Automatica 80 (2017) 200–209

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Sliding mode control of continuous-time Markovian jump systems with digital data transmission☆

Ming Liu[a], Lixian Zhang[a], Peng Shi[b,c], Yuxin Zhao[d]

[a] School of Astronautics, Harbin Institute of Technology, Harbin 150001, China

[b] School of Electrical and Electronic Engineering, University of Adelaide, Adelaide, S.A. 5005, Australia

[c] College of Engineering and Science, Victoria University, Melbourne, Vic. 8001, Australia

[d] College of Automation, Harbin Engineering University, Harbin 150001, China

ARTICLE INFO

Article history:
Received 11 May 2016
Received in revised form
3 October 2016
Accepted 11 January 2017

Keywords:
Sliding mode control
Markovian jump systems
Digital communication channel
Actuator degradation

ABSTRACT

This paper investigates the design problem of sliding mode control for a class of continuous-time Markovian jump systems with digital communication constraints. Two types of classical quantization schemes, that is, dynamical uniform quantizer and static logarithmic quantizer, are employed to perform the design work, respectively. In this study, a novel quantized sliding mode control design method is developed to stabilize the closed-loop systems in the presence of both state and input quantization and unknown time-varying actuator faults. Under the proposed quantized control strategies, the quantization effects can be completely compensated via injecting quantizer parameters into the controller gains. Moreover, in both quantization cases, the proposed robust quantized sliding mode controllers can guarantee the state trajectories of the closed-loop systems to be mean-square stable, and ensure the reachability of the specified sliding surface with probability 1 at the same time. Finally, a numerical example with an F-404 aircraft engine system is provided to show the effectiveness of the proposed digital control design approach.

1. Introduction

Markovian jump systems (MJSs) have received extensive attention in both theoretical research and practical applications due to its effective ability to describe the random switching phenomenon (abrupt variations) in plants structures (Huang, Long, & Li, 2015; Liu, Zhou, Liang, & Wang, 2017; Niu & Zhao, 2013; Wang, Zhang, & Yan, 2015). Over the past decade, the stability and stabilization issues have been investigated for a large class of MJSs such as singular MJSs (Wang et al., 2015), Itô stochastic MJSs (Basin, Ferreira, & Fridman, 2007; Basin, Fridman, Rodriguez-González, & Acosta, 2003), network-based MJSs (Liu, Ho, & Niu, 2009), etc. In particular, recently Li, Sui, and Tong (2016) have successfully dealt with the problem of adaptive fuzzy output feedback control for stochastic nonlinear switched systems with unmeasured states. It is well-known that the sliding mode control (SMC) is a powerful tool to eliminate so-called matched plant parameters and external disturbances with insensitivity property (Basin & Rodriguez-Ramirez, 2012; Liu, Gao, Tong, & Li, 2016; Liu & Tong, 2017; Wang, Gao, Qiu, & Ahn, 2016; Wu, Zheng, & Gao, 2013). In recent years, SMC method has been extended to MJSs and a series of results have been proposed in the existing literature (Liu, Shi, Bi, & Zhang, 2016; Zhou, Wu, & Shi, 2016). On the other hand, actuator degradation is always an inevitable phenomenon in realistic systems which may lead to performance degradation of the control systems or even instability (Yang, Jiang, & Staroswiecki, 2009). Therefore, reliable control problems, that is, control systems design against actuator degradation with maintaining an acceptable stability/performance, have been extensively investigated for different plants including linear systems (Cristofaro & Johansen, 2014), nonlinear systems (Liu, Niu, Lam, & Zhang, 2014; Rios, Kamal, Fridman, & Zolghadri, 2015), and MJSs (Wang et al., 2015), etc. In particular, SMC schemes have been applied to solve reliable control

☆ This work was supported in part by the National Natural Science Foundation of China (U1509217, 51379049, 41670688, 61322301, 61473096), Australian Research Council (DP170102644), the New Century Excellent Talents Program of the Ministry of Education of the People's Republic of China under Grant NCET-13-0170. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Michael V. Basin under the direction of Editor Ian R. Petersen.

E-mail addresses: mingliu23@hit.edu.cn (M. Liu), lixianzhang@hit.edu.cn (L. Zhang), peng.shi@adelaide.edu.au (P. Shi), zhaoyuxin@hrbeu.edu.cn (Y. Zhao).

http://dx.doi.org/10.1016/j.automatica.2017.02.002
0005-1098/

problems for MJSs, and some elementary results have been developed (Chen, Niu, & Zou, 2013).

On another research forefront, network technologies have achieved a rapid development in modern control systems and bring about the networked control systems (NCSs) (Yin, Li, Gao, & Kaynak, 2014). The main characterization of NCSs is that data transmission among components in feedback control loops is implemented via network communication links. Since the network transmission channel is of limited capacity, a few unexpected phenomena exist inevitably such as communication delay, data packet losses and signal quantization, etc. So far, a great number of results have been reported on NCSs, among which considerable attention has been paid on the quantized feedback control problem (Gao & Chen, 2008; Niu & Ho, 2014). It should be pointed out that, however, as a new challenging problem, quantized SMC design for MJSs with actuator degradation has attracted little research effort. In fact, this design problem is significant since, the corresponding results will combine the variable-structure control theory and network technologies together to realize possible practical application of SMC in modern control systems.

It is worth pointing out that, there are two difficulties required to be considered in this design issue (Hao & Yang, 2013; Zheng & Yang, 2012): (i) in this design, the switching term of the controller has to be synthesized by using quantized data, however in conventional SMC techniques it is based upon exact state (or output) measurements directly without quantization; (ii) the quantization effects will prevent the state trajectories of closed-loop systems from arriving on the pre-defined sliding surface exactly, while it may only reach certain neighborhood of the sliding surface if the quantization errors cannot be compensated. Hence, new effective SMC control methods are desirable to be developed to solve this design problem, which motivates the study of this paper.

In this paper, a new type of quantized SMC design approach is developed for MJSs with time-varying actuator faults. We employ two kinds of quantization schemes, that is, the dynamical uniform quantization and the static logarithmic quantization, respectively, to implement the design work. First, a dynamical uniform quantizer policy is developed to realize flexible adjustment for quantization sensitivity, where the online adjusting parameters for dynamical uniform quantizer are involved into the controller synthesis. Second, a static logarithmic quantization scheme is proposed and, it is shown that, the quantization effects can be completely compensated by the proposed sliding mode controller, provided the density of the logarithmic quantizer satisfies some prespecified constraints. In both quantization cases, the developed quantized SMC schemes can ensure the mean-square stability of the closed-loop systems and the finite-time reachability of the designed sliding surface. Finally, an illustrative numerical example with an F-404 aircraft engine system is given to verify the effectiveness and usefulness of the proposed control methodology.

2. Problem formulation

Considering the following continuous-time system

$$\dot{x}(t) = A(r_t)x(t) + B(r_t)\left[u^F(t) + d(t) + f_a(t)\right] + g(t, x_t, r_t) \qquad (1)$$

where $x(t) \in \mathbb{R}^n$ is the system state, $d(t) \in \mathbb{R}^m$ is the bounded disturbance, $f_a(t) \in \mathbb{R}^m$ denotes the unknown additive actuator fault, $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are system matrices, $u^F(t) \in \mathbb{R}^m$ is the fault control input described in (3), $g(t, x_t, r_t) \in \mathbb{R}^n$ is the unknown Lipschitz nonlinear vector function. $\{r_t, t \geq 0\}$ is a right-continuous Markov chain on the probability space $(\Omega, \mathcal{F}, \mathcal{P})$ taking values in a finite state space $\mathbb{S} = \{1, 2, \dots, N\}$. The mode transition probabilities $\Pi = (\pi_{ij})_{N\times N}$ of the Markov chain are given by $p_{ij} = \Pr\{r_{t+\Delta} = j | r_t = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta) & \text{if } i \neq j, \\ 1 + \pi_{ii}\Delta + o(\Delta) & \text{if } i = j, \end{cases}$ where $\Delta > 0$ and $\lim_{\Delta\to 0} o(\Delta)/\Delta = 0$, π_{ij} is the switching rate from i to j and satisfies: $\pi_{ij} > 0,\ i \neq j$, and $\pi_{ii} = -\sum_{j\neq i}\pi_{ij} < 0$ for $\forall\, i, j \in \mathbb{S}$. $A(r_t) \in \mathbb{R}^{n\times n}$ and $B(r_t) \in \mathbb{R}^{n\times m}$ are system matrices, for each $i \in \mathbb{S}$, $A(r_t) = A_i$, $B(r_t) = B_i$, where $A_i \in \mathbb{R}^{n\times n}$ and $B_i \in \mathbb{R}^{n\times m}$ are constant matrices. Suppose the external disturbance $d(t)$ and actuator fault $f_a(t)$ satisfy the following norm constraints $\|d(t)\| \leqslant d$, $\|f_a(t)\| \leqslant r$, where $d > 0$ and $r > 0$ are known constants. For the unknown Lipschitz nonlinear function $g(t, x_t, r_t)$, it is assumed that, for arbitrary state vectors $x(t)$ and $\hat{x}(t) \in \mathbb{R}^n$, the following property holds

$$\|g(t, x_t, i) - g(t, \hat{x}_t, i)\| \leqslant \ell_i\|x - \hat{x}\| \leqslant \ell\|x - \hat{x}\| \qquad (2)$$

where $\ell_i > 0$, $i \in \mathbb{S}$ and $\ell = \max_{i\in\mathbb{S}} \ell_i$ is a known constant parameter. For simplicity, we denote $g(t, x_t, i)$ as $g(t)$ in the following discussion.

Assume that the fault control input $u^F(t) \triangleq [u_1^F(t), \dots, u_m^F(t)]^T$ has the following form

$$u^F(t) = \rho^f(t)u(t), \qquad (3)$$

where $\rho^f(t) \triangleq [\rho_1^f(t), \dots, \rho_m^f(t)]^T$, $i = 1, 2, \dots, m$ denotes the *unknown* actuator efficiency factor. From a practical viewpoint, it is reasonable to suppose that $0 < \underline{\rho}_i^f \leqslant \rho_i^f(t) \leqslant \overline{\rho}_i^f \leqslant 1$ with $\underline{\rho}_i^f$ and $\overline{\rho}_i^f$ being known constants. We define the following parameters and matrices (Liu et al., 2014)

$$\sigma_i = \frac{\underline{\rho}_i^f - \overline{\rho}_i^f}{\underline{\rho}_i^f + \overline{\rho}_i^f}, \qquad e_i = \frac{1}{2}(\underline{\rho}_i^f + \overline{\rho}_i^f), \qquad \psi_i(t) = \frac{\rho_i^f(t) - e_i}{e_i},$$
$$\Gamma = \mathrm{diag}(\sigma_1,\ \sigma_2, \dots,\ \sigma_m) \leqslant 0,$$
$$E = \mathrm{diag}(e_1,\ e_2, \dots,\ e_m) > 0,$$
$$\Psi(t) = \mathrm{diag}(\psi_1(t),\ \psi_2(t), \dots,\ \psi_m(t)).$$

It can be verified that $\rho^f = E(I_m + \Psi(t))$, and thus $u^F(t)$ can be written as $u^F(t) = E(I_m + \Psi(t))u(t)$. In addition, we define

$$\alpha = \max_{\rho_i(t)\in[\underline{\rho}_i,\ \overline{\rho}_i]} \|(I_m + \Psi(t))\|, \qquad \beta = \|E^{-1}\|,$$
$$\eta = \|\Gamma\| < 1,$$

it can be checked that $\|(I_m + \Psi(t))\| \leqslant \beta$ and $\|\Psi(t)\| \leqslant \eta < 1$ hold.

Throughout this paper, the following definition is adopted.

Definition 1 (*Xiong, Lam, Gao, & Ho, 2005*)**.** The MJS (1) (with $u(t) \equiv 0$) is said to be mean-square stable if $\lim_{t\to\infty} \mathbb{E}\left\{\|x(t, x_0, r_0)\|^2\right\} = 0$ for any initial condition $x_0 \in \mathbb{R}^n$ and $r_0 \in \mathbb{S}$.

In practical engineering, the state variables (output measurements) are always required to be quantized and then transmitted to the controller side for synthesis. It is well known that there are two types of quantization strategies, namely static logarithmic quantization and dynamical uniform quantization (Elia & Mitter, 2001). In this paper, we will employ both of these two quantization strategies to perform the design work. The objective of this paper is thus to design the control input $v(t)$ with SMC approach by using quantized signals to stabilize the closed-loop systems.

3. Sliding surface design and sliding motion analysis

In this section, we will consider the problems of sliding surface design and sliding motion equation analysis for MJSs (1) with digital communication channels. Considering the system (1), we design the following integral-type sliding surface

$$s(t) = Gx(t) - \int_0^t G(A_i + B_iK_i)x(\tau)d\tau \qquad (4)$$