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Cascade high-gain observers in output feedback control ☆

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ABSTRACT

High-gain observers proved to be a useful tool in the design of output feedback control of nonlinear systems. However, the observer faces a numerical challenge when its dimension is high. For an observer of dimension ρ and a high-gain parameter k, the observer gain is of the order of k^ρ and the observer variables could be of the order of $k^{\rho-1}$ during the transient period. This paper presents a new high-gain observer that is based on cascading lower-dimensional observers with saturation functions in between them. The observer gain in the new observer is of the order of k and its variables are limited to be of the order of k during the transient period. It is shown that the cascade observer has properties similar to the standard one. In particular, a nonlinear separation principle is proved.

1. Introduction

For over quarter of a century, high-gain observers have been used extensively in the design of output feedback control of nonlinear systems; see Khalil and Praly (2014) and the references therein. They provided the first (non-local) nonlinear separation principle for nonlinear systems (Atassi & Khalil, 1999; Teel & Praly, 1994). Not only does the observer recover stability achieved under state feedback, it also recovers its performance in the sense that the trajectories of the system under output feedback approach those under state feedback as the observer gain increases. The separation principle holds not only because the observer gain is made high, but also by designing the feedback control as a globally bounded function of the observer estimates to overcome the peaking phenomenon of the observer (Atassi & Khalil, 1999). However, the standard high-gain observer faces a numerical challenge if its dimension, ρ, is high. The design of the observer is parameterized by a small parameter ε and we refer to $k = 1/\varepsilon$ as the high-gain parameter. The observer gains are proportional to powers of k, with k^ρ as the highest power. Moreover, during the transient period the internal states of the observer could peak to large values, which are proportional to powers of k, with $k^{\rho-1}$ as the highest power. These features pose a challenge in the numerical implementation of the observer when ρ is high because in digital implementation both parameters and signals have to be represented within the finite wordlength of the digital system. It is worthwhile to note that while it is typical to saturate the state estimates or the control signal before applying the control to the plant, such saturation takes place at the output side of the observer so it does not prevent peaking in the internal variables of the observer.

Peaking of the observer's internal variables is eliminated in Maggiore and Passino (2003) by using estimate projection. This technique was developed to deal with a class of nonlinear systems that are not uniformly completely observable. While the technique is elegant, most results that dealt with uniformly observable systems continued to use saturation as a tool to overcome the effect of peaking, as it can be seen from the references of the survey paper (Khalil & Praly, 2014). Moreover, the observer of Maggiore and Passino (2003) still suffers from the drawback that its gain is proportional to k^ρ. On the other hand, the recent paper (Astolfi & Marconi, 2015) has proposed a new high-gain observer where the observer gain is limited to be of the order of k^2. However, the internal states of that observer could still peak to $O(k^{\rho-1})$ values. The dimension of this observer is $2(\rho - 1)$ compared with dimension ρ for the standard observer. The observer of Astolfi and Marconi (2015) can be viewed as a cascade connection of $\rho - 1$ second-order high-gain observers with feedback injection from one stage to the previous one. This idea motivated the cascade observer presented here, which eliminates the feedback injection, replaces second-order observers by first-order ones, and inserts saturation functions between the cascaded observers to limit peaking. The more recent papers (Astolfi, Marconi, & Teel, 2016; Teel, 2016) limited peaking in the low-power observer of Astolfi and Marconi (2015) by using nested saturations. The

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cascade observer of this paper is different from the observers in Astolfi et al. (2016) and Teel (2016). Its dimension is ρ, compared with $2(\rho-1)$, and its design is simpler.

The paper is organized as follows. Section 2 reviews some basic knowledge about the standard observer. Section 3 presents the cascade observer and proves that the estimation errors decay to $O(\varepsilon)$ values within a time period $[0, T(\varepsilon)]$, where $\lim_{\varepsilon\to 0} T(\varepsilon) = 0$. Section 4 proves a nonlinear separation theorem when the observer is used in feedback control. The results of Sections 3 and 4 are illustrated by simulation. Finally, some concluding remarks are given in Section 5.

2. Preliminaries

A high-gain observer for the system

$$\dot{w} = f_0(w, x, u) \tag{1}$$

$$\dot{x}_i = x_{i+1}, \quad \text{for } 1 \le i \le \rho - 1 \tag{2}$$

$$\dot{x}_\rho = \phi(w, x, u) \tag{3}$$

$$y = x_1 \tag{4}$$

is given by

$$\dot{\hat{x}}_i = \hat{x}_{i+1} + \frac{\alpha_i}{\varepsilon^i}(y - \hat{x}_1), \quad \text{for } 1 \le i \le \rho - 1 \tag{5}$$

$$\dot{\hat{x}}_\rho = \phi_0(\hat{x}, u) + \frac{\alpha_\rho}{\varepsilon^\rho}(y - \hat{x}_1) \tag{6}$$

where ϕ_0 is a nominal model of ϕ, ε is a sufficiently small positive constant, and α_1 to α_ρ are chosen such that the roots of

$$s^\rho + \alpha_1 s^{\rho-1} + \cdots + \alpha_{\rho-1}s + \alpha_\rho = 0 \tag{7}$$

have negative real parts. We assume that f_0, ϕ, and ϕ_0 are locally Lipschitz in their arguments and

$$|\phi(w, x, u) - \phi_0(z, u)| \le L\|x - z\| + M \tag{8}$$

for all $z \in R^\rho$ and all bounded w, x, and u. Because

$$\phi(w, x, u) - \phi_0(z, u) = \phi(w, x, u) - \phi_0(x, u) + \phi_0(x, u) - \phi_0(z, u)$$

and $\phi_0(x, u)$ can be chosen to be globally Lipschitz in x by saturating its x-argument outside a compact set, (8) requires the modeling error $\phi(w, x, u) - \phi_0(x, u)$ to be bounded. We can choose $\phi_0 = 0$, which would be a natural choice if no information is available on ϕ. In this case, (8) holds with $L = 0$. On the other hand, if ϕ is known and either it is not function of w or w is measured, we can take $\phi_0 = \phi$ with the x-argument of ϕ saturated outside a compact set of interest. In this case, (8) holds with $M = 0$. The estimation error $\tilde{x}_i = x_i - \hat{x}_i$ satisfies the inequality.[1]

$$|\tilde{x}_i| \le \max\left\{\frac{b}{\varepsilon^{i-1}}\,||\tilde{x}(0)|e^{-at/\varepsilon},\ \varepsilon^{\rho+1-i}cM\right\} \tag{9}$$

for $0 < \varepsilon \le \varepsilon^*$ for some positive constants a, b, c, and ε^*. The two terms on the right-hand side of (9) show bounds on the estimation error due to two sources. The term $(b/\varepsilon^{i-1})\ \|\tilde{x}(0)\|e^{-at/\varepsilon}$ is due to the initial estimation error $\tilde{x}(0)$ and exhibits the peaking phenomenon. It illustrates the fact that the estimation error decays rapidly to small values. In particular, given any positive constant K, it can be seen that

$$\frac{b}{\varepsilon^{\rho-1}}e^{-at/\varepsilon} \le K\varepsilon, \quad \forall\, t \ge T(\varepsilon) \overset{\text{def}}{=} \frac{\varepsilon}{a}\ln\left(\frac{b}{K\varepsilon^\rho}\right). \tag{10}$$

[1] See for example (Khalil, 2015, Section 11.4).

Using l'Hôpital's rule, it can be seen that $\lim_{\varepsilon\to 0} T(\varepsilon) = 0$. The second term $\varepsilon^{\rho+1-i}cM$ is due to the error in modeling the function ϕ. This error will not exist if the observer is implemented with $\phi = \phi_0$.

In the special case where the task is to estimate the first derivative of a signal using a linear observer, the system (1)–(4) specializes to

$$\dot{x}_1 = x_2, \qquad \dot{x}_2 = \phi(x_1, x_2, u), \qquad y = x_1 \tag{11}$$

and the second-order linear observer is given by

$$\dot{\hat{x}}_1 = \hat{x}_2 + (\alpha_1/\varepsilon)(y - \hat{x}_1), \qquad \dot{\hat{x}}_2 = (\alpha_2/\varepsilon^2)(y - \hat{x}_1) \tag{12}$$

where α_1, α_2, and ε are positive constants. The observer's transfer function from y to $\hat{x}_2$ is given by

$$\frac{\alpha_2 s}{(\varepsilon s)^2 + \alpha_1(\varepsilon s) + \alpha_2}. \tag{13}$$

The observer gain in (12) is of the order of $1/\varepsilon^2$. By scaling the observer variables as $z_1 = \hat{x}_1$ and $z_2 = \varepsilon\hat{x}_2$, the observer is implemented by the equations

$$\dot{z}_1 = (1/\varepsilon)[z_2 + \alpha_1(y - z_1)], \qquad \dot{z}_2 = (\alpha_2/\varepsilon)(y - z_1) \tag{14}$$

$$\hat{x}_1 = z_1, \qquad \hat{x}_2 = z_2/\varepsilon \tag{15}$$

where now the highest gain is of the order of $1/\varepsilon$.

The first derivative can also be estimated using a reduced-order linear observer of the form

$$\dot{z} = -(\beta/\varepsilon)(z + y), \qquad \hat{x}_2 = (\beta/\varepsilon)(z + y) \tag{16}$$

where β and ε are positive constants. The observer's transfer function from y to $\hat{x}_2$ is given by

$$\frac{\beta s}{\varepsilon s + \beta}. \tag{17}$$

In both observers, the estimation error satisfies the inequality

$$|\tilde{x}_2| \le \max\left\{\frac{b}{\varepsilon}||\tilde{x}(0)|e^{-at/\varepsilon},\ \varepsilon cM\right\}$$

for some positive constants a, b, c. However, comparison of the transfer functions (13) and (17) shows that the second-order observer has low-pass filtering characteristics that filter out high-frequency noise.

3. Cascade observer

In this section, we present a modified high-gain observer for the system (1)–(4) that overcomes the numerical challenges in implementing the standard high-gain observer when ρ is high. We assume that f_0 and ϕ are locally Lipschitz in their arguments and $w(t)$, $x(t)$, and $u(t)$ are bounded for all $t \ge 0$. What is special about the system (1)–(4) is that the states x_2 to x_ρ are derivatives of x_1. The derivative of a signal can be estimated by a first-order or second-order linear observer as we saw in the previous section. By cascading such low-order observers we can estimate higher derivatives of the signal. We choose to estimate x_2 using a second-order observer because of its low-pass filtering characteristics, but use first-order observers to estimate x_3 to x_ρ so that the dimension of the cascade connection is ρ, as in the standard observer. The cascade connection is given by

$$\dot{z}_1 = (1/\varepsilon)[z_2 + \beta_1(y - z_1)] \tag{18}$$

$$\dot{z}_2 = (\beta_2/\varepsilon)(y - z_1) \tag{19}$$

$$\hat{x}_1 = z_1, \qquad \hat{x}_2 = z_2/\varepsilon \tag{20}$$

$$\dot{z}_i = -(\beta_i/\varepsilon)(z_i + \hat{x}_{i-1}), \quad \text{for } 3 \le i \le \rho \tag{21}$$

$$\hat{x}_i = (\beta_i/\varepsilon)(z_i + \hat{x}_{i-1}), \quad \text{for } 3 \le i \le \rho \tag{22}$$