in \mathbb{C}^{m\times m}$. Moreover, $^{\#}$ denotes the adjoint of a vector of operators or the complex conjugate of a complex matrix. Furthermore, † denotes the adjoint transpose of a vector of operators or the complex conjugate transpose of a complex matrix. In addition, $\mathcal{A}(t) = [\mathcal{A}_1(t) \dots \mathcal{A}_m(t)]^T$ is a vector of external independent quantum field operators and $\mathcal{A}^{out}(t)$ is the corresponding vector of output field operators.

Theorem 2.1 (*See* Petersen, 2013b; Shaiju & Petersen, 2012). *A complex linear quantum system of the form* (1), (2) *is physically realizable, if and only if there exists a complex commutation matrix* $\Theta = \Theta^{\dagger}$ *satisfying the following commutation relation*

$$\Theta = \left[\begin{bmatrix} a \\ a^{\#} \end{bmatrix}, \begin{bmatrix} a \\ a^{\#} \end{bmatrix}^{\dagger} \right] \\ = \begin{bmatrix} a \\ a^{\#} \end{bmatrix} \begin{bmatrix} a \\ a^{\#} \end{bmatrix}^{\dagger} - \left(\begin{bmatrix} a \\ a^{\#} \end{bmatrix}^{\#} \begin{bmatrix} a \\ a^{\#} \end{bmatrix}^{T} \right)^{T}, \tag{3}$$

such that

$$F\Theta + \Theta F^{\dagger} + GJG^{\dagger} = 0, \\ G = -\Theta H^{\dagger} J, \\ K = I, \tag{4}$$

where $J = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$.

2.1. Annihilation operator only systems

Annihilation operator only linear quantum systems are a special case of the above class of linear quantum systems, where the QSDEs (1) can be described purely in terms of the vector of annihilation operators (Maalouf & Petersen, 2011a,b):

$$da(t) = F_1 a(t)dt + G_1 d\mathcal{A}(t); \\ d\mathcal{A}^{out}(t) = H_1 a(t)dt + K_1 d\mathcal{A}(t). \tag{5}$$

Theorem 2.2 (*See* Maalouf & Petersen, 2011a; Petersen, 2013b). *An annihilation operator only linear quantum system of the form* (5) *is physically realizable, if and only if there exists a complex commutation matrix* $\Theta = \Theta^{\dagger} > 0$, *satisfying*

$$\Theta = \left[a, a^{\dagger} \right], \tag{6}$$

such that

$$F_1\Theta + \Theta F_1^{\dagger} + G_1 G_1^{\dagger} = 0, \\ G_1 = -\Theta H_1^{\dagger}, \\ K_1 = I. \tag{7}$$

2.2. Linear quantum system from quantum optics

An example of a linear quantum system is a linearized dynamic optical squeezer. This is an optical cavity with a non-linear optical element inside as shown in Fig. 1. Such a dynamic squeezer can be described by the quantum stochastic differential equations (Petersen, 2013a):

$$da = -\frac{\gamma}{2} a dt - \chi a^* dt - \sqrt{\kappa_1} d\mathcal{A}_1 - \sqrt{\kappa_2} d\mathcal{A}_2; \\ d\mathcal{A}_1^{out} = \sqrt{\kappa_1} a dt + d\mathcal{A}_1; \\ d\mathcal{A}_2^{out} = \sqrt{\kappa_2} a dt + d\mathcal{A}_2, \tag{8}$$

where $\kappa_1, \kappa_2 > 0$, $\chi \in \mathbb{C}$, and a is a single annihilation operator of the cavity mode (Bachor & Ralph, 2004; Gardiner & Zoller, 2000). This leads to a linear quantum system of the form (1) as follows:

$$\begin{bmatrix} da(t) \\ da(t)^* \end{bmatrix} = \begin{bmatrix} -\frac{\gamma}{2} & -\chi \\ -\chi^* & -\frac{\gamma}{2} \end{bmatrix} \begin{bmatrix} a(t) \\ a(t)^* \end{bmatrix} dt \\ - \sqrt{\kappa_1} \begin{bmatrix} d\mathcal{A}_1(t) \\ d\mathcal{A}_1(t)^* \end{bmatrix} - \sqrt{\kappa_2} \begin{bmatrix} d\mathcal{A}_2(t) \\ d\mathcal{A}_2(t)^* \end{bmatrix}; \\ \begin{bmatrix} d\mathcal{A}_1^{out}(t) \\ d\mathcal{A}_1^{out}(t)^* \end{bmatrix} = \sqrt{\kappa_1} \begin{bmatrix} a(t) \\ a(t)^* \end{bmatrix} dt + \begin{bmatrix} d\mathcal{A}_1(t) \\ d\mathcal{A}_1(t)^* \end{bmatrix}; \\ \begin{bmatrix} d\mathcal{A}_2^{out}(t) \\ d\mathcal{A}_2^{out}(t)^* \end{bmatrix} = \sqrt{\kappa_2} \begin{bmatrix} a(t) \\ a(t)^* \end{bmatrix} dt + \begin{bmatrix} d\mathcal{A}_2(t) \\ d\mathcal{A}_2(t)^* \end{bmatrix}. \tag{9}$$

The above quantum system requires $\gamma = \kappa_1 + \kappa_2$ in order for the system to be physically realizable.

Also, the above quantum optical system can be described purely in terms of the annihilation operator, if and only if $\chi = 0$, i.e. there is no squeezing, in which case it reduces to a passive optical cavity. This leads to a linear quantum system of the form (5) as follows:

$$da = -\frac{\gamma}{2} a dt - \sqrt{\kappa_1} d\mathcal{A}_1 - \sqrt{\kappa_2} d\mathcal{A}_2; \\ d\mathcal{A}_1^{out} = \sqrt{\kappa_1} a dt + d\mathcal{A}_1; \\ d\mathcal{A}_2^{out} = \sqrt{\kappa_2} a dt + d\mathcal{A}_2, \tag{10}$$

where again the system is physically realizable when we have $\gamma = \kappa_1 + \kappa_2$.