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Brief paper

Control co-design for discrete-time switched linear systems[☆]

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ABSTRACT

The paper deals with the co-design of a control policy, composed by both the state feedback and the switching control law, for discrete-time switched linear systems. Constructive conditions are given that are necessary and sufficient for the stabilizability of systems which are periodic stabilizable. The conditions are in form of a Linear Matrix Inequality (LMI) problem whose solution provides the switching law and a family of state feedback gains stabilizing the system as well as a bound on the exponential decreasing rate. The effectiveness of the proposed technique is illustrated by comparison with results from the literature.

1. Introduction

Switched systems are characterized by a dynamics that changes with time among a finite number of different modes (Liberzon, 2003). Switched systems attracted a notable research interest due, on the one hand, to their capability of modeling complex real systems, such as networked and embedded systems, and on the other hand to their dynamical properties, non-trivial to analyze and to design (Liberzon, 2003; Sun & Ge, 2011).

Stability and stabilizability are central issues of the literature on switched systems, see Sun and Ge (2011) and the survey Lin and Antsaklis (2009). Many results are available for the problem of stability of autonomous switched systems with arbitrary switching law, like the joint spectral radius analysis (Jungers, 2009), and the necessary and sufficient conditions given in Molchanov and Pyatnitskiy (1989). The latter work in particular assessed that the existence of polyhedral, hence convex, Lyapunov functions is necessary and sufficient for the stability. On the other hand convex functions are proved to be conservative for switched systems with switching law as control input, see Blanchini and Savorgnan (2008). In this context many results are based on the min-switching policy, see Liberzon (2003), that leads to nonconvex control Lyapunov functions that are minimum of quadratics. Such functions are obtained as solutions to LMI conditions in Daafouz, Riedinger, and Iung (2002), to Lyapunov–Metzler BMI conditions in Geromel and Colaneri (2006), Heemels, Kundu, and Daafouz (2016) and through an LQR iterative procedure in Sun and Ge (2011). The latter also proved that the existence of a minimum quadratic Lyapunov function is necessary and sufficient for stabilizability. Another necessary and sufficient condition, based in set-theory, appeared in Fiacchini and Jungers (2014). Some of the cited conditions and novel LMI ones are analyzed and compared in Fiacchini, Girard, and Jungers (2016). The problem of co-designing both the switching law and the control input, is even more involved than the problem of stabilizability of autonomous switched systems. This kind of problem has been addressed in several works. Some approaches consist in fixing the complexity of the Lyapunov function candidates and of the control policy in function of the number of modes, as in Daafouz et al. (2002) and Deaecto, Geromel, and Daafouz (2011); Deaecto, Souza, and Geromel (2015). Techniques based on approximating the LQR control are presented in Antunes and Heemels (2016) and Zhang, Abate, Hu, and Vitus (2009); Zhang, Hu, and Abate (2012).

This paper deals with the co-design of the switching law and the feedback control for non-autonomous switched linear systems. The results are based on the convex conditions for stabilization of autonomous systems presented in Fiacchini et al. (2016), that are necessary and sufficient for periodic stabilizable systems. The problem is treated by providing an analogous LMI condition for stabilizability that is proved to be necessary and sufficient for systems that are periodic stabilizable through co-design. The LMI

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condition is constructive and its solution provides the control policy. The main limitation of the approach lies in its complexity that depends on the number of sequences of modes, which grows combinatorially with their maximal length considered. The method is compared with the approach presented in Zhang et al. (2009, 2012) and with Lyapunov–Metzler approach.

Notation: Given $n \in \mathbb{N}$, define $\mathbb{N}_n = \{j \in \mathbb{N} : 1 \leq j \leq n\}$. The Euclidean-norm in $\mathbb{R}^n$ is $\|x\|$. The ith element of a finite set of matrices is denoted as A_i. The set of q switching modes is $\mathscr{I} = \mathbb{N}_q$, all the possible sequences of modes of length N is $\mathscr{I}^N = \prod_{j=1}^N \mathscr{I}$, and $|\sigma| = N$ if $\sigma \in \mathscr{I}^N$. Given $N \in \mathbb{N}$, $N_{\mathscr{I}} = \sum_{k=1}^N q^k$ is the number of elements in $\mathscr{I}^{[1:N]}$. Given $\sigma \in \mathscr{I}^N$, define: $\mathbb{A}_\sigma = \prod_{j=1}^N A_{\sigma_j} = A_{\sigma_N} \cdots A_{\sigma_1}$, and define $\prod_{j=M}^N A_{\sigma_j} = I$ if $M > N$. Given $a \in \mathbb{R}$, the maximal integer smaller than or equal to a is $\lfloor a \rfloor$.

2. Preliminaries and problem formulation

Consider the discrete-time switched linear system

$$x_{k+1} = A_{\sigma_k} x_k + B_{\sigma_k} u_k, \tag{1}$$

where $x_k \in \mathbb{R}^n$ and $u_k \in \mathbb{R}^m$ are the state and the control input at time $k \in \mathbb{N}$, respectively; $\sigma : \mathbb{N} \to \mathscr{I}$ is the switching law and $\{A_i\}_{i\in\mathscr{I}}$ and $\{B_i\}_{i\in\mathscr{I}}$, with $A_i \in \mathbb{R}^{n\times n}$ and $B_i \in \mathbb{R}^{n\times m}$ for all $i \in \mathscr{I}$. A time-varying control policy $\nu : \mathbb{R}^n \times \mathbb{N} \to \mathscr{I} \times \mathbb{R}^{m\times n}$, is such that $\nu(x, k) = \big(\sigma(x, k),\ K(x, k)\big) \in \mathscr{I} \times \mathbb{R}^{m\times n}$, where $K(x, k)$ is the state feedback gain, i.e. such that $u_k(x_k) = K(x_k, k)x_k$ and then the feedback law may change at every instant.

Remark 1. As proved in Zhang et al. (2009), see Theorems 5 and 7 in particular, the attention can be restricted without loss of generality to static control policies of the form

$$\nu(x) = \big(\sigma(x),\ K(x)\big) \in \mathscr{I} \times \mathbb{R}^{m\times n}, \tag{2}$$

such that $\nu(ax) = \nu(x)$ for all $x \in \mathbb{R}^n$ and $a \in \mathbb{R}$, and to piecewise quadratic Lyapunov functions. Moreover $K(x)$ belongs to a finite set i.e. $K(x) \in \mathscr{K} = \{\kappa_i\}_{i\in\mathbb{N}_M}$, with $M \in \mathbb{N}$.

The switched system in closed loop with (2) reads

$$x_{k+1} = \big(A_{\sigma(x_k)} + B_{\sigma(x_k)} K(x_k)\big)\, x_k, \tag{3}$$

where $\sigma(x_k) = \sigma_k$. We denote with $x_k^\nu(x_0) \in \mathbb{R}^n$ the state of the system (1) at time k starting from $x(0) = x_0$ by applying the control policy ν. Given $\sigma \in \mathscr{I}^D$ we denote with $x_k^\sigma(x_0)$ the state of (3) at time $k \leq D$ starting at x_0 under the switching sequence σ. The dependence of x_k^ν and x_k^σ on the initial conditions will be dropped when clear from the context.

Definition 1. The system (1) is globally exponentially stabilizable if there are a control policy $\nu(x)$ as in (2), $c \geq 0$ and $\lambda \in [0, 1)$ such that $\|x_k^\nu(x_0)\| \leq c\lambda^k \|x_0\|$, for all $x_0 \in \mathbb{R}^n$, with x_k state of (3).

Some recent results from Fiacchini et al. (2016) concerning the stabilizability of autonomous switched linear systems $x_{k+1} = A_{\sigma_k} x_k$, with $\sigma_k \in \mathscr{I}$, are recalled hereafter since widely employed in the following. A periodic switching law for the system $x_{k+1} = A_{\sigma_k} x_k$ is given by $\sigma(k) = i_{p(k)}$ and $p(k) = k - D\lfloor k/D \rfloor + 1$, with $D \in \mathbb{N}$ and $i \in \mathscr{I}^D$, which means that the sequence of modes given by i repeats cyclically in time.

Definition 2. The system $x_{k+1} = A_{\sigma_k} x_k$ is periodic σ-stabilizable if there exist a periodic switching law $\sigma : \mathbb{N} \to \mathbb{N}_q$, $c \geq 0$ and $\lambda \in [0, 1)$ such that $\|x_k^\sigma(x)\| \leq c\lambda^k \|x\|$ holds for all $x \in \mathbb{R}^n$.

For periodic σ-stabilizability a periodic, state-independent stabilizing switching law must exists, whereas it could not exist for generic σ-stabilizability. One of the main results provided in Fiacchini et al. (2016) is a necessary and sufficient condition for periodic σ-stabilizability in form of LMI.

Theorem 1. *A periodic σ-stabilizing switching law for the system (1) exists if and only if there exist $N \in \mathbb{N}$ and $\eta \in \mathbb{R}^{N_{\mathscr{I}}}$, with $\eta \geq 0$, such that $\sum_{i\in\mathscr{I}^{[1:N]}} \eta_i = 1$ and*

$$\sum_{i\in\mathscr{I}^{[1:N]}} \eta_i \mathbb{A}_i^T \mathbb{A}_i < I. \tag{4}$$

In this paper, we are not interested in determining periodic stabilizing switching laws but on computing a state-dependent control policy whenever the system admits a periodic stabilizing switching sequence.

Remark 2. The condition (4) can be used to determine if a periodic σ-stabilizing switching law exists, but such a switching law could be very poor in terms of convergence and very complex, as its length can be very high. In fact, supposing that (4) is satisfied or equivalently that there exists $\mu \in [0, 1)$ such that $\sum_{i\in\mathscr{I}^{[1:N]}} \eta_i \mathbb{A}_i^T \mathbb{A}_i \leq \mu I$, the periodic sequence length is bounded by pN with p such that $\mu^p n < 1$ (see the proof of Theorem 22 in Fiacchini et al., 2016), which can be very big for high values of μ. Moreover, the convergence can be very slow (see examples in Fiacchini et al., 2016).

Thus, if, on the one hand, periodic σ-stabilizability is more conservative than generic σ-stabilizability, on the other hand, the equivalent condition is much more computationally tractable, see Section 4. Indeed, the condition in case of periodic σ-stabilizability is an LMI in the parameter N that might be much smaller than the periodic cycle length. In this paper we focus on a condition analogous to the LMI one (4) for the controlled switched system (1). The aim is to provide an LMI problem whose solution determines a stabilizing control policy (2) for periodic stabilizable systems.

3. Switched state-dependent control policy

The following lemma is functional for the main results presented in this paper. Its proof is based on the elimination (or projection) lemma, see Boyd, El Ghaoui, Feron, and Balakrishnan (1994), analogously to what done in Pipeleers, Demeulenaere, Swevers, and Vandenberghe (2009). The elimination lemma claims that there exists $X \in \mathbb{R}^{m\times m}$ satisfying $U^T XV + V^T X^T U + Z > 0$ with $Z \in \mathbb{R}^{n\times n}$ symmetric, if and only if

$$N_u^T Z N_u > 0, \qquad N_v^T Z N_v > 0, \tag{5}$$

with $N_u, N_v \in \mathbb{R}^{n\times m}$ such that $UN_u = 0$ and $VN_v = 0$.

Lemma 1. *Given $M_i \in \mathbb{R}^{n\times n}$, with $i \in \mathbb{N}_p$, and the nonsingular matrix $P \in \mathbb{R}^{n\times n}$, the inequality*

$$\eta P^T M_1^T \ldots M_p^T M_p \ldots M_1 P < I,$$

with $\eta > 0$, holds if and only if there exist $G_i \in \mathbb{R}^{n\times n}$, with $i \in \mathbb{N}_{p-1}$, such that

$$\begin{bmatrix} \eta I & M_p G_{p-1} & \cdots & 0 & 0 & 0 \\ G_{p-1}^T M_p^T & G_{p-1} + G_{p-1}^T & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & G_2 + G_2^T & M_2 G_1 & 0 \\ 0 & 0 & \cdots & G_1^T M_2^T & G_1 + G_1^T & \eta M_1 \\ 0 & 0 & \cdots & 0 & \eta M_1^T & (PP^T)^{-1} \end{bmatrix} > 0 \tag{6}$$

is satisfied.