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Event-triggered control systems under packet losses[☆]

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ABSTRACT

Networked control systems (NCSs) offer many benefits in terms of increased flexibility and maintainability but might also suffer from inevitable imperfections such as packet dropouts and limited communications resources. In this paper, (static and dynamic) event-triggered control (ETC) strategies are proposed that aim at reducing the utilization of communication resources while guaranteeing desired stability and performance criteria and a strictly positive lower bound on the inter-event times despite the presence of packet losses. For the packet losses, we consider both configurations with an acknowledgement scheme (as, e.g., in the transmission control protocol (TCP)) and without an acknowledgement scheme (as, e.g., in the user diagram protocol (UDP)). The proposed design methodology will be illustrated by means of a numerical example which reveals tradeoffs between the maximum allowable number of successive packet dropouts, (minimum and average) inter-event times and $\mathcal{L}_p$-gains of the closed-loop NCS.

1. Introduction

Networked control systems (NCSs) differ from traditional control setups as they rely on shared communication media instead of dedicated point-to-point connections to transmit the sensor and actuation data. This offers many benefits as NCSs are typically easier to install and maintain. Moreover, in case the communication is wireless, the physical limitations of wired links are not present. Nonetheless, before all the benefits of NCSs can be fully exploited, many issues regarding the inherent imperfections of (packet-based) networked communication, such as, limited communication resources and packet dropouts, need to be resolved.

To deal with the fact that, in the context of NCSs, communication resources are often limited and possibly shared with other users, new control strategies need to be developed that do not only guarantee desired stability and closed-loop performance properties but also aim to reduce the utilization of the communication channel. In addition, these control strategies should also guarantee the desired closed-loop behaviour in case packet dropouts are present. Traditional (digital) control setups, in which data packages are typically sent in a *time-triggered* fashion according to a fixed sampling rate often lead to inefficient use of communication resources as the scheduling of transmission instants is purely based on time and not on the actual status of the plant. Hence, it seems more natural to use resource-aware control methodologies that determine the transmission instants on the basis of state or output information to allow a better balance between communication efficiency and control performance. Examples of resource-aware control methods include *event-triggered* control and *self-triggered* control, see Heemels, Johansson, and Tabuada (2012) for a recent overview.

In event-triggered control (ETC) strategies, transmission times are determined by means of a triggering rule that depends on, *e.g.*, state or output measurements of the system. This enables ETC strategies to reduce the number of transmissions while maintaining desired stability and performance criteria. Although many ETC strategies were proposed before, the majority of them do not consider the occurrence of packet losses despite the facts that these packet losses are often present in practical NCSs and that they deteriorate the performance and might even lead to instability of the closed-loop system. Obviously, due to the latter, the performance and stability results of existing ETC strategies in which the occurrence of packet losses are not taken into account are not valid in the presence of packet losses. In addition, in the context of ETC systems, the presence of packet losses might annul the existence of a *positive minimum inter-event time* (MIET). The latter property is essential for enabling practical implementation of the ETC strategy. Because of the above mentioned reasons, it is of interest to study ETC strategies that do take into account

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the presence of packet losses. Examples of such ETC strategies include Bommannavar and Basar (2008), Dolk and Heemels (2015), Mamduhi, Tolic, Molin, and Hirche (2014), Molin and Hirche (2013) and Molin and Hirche (2014) in which stochastic optimal control approaches are used to minimize a cost function consisting of a quadratic control cost and a communication cost. A key assumption there is that acknowledgement signals are available, *e.g.* as in transmission control protocols (TCP), such that it is known whether a transmitted package has been received or not. In Guinaldo, Lehmann, Sánchez, Dormido, and Johansson (2012), Guinaldo, Lehmann, Sánchez, Dormido, and Johansson (2014), Lehmann and Lunze (2012) and Yu, Garcia, and Antsaklis (2013), a different approach is presented which combines time-triggered and event-triggered solutions in the sense that in case a packet loss is detected, the ETC scheme is interrupted and transmissions are scheduled according to time-based specifications until the controller successfully receives the plant measurements. Clearly, this approach requires an acknowledgement scheme as well. In Wang and Lemmon (2011) it was shown that the design of a triggering rule of the form as in Tabuada (2007) can be adapted such that a maximum allowable number of successive packet drops (MANSD) can be tolerated. This setup does not require any acknowledgement scheme and is thereby compatible with, *e.g.*, the user diagram protocol (UDP). However, as shown in Borgers and Heemels (2014), this approach does not guarantee a strictly positive lower bound on the inter-event times in case disturbances are present. In Peng and Yang (2013) a periodic event-triggered control (PETC) scheme is considered in the sense that the triggering condition is only evaluated at equidistant instances in time. As such, a lower-bound on the inter-event times is enforced despite the presence of disturbances. In a similar spirit as in Wang and Lemmon (2011), it was shown that the design of such a PETC rule can be adapted to tolerate a MANSD without the need for an acknowledgement scheme.

A significant drawback of the aforementioned approaches is that they rely on the availability of full state information which may not be the case in practice. Since, especially in the presence of disturbances, it is far from trivial to modify existing *state-based* ETC schemes to *output-based* ETC schemes as shown in Abdelrahim, Postoyan, Daafouz, and Nešić (2016), Borgers and Heemels (2014) and Donkers and Heemels (2012), it is of interest to study *output-based* ETC schemes subject to packet losses. To the best of our knowledge, the output-based case in the context of packet dropouts has not been addressed in literature so far. Therefore, we propose a new design framework for *output-based* event-triggering strategies for NCSs that are subject to packet losses and disturbances. Motivated by UDP and TCP protocols, we consider both the case with acknowledgements and the case without acknowledgements. Interestingly, the design framework proposed in this paper can lead to both *dynamic* event-triggering mechanisms (ETMs), see also Dolk, Borgers, and Heemels (2014), Dolk, Borgers, and Heemels (2017), Girard (2015) and Postoyan, Tabuada, Nešić, and Anta (2015), and the more commonly studied *static* ETMs.

The remainder of this paper is organized as follows. First, we present the necessary preliminaries and notational conventions in Section 2, followed by the introduction of the event-triggered NCS setup considered in this paper and the problem statement in Section 3. In Section 4, we describe the event-triggered NCS by means of the hybrid modelling framework as presented in Goebel, Sanfelice, and Teel (2012) leading to a more mathematically rigorous problem formulation. In Sections 5 and 6 we present design conditions for the proposed *static* and *dynamic* event-triggering strategies for the case with and without acknowledgements, respectively. Finally, we demonstrate how the presented theory leads to tradeoffs between the maximum allowable number of successive packet dropouts (MANSD), (minimum and average) inter-event times and $\mathcal{L}_p$-gains by means of a numerical example in Section 7. We provide concluding remarks in Section 8.

2. Definitions and preliminaries

$\mathbb{N}$ denotes the set of all non-negative integers, $\mathbb{N}_{>0}$ the set of positive integers, $\mathbb{R}$ the field of real numbers and $\mathbb{R}_{\geqslant 0}$ the set of all non-negative reals. For N vectors $x_i \in \mathbb{R}^{n_i}$, $i \in \bar{N}$, we denote the vector obtained by stacking all vectors in one (column) vector $\bar{x} \in \mathbb{R}^n$ with $n = \sum_{i=1}^{N} n_i$ by $(x_1, x_2, \ldots, x_N)$, *i.e.*, $(x_1, x_2, \ldots, x_N) = \begin{bmatrix} x_1^\top & x_2^\top & \cdots & x_N^\top \end{bmatrix}^\top$. By $|\cdot|$ and $\langle \cdot, \cdot \rangle$ we denote the Euclidean norm and the usual inner product of real vectors, respectively. I denotes the identity matrix of appropriate dimensions. A function $\alpha : \mathbb{R}_{\geqslant 0} \to \mathbb{R}_{\geqslant 0}$ is said to be of class $\mathcal{K}$ if it is continuous, strictly increasing and $\alpha(0) = 0$. It is said to be of class $\mathcal{K}_\infty$ if it is of class $\mathcal{K}$, and in addition, it is unbounded. A function $\beta : \mathbb{R}_{\geqslant 0} \times \mathbb{R}_{\geqslant 0} \to \mathbb{R}_{\geqslant 0}$ is said to be a $\mathcal{KL}$ function if it is continuous, $\beta(\cdot, t)$ is of class $\mathcal{K}$ for each $t \geqslant 0$ and $\beta(s, \cdot)$ is nonincreasing and satisfies $\lim_{t\to\infty} \beta(s, t) = 0$. A function $f : \mathbb{R}^n \to \mathbb{R}^n$ is said to be locally Lipschitz continuous if for each $x_0 \in \mathbb{R}^n$ there exist constants $\delta > 0$ and $L > 0$ such that $|x - x_0| \leqslant \delta \Rightarrow |f(x) - f(x_0)| \leqslant L|x - x_0|$.

In this paper, we model NCSs as hybrid systems $\mathcal{H}$ of the form

$$\dot{\xi} = F(\xi, w), \text{ when } \xi \in C, \tag{1a}$$

$$\xi^+ \in G(\xi), \qquad \text{when } \xi \in D \tag{1b}$$

where F describes the flow dynamics, G the jump dynamics, C the flow set and D the jump set. We denote the hybrid system as in (1) with $\mathcal{H} = (C, D, F, G)$ or by $\mathcal{H}$ in short. We now recall some definitions given in Goebel et al. (2012) on the solutions of such hybrid system.

A *compact hybrid time domain* is a set $\mathcal{D} = \bigcup_{j=0}^{J-1} [t_j, t_{j+1}] \times \{j\} \subset \mathbb{R}_{\geqslant 0} \times \mathbb{N}$ with $J \in \mathbb{N}_{>0}$ and $0 = t_0 \leq t_1 \ldots \leq t_J$. A *hybrid time domain* is a set $\mathcal{D} \subset \mathbb{R}_{\geqslant 0} \times \mathbb{N}$ such that $\mathcal{D} \cap ([0, T] \times \{0, \ldots, J\})$ is a compact hybrid time domain for each $(T, J) \in \mathcal{D}$. A *hybrid signal* is a function defined on a hybrid time domain. In this paper, the hybrid signal $w : \operatorname{dom} w \to \mathbb{R}^n_w$ is referred to as a *hybrid input*. A hybrid signal $\xi : \operatorname{dom} \xi \to \mathbb{R}^n$ is called a *hybrid arc* if $\xi(\cdot, j)$ is locally absolutely continuous for each j.

For the hybrid system $\mathcal{H}$ given by the state space $\mathbb{R}^n$, the input space $\mathbb{R}^{n_w}$ and the data (F, G, C, D), where flow map $F : \mathbb{R}^n \times \mathbb{R}^{n_w} \to \mathbb{R}^n$ is continuous, the jump map $G : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ is a set-valued map, and the flow set C and jump set D are subsets of $\mathbb{R}^n$, a hybrid arc $\xi : \operatorname{dom} \xi \to \mathbb{R}^n$ and a hybrid input $w : \operatorname{dom} w \to \mathbb{R}^{n_w}$ is a *solution pair* (ξ, w) to $\mathcal{H}$ if

(1) $\operatorname{dom} \xi = \operatorname{dom} w$.
(2) For all $j \in \mathbb{N}$ and for almost all t such that $(t, j) \in \operatorname{dom} \xi$, we have $\xi(t, j) \in C$ and $\dot{\xi}(t, j) = F(\xi(t, j), w(t, j))$.
(3) For all $(t, j) \in \operatorname{dom} \xi$ such that $(t, j + 1) \in \operatorname{dom} \xi$, we have $\xi(t, j) \in D$ and $\xi(t, j + 1) \in G(\xi(t, j))$.

Let us remark that the hybrid systems considered in this paper have time regularization (or dwell time) and external inputs only appearing in the flow map. The latter allow us to employ the following signal norm definitions inspired by Khalil (2002). For $p \in [1, \infty)$, we introduce the $\mathcal{L}_p$-norm of a function ξ defined on a hybrid time domain $\operatorname{dom} \xi = \bigcup_{j=0}^{J-1} [t_j, t_{j+1}] \times \{j\}$ with J possibly ∞ and/or $t_J = \infty$ by

$$\|\xi\|_p = \left(\sum_{j=0}^{J-1} \int_{t_j}^{t_{j+1}} |\xi(t, j)|^p dt \right)^{1/p} \tag{2}$$

provided the right-hand side is well-defined and finite. In case $\|\xi\|_p$ is finite, we say that $\xi \in \mathcal{L}_p$.

3. NCS model and problem statement

In this section, we present the event-triggered NCS setup considered in this paper and discuss how this NCS is affected by packet losses. Based on these descriptions, we provide an initial problem formulation, which will be formalized later in Section 4.