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Brief paper

Constraint admissible state sets for switched systems with average dwell time ☆

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ABSTRACT

The exact computational problem of λ-constrained admissible average dwell time (CAADT) contractive set and CAADT invariant set for discrete-time switched systems with and without persistent amplitude-bounded additive disturbance is investigated in this paper. The considered switching signal satisfies average dwell time (ADT) and is discussed in terms of stage which is both length-based and sequence-based for the first time. Two more general criteria are established for checking the existence of the λ-average dwell time (ADT) contractive set and the ADT invariant set of the considered system by assuming the value of stage length is taken from an arbitrary finite set. A detailed discussion is included to identify a proper value for stage length by taking the computational complexity and the conservatism of the designed finite-step computational algorithms of the λ-CAADT contractive set and the CAADT invariant set into consideration. Some characteristics of the admissible λ-CAADT contractive set and CAADT invariant set are derived. An illustrative example illustrates the effectiveness of the developed results.

1. Introduction

A switched system, consists of at least two subsystems and a rule that governs the jumping behavior among them, has attracted a great deal of attention of researchers over the past decades due to its effectiveness in modeling large numbers of complex practical systems such as process control (Mhaskar, El-Farra, & Christofides 2005), transportation systems (Farina & Rinaldi, 2000), automobile industry (Morselli, Zanasi, & Ferracin, 2006), etc. Due to the conservatism of arbitrary switching signal is highlighted, some other kinds of switching signals which satisfy dwell time (DT) (Dehghan & Ong, 2012a, 2012b), modal DT (MDT) (Dehghan & Ong, 2013; Zhang, Zhuang, & Braatz, 2016), average DT (ADT) (Zhang, Cui, Liu, & Zhao, 2011; Zhao, Yu, Yang, & Li, 2014), modal ADT (MADT) (Wang, Zhang, Wang, Dang, & Zhong, 2014), persistent DT (PDT) (Zhang, Zhuang, & Shi, 2015), and modal PDT (MPDT) (Zhang, Zhuang, Shi, & Zhu, 2015) were investigated systematically.

Recently, the set invariant theory has been to be an important tool in designing robust control algorithms such as tube-based robust model predictive control (MPC) (Mayne, Seron, & Raković, 2005) and homothetic tube optimal control (Raković, Kouvaritakis, & Cannon, 2013). As for the practical application of the invariant set theory, take the formation problem of unmanned aircraft vehicles (UAVs) as an example, the invariant set theory allows one to compute the maximum range of the trajectory of each individual UAV and this is helpful for avoiding collisions among individual UAVs. An ellipsoidal invariant set is always easy to calculate for a system which has been checked or stabilized to be stable by solving a semi-definite programming (SDP) problem (Blanchini & Miani, 2007). However, as pointed out in Blanchini and Miani (2007), the polyhedral invariant set attracts more attention recently because of the advantage that they form a closed family with respect to the operations such as the intersection operation, the union operation and the convex hull operation, etc. Although a well known procedure, which is used to estimate the polyhedral invariant set membership for a common linear system, as it is declared in Blanchini and Miani (2007), the complexity of the set increases with the dimension of the considered system and it is often impossible to be used in most real-time practical applications. In other words, the complexity of an exact solution will increase arbitrarily. Hence, some off-line computational algorithms have been proposed to derive an approximation polyhedral invariant set for such systems. To name

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a few, by employing an iterative procedure, in Raković, Kerrigan, Kouramas and Mayne (2005), a ϵ-approximation algorithm is proposed to compute the minimal robust positive invariant (mRPI) set such that the Pontryagin difference between the approximation one and the exact one is a subset of a predefined sufficiently small set. Raković et al. proposed another approximation algorithm of mRPI set in Raković et al. (2013) by selecting the shape of the final mRPI set as a priori and adjusting its size via iterating. To reduce the computation burden of the iterative algorithm and circumvent the computation of Minkowski sum, an approach was proposed in Trodden (2016) to transform the min–max problem into a linear programming problem and the new approach demonstrates a significantly reduction of the computation load. In the context of switched system, some milestone contributions are focused on exact computation of the invariant set (see Blanchini, Casagrande, & Miani, 2010; Blanchini & Miani, 2007; Kolmanovsky & Gilbert, 1998 and references therein). To mention some newest contributions, in Dehghan and Ong (2012a, 2012b), the computational algorithms of constraint admissible DT (CADT) invariant set of DT switched systems are designed by employing a length-based technique in which the switching signal is divided into stages with different lengths such that only one subsystem is active in each stage. In Zhang et al. (2016), discussed the issues of feasibility, stability and robustness for the switched MPC of switched systems in detail. The technique utilized in Dehghan and Ong (2012a, 2012b) is extended to compute the minimal RPI set for switched systems with MPDT by Zhang et al. in Zhang et al. (2015). However, on one hand, the length-based technique proposed in Dehghan and Ong (2012a) and Dehghan & Ong (2012b) cannot be used to compute the invariant set for ADT switched system since it is significantly difficult to find a set of stage length such that only one subsystem is active in each stage. On the other hand, to the best of our knowledge, no similar result has been published for ADT switched systems. This motivates our current work.

In this work, we investigate the computational problem and characteristics of contractive set and invariant set for switched systems with ADT and state constraint. Firstly, unlike the length-based technique proposed in Dehghan and Ong (2012a) and Dehghan and Ong (2012b), the switching signal is divided into stages which are both length-based and sequence-based, i.e. not only the stage length is allowed to take different values but also switchings are allowed to exist in any stage. The characteristics of the ADT switching signal are identified in each stage. By assuming that the stage lengths take values from a finite set, two criteria, which are more general than that established for DT switching signal in Dehghan and Ong (2012b), are established for checking the existence of the λ-ADT contractive of an autonomous switched system and the ADT invariant set of a disturbed switched system respectively. Thereafter, a detailed discussion is demonstrated to identify an adequate stage length in which both the conservatism and the computational complicity of the algorithms of the two sets are taken into consideration. Upon the identified stage length, computational algorithms of the λ-CAADT contractive set of the autonomous switched system and the CAADT invariant set of the disturbed switched system are designed and proved to be terminated in finite steps. Next, some characteristics of the two sets are obtained from the designed computational algorithms.

Notations. Notations employed in this paper are standard. The terms $\mathbb{R}^n$ and $\mathbb{Z}_{\geq \ell}$ denote the n dimensional Euclidean space and the integer set whose elements are no smaller than ℓ. The set $\mathbb{Z}_{[\ell_1,\ell_2]} = \{\ell : \ell \in \mathbb{Z}_{\geq 0}, \ell_1 \leq \ell \leq \ell_2\}$. The (i, j) entry of matrix P and the i_{th} entry of vector q are denoted by P_{ij} and q_i respectively. The operator $\rho(\cdot)$ computes the spectral radius of a matrix. The Minkowski sum and Pontryagin difference of two compact sets $\Theta_1 \subseteq \mathbb{R}^n,\ \Theta_2 \subseteq \mathbb{R}^n$ are $\Theta_1 \oplus \Theta_2 = \{\theta_1 + \theta_2 | \theta_1 \in \Theta_1,\ \theta_2 \in \Theta_2\}$ and $\Theta_1 \ominus \Theta_2 = \{\theta \in \mathbb{R}^n | \theta + \theta_2 \in \Theta_1,\ \theta_2 \in \Theta_2\}$. The operator $\bigcap(\cdot)$ denotes the intersection of at least two nonempty sets. The term $\mathrm{diag}(v)$ stands for a diagonal matrix whose diagonal elements are given by the vector v.

2. Preliminaries and problem formulation

Consider the following discrete-time switched system

$$x(k+1) = A_{\sigma(k)}x(k) + w(k) \tag{1}$$

$$x(k) \in \mathcal{X},\ w(k) \in \mathcal{W},\ k \in \mathbb{Z}_{\geq 0} \tag{2}$$

where $x(k)$ is the state vector within a constraint set $\mathcal{X}$. The term $w(k)$ denotes a persistent amplitude bounded additive disturbance signal whose value is taken in a bounded set $\mathcal{W}$. Both the sets $\mathcal{X}$ and $\mathcal{W}$ are polytope and contain the origin in their interior. The term $\sigma(k)$, whose value is taken in a finite set $\mathbb{Z}_{[1,M]}$ where M is the number of subsystems, denotes an ADT switching signal whose definition is reproduced here.

Definition 1 (*Hespanha, 2004*)**.** For a switching signal $\sigma(k)$ and any discrete time instants $k^{[2]} \geq k^{[1]} \geq 0$, let $N_\sigma(k^{[2]}, k^{[1]})$ be the switching times over the interval $[k^{[1]}, k^{[2]})$. If for some given positive integers N_0 and τ_a, it holds that

$$N_\sigma(k^{[2]}, k^{[1]}) \leq N_0 + \frac{k^{[2]} - k^{[1]}}{\tau_a}. \tag{3}$$

Here, N_0 and τ_a are called the chatter bound and the ADT, respectively.

For an ADT switching signal of a discrete-time system, the chatter bound N_0 means the maximum number of consecutive discrete time instants on which switchings are allowed.

When the disturbance $w(k) \equiv 0,\ \forall\, k \in \mathbb{Z}_{\geq 0}$, switched system (1)–(2) becomes autonomous system

$$x(k+1) = A_{\sigma(k)}x(k) \tag{4}$$

$$x(k) \in \mathcal{X},\ k \in \mathbb{Z}_{\geq 0}. \tag{5}$$

Generally speaking, a polytope set $\mathcal{Y}$ is formulated by a set of inequalities and can be represented by $\mathcal{Y} = \{x : Px \leq q\}$ where P is a matrix with proper dimension and q is a positive vector if and only if the polytope contains origin in its interior. Denote $\bar{q} = [\bar{q}_1,\ \bar{q}_2, \ldots,\ \bar{q}_{\mathfrak{N}}]$ where $\mathfrak{N}$ is the number of inequalities included in $\mathcal{Y}$ and $\bar{q}_i * q_i = 1$. Hence, $\mathcal{Y} = \{y : Py \leq q\} = \{y : \mathrm{diag}\{\bar{q}\}Py \leq \mathrm{diag}\{\bar{q}\}q\} = \{y : Ry \leq \vec{1}\}$ where $R = \mathrm{diag}\{\bar{q}\}P$. Therefore, $\mathcal{Y} = \{y : Ry \leq \vec{1}\}$ is considered to be a standard presentation of sets $\mathcal{X}$ and $\mathcal{W}$ in the sequel. Without loss of generality, we reproduce the following assumption.

Assumption 1. For autonomous switched system (4), we assume that an ADT τ_a has been identified such that system (4) is asymptotically stable.

Remark 2. Assumption 1 is reasonable since there are a lot of existing results that (such as Zhao, Zhang, Shi and Liu, 2012 and references therein) can be used to check or stabilize an ADT switched system and to identify a τ_a such that the considered system is stable. Moreover, once the assumption holds, the spectral radius of its system matrix is less than one automatically which is a common assumption when considering the invariant set theory.

To facilitate the following development, we imitate the corresponding definitions for the DT switched system of Dehghan and Ong (2012b) to propose the definitions of λ-ADT contractive set (for system (4)) and ADT invariant set (for system (1)) as follows: