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Brief paper

Position estimation from direction or range measurements[☆]

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ABSTRACT

This paper revisits the problems of estimating the position of an object moving in n ($\geq$ 2)-dimensional Euclidean space using velocity measurements and either direction or range measurements of one or multiple source points. The proposed solutions exploit the Continuous Riccati Equation (CRE) to calculate observer gains yielding global uniform exponential stability of zero estimation errors, also when the measured velocity is biased by an unknown constant vector or when range measurements are corrupted by an unknown constant bias. With respect to prior contributions on these subjects they provide a coherent generalization of existing solutions with the preoccupation of pointing out general and explicit persistent excitation (p.e.) conditions whose satisfaction ensures uniform exponential stability of the observers.

1. Introduction

The general problem of estimating the position, or the complete pose (position and orientation), of a body relatively to a certain spatial frame is central for a multitude of applications. The present paper focuses on the sole estimation of the body position using velocity measurements and either direction or range measurements of one or multiple source points. This corresponds to applications for which the body's attitude is either of lesser importance or is estimated by using other sensing modalities. In this case, iterative (gradient search) methods are all the more interesting that their domain of convergence can be global. Another advantage of iterative methods is that they are naturally suited to handle the non-static case, i.e. when either the body or the point source(s) move(s), by using on-line the extra data and information resulting from motion. In particular, the observation of a single source point may be sufficient in this case, provided that the body motion regularly grants a sufficient amount of "observability". In the case of direction measurements this possibility has been studied recently in Batista, Silvestre, and Oliveira (2013) by rendering the output equation linear in the state and applying Kalman filtering. A different solution not resorting on the use of a CRE is proposed in Le Bras, Hamel, Mahony, and Samson (2015). Discrete-time extensions of the solution derived in Batista et al. (2013) to the case of multiple source points are derived in Batista, Silvestre, and Oliveira (2015). The present paper borrows solutions from these latter references, regroups them in a single general framework, and complements them with the characterization of new general observability conditions whose satisfaction grants good-conditioning and global exponential stability to the proposed estimators.

Global Navigation Satellite Systems (GNSS), and the American *Global Positioning System* (GPS) (Dixon, 1991) in particular, have familiarized the larger public with the problem of body position estimation from source points distance (or range) measurements. In the static case (motionless body) three point sources (satellites) are required to algebraically calculate a finite number (equal to two) of theoretical solutions, with an extra source point (non-coplanar with the other points) needed to eliminate the non-physical solution and overcome the problem of desynchronized clocks resulting in constant range measurement bias. Studies of the non-static case are much less numerous and more recent. The observer solutions proposed in the present paper are inspired from the pioneering works of Batista, Silvestre, and Oliveira (2009, 2011, 2014) on the subject who exploit the possibility of linearizing the estimation problem via state augmentation. This possibility is also used here, but with some noticeable differences concerning in particular the augmented state definition and the formulation of persistent excitation (p.e.) conditions ensuring uniform exponential stability (not just convergence) of the observers. For instance, preventing the

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state dimension from growing with the number of source points yields simpler observers and reduced computational weight. From our perspective, the p.e. conditions here proposed are also more natural and easily interpreted, and better assess the gradual observability increase resulting from using more source points. It is here assumed that the object's velocity is available to measurement, except for a possible unknown constant bias. A way to avoid this assumption via filtering is proposed in Dandach, Fidan, Dasgupta, and Anderson (2009).

For five decades, Kalman filters for linear systems, and their extensions to non-linear systems known as Extended Kalman Filters (EKF), have consistently grown in popularity near engineers with various backgrounds (signal processing, artificial vision, robotics, etc.) to address a multitude of iterative state estimation problems involving additive "noise" upon the state and/or the measurements. The optimality of these filters in a stochastic framework under specific noise conditions and assumptions, and their direct applicability to Linear Time-Varying (LTV) systems, have undoubtedly contributed to this popularity. It is however important to keep in mind, or to recall, that the stability and robustness properties associated with them, i.e. features that supersede conditional stochastic optimality in practice, are not related to stochastic issues. They result from properties of the associated deterministic continuous-time (or discrete-time, depending on the chosen computational framework) Riccati equation that underlies a (locally) convex estimation error index (or Lyapunov function) and a way of forming recursive estimation algorithms that uniformly decrease this index exponentially (under adequate observability conditions). With this perspective, Kalman filters belong to the (slightly) larger set of Riccati observers that we intentionally derive here in a deterministic framework, knowing that a complementary stochastic interpretation may be useful to subsequently tune the Riccati equation parameters and observer gains. This tuning issue is important for practical purposes and deserves to be studied in its own right. However, it is out of the present paper's scope and is thus not pursued further here. We also believe that, by contrast with standard Kalman filter derivations performed in a stochastic framework, the deterministic approach here considered allows one to more directly comprehend how the system observability properties (uniform observability resulting from persistent excitation, in particular) are related to the good conditioning of the Riccati equation solutions and to the observer's performance (the rate of convergence to zero of the estimation errors, in particular) via a Lyapunov analysis.

The research themes addressed in the present paper are not new, nor are the basic conceptual tools (Riccati equation, Lyapunov stability, uniform observability and persistent excitation, etc.) used to derive the propose observers. However, we believe that the reported global approach to the problems, the proposed observers derived for both direction measurements and range measurements, in n ($\geq$2)-dimensional Euclidean space with an arbitrary number of source points, and the worked out p.e. conditions ensuring uniform exponential stability of these observers are original.

The paper is organized along six sections. Following the present introduction, Section 2 recalls basic observability concepts and central properties of the CRE, complemented with technical results used for stability and convergence analysis of the observers. Direction measurements and range measurements cases are treated in Sections 3 and 4 respectively. Illustrative simulations results are presented in Section 5, followed by a short Section 6 of concluding remarks. The proofs of several technical results are reported in Appendix.

2. Recalls

This section provides the reader with a short self-contained overview of basic observability concepts and of state observers whose gains are calculated from solutions to the Continuous Riccati Equation (CRE). This overview is also an opportunity to recall Lyapunov function candidates associated with these observers for stability and convergence analysis.

Throughout the paper the following notation is used:

- $A(t)$, $B(t)$, $C(t)$ are finite-dimensional matrix-valued functions depending on time. They are continuous, bounded, and r (≥ 0) times differentiable with bounded derivatives, with r specified (sometimes implicitly) in subsequent developments.
- The abbreviation p.s.d. (resp. p.d.) is used to denote positive semidefinite (resp. positive definite) square matrices that are also symmetric. Identity matrices are p.d. matrices and denoted as I_d independently of their dimensions.
- $Q(t)$ and $V(t)$ are p.s.d. finite-dimensional matrix-valued functions of time. They are also continuous and bounded. When no specific indication is provided in the text these matrix-valued functions are chosen strictly positive and greater than ϵI_d with $\epsilon > 0$.
- The infimum (resp. supremum) over time of the smallest (resp. largest) eigenvalue of a p.s.d. or p.d. matrix-valued function $P(t)$ is denoted as p_m (resp. p_M). For the matrix-valued function $V(t)$ these infimum and supremum values are accordingly denoted as v_m and v_M.

2.1. Observability definitions and conditions

Consider a generic linear time-varying (LTV) system

$$\begin{cases} \dot{x} = A(t)x + B(t)u \\ y = C(t)x \end{cases} \tag{1}$$

with $x \in \mathbb{R}^n$ the system state, $u \in \mathbb{R}^s$ the system input, and $y \in \mathbb{R}^m$ the system output. The following definitions and properties of observability associated with this system are borrowed from Aeyels, Sepulchre, and Peuteman (1998) and Chen (1984).

Definition 2.1 (*Instantaneous Observability*)**.** System (1) is *instantaneously observable* if $\forall t, x(t)$ can be calculated from the input $u(t)$, the output $y(t)$, and the time-derivatives $u^{(k)}(t), y^{(k)}(t), k \in \mathbb{N}$.

Lemma 2.2. *Define the observation space at the time-instant t as the space generated by*

$$\mathcal{O}(t) := \begin{pmatrix} N_0(t) \\ N_1(t) \\ \vdots \end{pmatrix}$$

with $N_0 = C, N_{k+1} = N_k A + \dot{N}_k, k = 1, \dots$ Then System (1) is instantaneously observable if $\operatorname{rank}(\mathcal{O}(t)) = n$.

Theorem 2.3 (*Uniform Observability*)**.** *System (1) is* uniformly observable *if there exist $\delta > 0, \mu > 0$ such that $\forall t \geq 0$:*

$$W(t, t+\delta) := \frac{1}{\delta} \int_t^{t+\delta} \Phi^\top(s,t) C^\top(s) C(s) \Phi(s,t) ds \geq \mu I_d > 0 \tag{2}$$

with $\Phi(t,s)$ the transition matrix associated with $A(t)$, i.e. such that $\frac{d}{dt}\Phi(t,s) = A(t)\Phi(t,s)$ with $\Phi(t,t) = I_d$.

The matrix valued-function $W(t, t+\delta)$ is called the *observability Gramian* of System (1). This definition of uniform observability is different from other definitions proposed in the literature, *e.g.* Besançon (2007) or Gauthier and Kupka (1994). What matters here is that the condition (2) is the one needed to establish good conditioning and exponential stability of the estimators derived in the present paper. The following lemma, taken from Scandaroli (2013), gives a sufficient condition for uniform observability in terms of the properties of the matrices $A(t)$ and $C(t)$ and their time-derivatives: