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Identifiability of affine linear parameter-varying models[☆]

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ABSTRACT

In this paper, the identifiability of discrete-time Affine Linear Parameter-Varying (ALPV) models is studied. Examples are presented to show that, in general, the identifiability of ALPV model parameterizations does not guarantee the identifiability of the LTI parameterizations composed of frozen LTI models. A new sufficient and necessary condition is then introduced in order to guarantee the structural identifiability for ALPV parameterizations. The identifiability of this class of parameterizations is related to the lack of state–space isomorphisms between any two models corresponding to different parameter values. In addition, we present a sufficient and necessary condition for local structural identifiability, and a sufficient condition for (global) structural identifiability which are both based on the rank of a user-defined matrix. These latter conditions allow systematic verification of identifiability. Numerical examples are finally presented to illustrate our results.

1. Introduction

Linear parameter-varying (LPV) models are a special class of nonlinear models. They can be seen as an extension of Linear Time-Invariant (LTI) representations where the model parameters are functions of measurable time-varying signals, which are usually called the *scheduling signals* (Shamma, 2012; Wood, Goddard, & Glover, 1996). While control theory for LPV systems is rather complete (Balas, 2002; Lee & Poolla, 1996; Mohammadpour & Scherer, 2012; Sename, Gaspar, & Bokor, 2013), the theoretical foundation of LPV system identification still lacks basic tools which allow the identification to be carried out in a well established theory. In particular, one important concept (hardly studied in the literature dedicated to LPV model identification according to the authors' knowledge) is the notion of identifiability. This paper is devoted to characterizing identifiability of a subclass of discrete-time LPV models, known as affine LPV models (abbreviated as ALPV). An ALPV model is a discrete-time state–space LPV model whose matrices depend on the scheduling variable in an affine way. Informally, identifiability of a parametrized family of ALPV models (in the sequel referred to as an *ALPV parametrization*) means that there exist no two distinct parameter values such that the corresponding models have the same input–output behavior.

Motivation for studying identifiability. Whatever the model structure is (linear time-invariant, linear parameter-varying, nonlinear), the identifiability of a parametrized model should be studied before designing and performing any identification experiment in order to determine whether this is a well-posed problem or not. By well-posedness, we mean that there is a unique tuple of model parameters, which can be found from the input–output data using some user-defined identification method (Ljung, 1999; Söderström & Stoica, 1989). There are several reasons for preferring well-posed identification problems. This is the case, for instance, when the problem of system identification is transformed into a problem of minimizing a cost function. With such an approach, for every parameter value, the cost function thus describes the discrepancy between the model outputs and the actual ones. The desired parameter value is then chosen as the optimal point of the cost function. As illustrated, *e.g.*, in Verhaegen and Verdult (2007, Chapter 7), when using non-injective parameterizations, we can find more than one parameter vector θ which results in the same input–output behavior and, by extension, in the same

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optimal value of the minimized cost function. As shown in McKelvey, Helmersson, and Ribarits (2004), Vayssettes and Mercère (2014) and Wills and Ninness (2008), this lack of injectivity can be problematic when gradient-based optimization algorithms are used, leading to erratic behavior of these algorithms. That is, non-identifiable models should be discarded to avoid erratic behavior of identification algorithms. Another example illustrating the importance of the identifiability concept is when the sought after parameters represent physical attributes. In this case, it is compulsory to ensure that the user-defined model structure is identifiable to guarantee the convergence towards the real parameters values. Finally, evaluating the performance of identification algorithms by comparing their outcomes with the real parameters makes sense only if the parametrization is identifiable, *i.e.*, only if the solution is unique (which should be then the real parameter values). For this reason, we believe that identifiability of ALPV parameterizations is an important problem for system identification.

Since an ALPV model can be viewed as a collection of LTI models, it would be tempting to reduce the problem of identifiability of ALPV parameterizations to the identifiability of LTI parameterizations obtained for constant values of the scheduling signal. However, in general this cannot be done, as it can happen that an ALPV parametrization is identifiable, while the corresponding LTI parametrization is not identifiable. Such an example will be presented in Section 2. Hence, we need to develop identifiability analysis of ALPV models from the beginning. This paper is devoted to this task.

Related work. For LTI state–space models, identifiability was characterized in Glover and Willems (1974), Hanzon (1989) and van den Hof (1998). Identifiability for LTI systems was studied from a slightly different angle in Bazanella, Bombois, and Gevers (2012) and Dötsch and Van Den Hof (1996). In Bazanella et al. (2012), local identifiability of an LTI parametrization and informativity of a data set were studied. Roughly speaking, informativity of a data set means that the data is rich enough to yield different prediction errors for different models. Informativity of a data set and local identifiability were shown to be equivalent to the information matrix being of full rank. Note that the information matrix depends on the data set and on the parametrization. In particular, Bazanella et al. (2012) imply that if for some data set the information matrix is full rank, then the parametrization is locally identifiable. In Dötsch and Van Den Hof (1996) the latter result was shown for a specific data set which corresponds to unit pulse signal. We conjecture that the results of this paper will also allow to come up with sufficient conditions for informativity of a data set and identifiability for ALPV parameterizations in terms of information matrices. However, this remains a topic of future research. The identifiability definition of Bazanella et al. (2012) and Dötsch and Van Den Hof (1996) is the same as that of this paper, when the latter is applied to LTI systems, and it coincides with the modern usage of the term. Note that this definition of identifiability depends only on the structure of the parametrization, and not on the experimental data. Historically, the term 'identifiability' used to include conditions on the experimental data, see Bazanella et al. (2012, Section 3) for a historical overview of the subject.

For various subclasses of nonlinear state–space representations, identifiability was investigated in Němcová (2010), Peeters and Hanzon (2005), Vajda (1987), Vajda, Godfrey, and Rabitz (1989) and Walter and Lecourtier (1982). More recently, identifiability of linear switched state–space representations was studied in Petreczky, Bako, and Van Schuppen (2010).

Note that none of the cited papers deal with LPV state–space representations. For LPV models, we are aware of only two major results on identifiability: Dankers, Toth, Heuberger, Bombois, and Van Den Hof (2011) and Lee and Poolla (1997).

The paper (Dankers et al., 2011) deals with parametrization of input–output LPV-ARX (Autoregressive exogenous) models with a static dependence on the scheduling variable. It investigates the effect of identifiability and informativity of the data set for prediction error identification of LPV-ARX models. On the one hand, we do not study informativity of data sets, which is an important problem for practical purposes. On the other hand, in contrast to Dankers et al. (2011), in this paper we deal with LPV state–space representation. Even in the LTI case, analysis of identifiability of state–space representations is more challenging than identifiability analysis of input–output representations. Moreover, it is not clear if every ALPV model gives rise to an LPV-ARX model with static dependence on the scheduling variable. The connection between these model classes is not fully understood. This means that it is unclear how to use identifiability analysis of LPV-ARX models for identifiability analysis of ALPV models. That is, Dankers et al. (2011) do not solve the problem studied in this paper.

In Lee and Poolla (1997), the authors studied certain aspects related to identifiability of LPV state–space representations in LFR form. However, Lee and Poolla (1997) do not present a characterization of identifiability of LFRs. Instead, it studies LFR parameterizations which may contain isomorphic systems. It points out that such parameterizations are not identifiable, and that parametric identification algorithms need not work for such parameterizations. In order to deal with the presence of isomorphic LFRs, Lee and Poolla (1997) propose an "adapted" Gauss–Newton-style parameter estimation method for LFRs. However, the precise conditions for convergence of the proposed algorithm remain an open problem. Finally, Lee and Poolla (1997) study LFRs, and while ALPV models can be transformed to LFRs, the impact of this transformation on identifiability is not well understood. That is, Lee and Poolla (1997) do not solve the problem studied in this paper. However, Lee and Poolla (1997) clearly show that identifiability is an important problem in parametric identification of LPV models.

In this paper, we will use realization theory of ALPV models for the identifiability of ALPV parametrizations. We will follow the same idea as in Glover and Willems (1974), Hanzon (1989), van den Hof (1998), Němcová (2010), Peeters and Hanzon (2005), Petreczky et al. (2010), Vajda (1987), Vajda et al. (1989) and Walter and Lecourtier (1982). More precisely, we will use the recent results suggested in Petreczky and Mercère (2012) on realization theory, according to which any two minimal ALPV models realizing the same input–output behavior are related by state–space isomorphism. This remark will allow us to formulate necessary and sufficient conditions for identifiability, similar to the ones in Glover and Willems (1974) and van den Hof (1998).

Outline of the paper. The rest of this paper is organized as follows. We start by presenting a motivating example in Section 2. Then, in Section 3, we recall basic definitions and properties of ALPV models. In Section 4, we introduce the definitions of structural and local structural identifiability of a parametrization. Then we present necessary and sufficient conditions for both structural and local structural identifiability. Illustrative examples are presented in Section 5. Finally, Section 6 concludes the paper. The proofs of the results are presented in Appendix.

2. Motivating example

Below we present an example of an ALPV parametrization which is identifiable, but for which the corresponding LTI parametrization obtained by taking constant scheduling signals is not identifiable. Consider now the following parametrization of discrete-time ALPV models

$$\begin{aligned} x(k+1) &= A(p(k),\theta)x(k) + B(p(k))u(k), \\ y(k) &= C(p(k),\theta)x(k), \end{aligned} \tag{1}$$