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Brief paper

Filter design based on characteristic functions for one class of multi-dimensional nonlinear non-Gaussian systems☆

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ABSTRACT

A filter based on characteristic functions is developed in this paper, to fit to a class of non-Gaussian dynamical systems, which state models and measurement models are all nonlinear and multi-dimensional. The new filter overcomes limitations and expands the application of this kind of filter, which is proved to just fit to one special kind of systems with multi-dimensional linear state models and one-dimensional nonlinear measurement models. Firstly, the filter using characteristic function is introduced and its limitation is analysed. Then, we design the new filter to fit to nonlinear states and multi-dimensional measurements. Thirdly, the matrix format of performance index is presented to match to the new filter gain, and the weighting function vector is given to ensure the uniform boundedness of such a performance index. Finally, the new filter gain can be obtained by minimizing this performance index, and the process of filtering design is accomplished. Simulation examples are given to illustrate the effectiveness of the proposed filter design scheme.

1. Introduction

In Guo and Wang (2006); Zhou, Wang, and Zhou (2008); Zhou, Zhou, Wang, Guo, and Chai (2010), authors have displayed the distribution function and PDF tracking filters design, which are based on characteristic functions and fit to the MIMO (Multiple-Input Multiple-Output) systems. Analytic study shows that this kind of filter only fits to systems with the one-dimensional measurement, and incapacitates for multi-dimensional measurement. However, the systems with multi-dimensional measurements are always encountered in practice. Motivated by those reasons, a new filter based on characteristic function is designed to overcome the limitation in this paper. It fits to the multi-dimensional measurement models as well as the multi-dimensional nonlinear state models. There are some existing filters in Gaussian and no-Gaussian systems, which can be reviewed as follows.

For Gaussian systems, among a variety of existing filtering methods, the Kalman filtering approach has been widely adopted (Anderson & Moore, 1979; Goodwin & Sin, 1984; Jazwinski, 1970). It is well known that the key of KF is to design an appropriate filter gain matrix to ensure that the covariance matrix of the estimation error is minimum (Francois et al., 2013; Kalman, 1960). KF is just suitable to the linear Gaussian systems, in the case that process noises and measurement noises are white Gaussian noise (Wen & Zhou, 2002). In order to extend the scope of application of the filter, many filters have been designed, such as Extended Kalman Filter (EKF), Unscented Kalman Filter (UKF), Cubature Kalman Filter (CKF), Strong Tracking Filter (STF), H_∞/H_2 filter (Bucy & Renne, 1971; Hou, Jing, & Yang, 2013; Li, Peng, Chen, & Liu, 2014; Li & Sun, 2013; Sunahara, 1970; Wang, Liu, & Liu, 2008). In the derivation process of EKF, the nonlinear systems are linearized, and fits to the theory of KF. However, EKF is linearized based on the first-order Taylor expansion, which just fits to weak nonlinear Gaussian systems. UKF is a method based on Unscented Transformation (UT) and the Kalman Filtering theory, which fits to stronger nonlinear Gaussian systems. By Cubature Transformation (CT), CKF has been proposed via rigorous derivation to improve the stabilization and accuracy comparing with UKF for high-dimensional systems. By introducing the fading factor, STF has been designed according to EKF to fit to nonlinear systems with uncertainty models (Hu, Wang, Gao, & Stergioulas, 2012; Li & Sun, 2013). Also polynomial

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filter has been proposed to solve fault detection problems for a class of nonlinear systems subjected to additive noises and faults (Liu, Wang, He, & Zhou, 2015). However, all above filters are all incapable in non-Gaussian systems.

For non-Gaussian systems, filters have received considerable attention for stochastic systems, which is based on different characterizations of process noises and measurement noises, such as distribution function (DF), probability density function (PDF) and characteristic function (CF). Based on the PDF of noises, Particle filter (PF) has been proposed to solve the filtering problem in the stochastic non-Gaussian system (Deng, Lu, Yue, & Zhang, 2011; Gordon, Salmond, & Smith, 1993; Hu & Jing, 2005). Furthermore, CF has been developed by Xu to overcome the highly conservative by making symmetric K–L distance as a new performance index function and has been given constraint range of the weighting function (Xu, Wen, & Feng, 2013). The filter in Zhou et al. (2008, 2010), just at nominal, is suitable to MIMO systems, which only fits to systems with one-dimensional measurement.

Driven by advanced technique and existing problems, the new filter method is proposed to fit to MIMO systems, in this paper. This appears to be a challenging task with three essential difficulties identified as follows: (1) how to obtain the recursive characteristic function of estimation errors for nonlinear states equations? (2) What kind of methods can be developed to design the performance index for multi-dimensional measurement? (3) How to obtain the desired filter gain by the performance index?

In this paper, the main contributions of this paper are outlined as follows: (1) a new form of filter is designed, and the gain matrix is given in the form of matrix; (2) the matrix format of performance index and filter gain are designed respectively, to fit to multi-dimensional measurement, and the selecting range of the weighting function vector is given to ensure the uniform boundedness of the designed performance index; (3) the gain matrix has been solved by minimizing the performance index. And the new filter is suitable for recursive computation in online applications; (4) one kind of Gaussian and two different kinds of non-Gaussian nonlinear systems are employed to illustrate the effectiveness of the filter by computer simulation.

The remainder of this paper is organized as follows. Section 2 reviews the filter design in Zhou et al. (2008, 2010) and analyses the limitation of such a filter. Section 3 designs the new filter for one class of system with multi-dimensional nonlinear state models and measurement models. Then, we present some key problems which will be encountered in the design process. In Section 4, the state model is linearized to obtain the recursive expression of the estimation error. Section 5 designs the performance index in a matrix form and obtains the gain matrix of the filter designed in Section 3. In Section 6, simulation examples are used to illustrate the effectiveness of the proposed methods. The paper is concluded and given outlook in Section 7.

The notations used throughout the paper are standard. For a matrix $\mathbf{M}$, $\mathbf{M}^T$ represents its transpose. $\mathrm{diag}\{X_1, X_2, \ldots, X_n\}$ stands for a block-diagonal matrix. $E\{x\}$ and $\mathrm{Var}\{x\}$ stand for expectation and variance of random variable x, respectively. $\varphi_{\mathbf{z}}\{\cdot\}$ stands for the (hybrid) characteristic function of the vector $\mathbf{z}$. $\mathbf{I}_n \in R^{n\times n}$ represents the identity.

2. Tracking filter design using hybrid characteristic functions

In this section, the filter method based on characteristic functions in Zhou et al. (2008, 2010) will be introduced, and the limitation, existing in the process of designing and derivation, will be analysed.

2.1. System description

Consider the following system

$$\mathbf{x}_k = \mathbf{A}_{k-1}\mathbf{x}_{k-1} + \mathbf{G}_k\mathbf{w}_{k,k-1}, \tag{1}$$

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k, \tag{2}$$

where $\mathbf{x}_k \in \Re^m$ is the state and $\mathbf{z}_k \in \Re^l$ is the output, $\mathbf{w}_{k,k-1}$ and $\mathbf{v}_k$ are process noise and measurement noise, which are random disturbances. $\mathbf{A}_{k-1}$ and $\mathbf{G}_k$ are two known time-varying system matrices.

Assumption I. The random variables $\left\{\mathbf{w}_{k,k-1}\right\}$ and $\{\mathbf{v}_k\}$ are bounded and stationary processes. $\left\{\mathbf{w}_{k,k-1}\right\}$, $\{\mathbf{v}_k\}$ and $\mathbf{x}_0$ are mutually independent. $\left\{\mathbf{w}_{k,k-1}\right\}$ has a known characteristic function denoted by $\varphi_{\mathbf{w}}(x)$ with $\left|E(\mathbf{w}_{k,k-1})\right| < +\infty$, $\mathrm{Var}(\mathbf{w}_{k,k-1}) < +\infty$, and $\{\mathbf{v}_k\}$ has a known bounded mean value $|E(\mathbf{v}_k)| < +\infty$, Without loss of generality, it is assumed that $E(\mathbf{v}_k) = 0$.

Assumption II (*Zhou et al., 2010*)**.** $\mathbf{h}(\cdot)$ is a known Borel measurable and smooth vector-value nonlinear function of its arguments.

2.2. Filters design

For the system given by Eqs. (1)–(2), a filter can be described as follows (Zhou et al., 2008, 2010).

$$\hat{\mathbf{x}}_{k|k} = \mathbf{A}_{k-1}\hat{\mathbf{x}}_{k-1|k-1} + \mathbf{U}_k\tilde{\mathbf{z}}_k, \tag{3}$$

$$\hat{\mathbf{z}}_k = \mathbf{h}(\hat{\mathbf{x}}_{k|k}), \tag{4}$$

where $\tilde{\mathbf{z}}_k = \mathbf{z}_k - \hat{\mathbf{z}}_k$, and $\mathbf{U}_k \in \Re^{n\times 1}$ is the filter gain to be determined, which is a critical step of the filter design. The filter gain can be designed as follows.

Let $\mathbf{e}_{k-1} = \mathbf{x}_{k-1} - \hat{\mathbf{x}}_{k-1|k-1}$, then from Eqs. (1) and (3), the function of the estimation error is

$$\mathbf{e}_k = \mathbf{A}_{k-1}\mathbf{e}_{k-1} + \mathbf{G}_k\mathbf{w}_{k,k-1} - \mathbf{U}_k\tilde{\mathbf{z}}_k. \tag{5}$$

In Zhou et al. (2010), there is a fact: when $\mathbf{A}_k$, $\mathbf{G}_k$, $\mathbf{z}_k$, $\hat{\mathbf{z}}_k$ and $\mathbf{U}_k$ are given, $\mathbf{e}_k$ can be obtained from two independent vectors $\mathbf{A}_{k-1}\mathbf{e}_{k-1}$ and $\mathbf{G}_k\mathbf{w}_{k,k-1}$ and the measurement error $\mathbf{U}_k\tilde{\mathbf{z}}_k$. So, the probability density function of $\mathbf{e}_k$ is the conditional probability density function of the probability density function of $\mathbf{e}_{k-1}$ and $\mathbf{w}_{k,k-1}$. For simplicity, $\gamma_{\mathbf{e}_k}$ and $\varphi_{\mathbf{e}_k}$ are used to represent the conditional joint probability density function and the conditional characteristic function of $\mathbf{e}_k$, respectively.

The performance index of this filter is

$$J = \int g(x)\log\frac{g(x)}{\gamma_{\mathbf{e}_k}}dx + \mathbf{U}_k^T\mathbf{R}_k\mathbf{U}_k, \tag{6}$$

where $g(x)$ is a pre-specified probability density function of the target vector, which can be obtained by prior information of system (Zhou et al., 2008, 2010), and $\mathbf{R}_k$ is the given positive definite weighting matrix.

In Eq. (6), the first term provides a direct metric to measure the difference between $\gamma_{\mathbf{e}_k}(x)$ and $g(x)$, and the second term reflects the constraints of the filter gain. Minimizing performance index means that the actual error distribution is made as close as possible to its desired distribution $g(x)$, whilst the magnitude of the filter gain matrix is minimized. This performance index has been used by Wang in stochastic distribution control, and the filter is called an optimal PDF tracking filter (Zhou et al., 2010). However, from Eq. (6), we can learn that, the calculation of $\gamma_{\mathbf{e}_k}(x)$ and $\log(\cdot)$ leads to high computation complexity in this performance index. In addition, it is very difficult to directly obtain a compact mathematical form that links the PDF $\gamma_{\mathbf{e}_k}(x)$ to that of the noise.