Automatica 80 (2017) 284–294

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Multi-robots Gaussian estimation and coverage control: From client–server to peer-to-peer architectures ☆

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ARTICLE INFO

Article history:
Received 30 October 2015
Received in revised form
10 February 2017
Accepted 13 February 2017

Keywords:
Gaussian estimation
Coverage control
Robotic networks
Centralized communications
Distributed communications

ABSTRACT

In this work we study the problem of multi-robot coverage of a planar region when the sensory field used to approximate the density of event appearance is not known in advance. We address the problem by considering two different communication architectures: *client–server* and *peer-to-peer*. In the first architecture the robots are allowed to communicate with a central server/base station. In the second the robots communicate among neighboring peers by means of a *gossip* protocol in a distributed fashion. For both the architectures, we resort to nonparametric Gaussian regression approach to estimate the unknown sensory field of interest from a collection of noisy samples. We propose a probabilistic control strategy based on the posterior of the estimation error variance, which lets the robots to estimate the true sensory field with any arbitrary accuracy while simultaneously computing and exploiting the corresponding centroidal Voronoi partitions. We also present a numerically efficient approximation based on a spatial discretization to trade-off the accuracy of the estimated map against the required computational complexity. This trade-off can be tuned based on explicit estimation error bounds which depend on the spatial resolution and the Gaussian kernel parameters. Finally, we test the proposed solutions via extensive numerical simulations.

1. Introduction

The growing sensing capabilities and the development of autonomous robot vehicles able to coordinate themselves to achieve desired tasks are expected to revolutionize our capability to control the physical environment (Leonard et al., 2007). In this context, the coverage of an area of interest is one important and interesting task. In many applications the ability of a group of robots to sense and automatically cover the surrounding environment to maximize the likelihood of detecting an event of interest is appealing. On the other hand, knowledge about the spatial distribution of the event of interest is needed. As an example, consider a group of robots monitoring a forest to detect possible wildfires. Since the probability of a wildfire is likely to be proportional to the temperature, the robots should more densely cover areas with higher temperature which, if not known in advance, must be reconstructed from collected samples. At the same time, to minimize the time to reach a wildfire, the robots should station near the centroids of the partitioned area. This highlights the issue of *simultaneous estimation and coverage* associated with the problem of interest.

In this work we analyze the problem of *covering* the area of interest while *estimating* the non-uniform measurable field of event appearance from *noisy measurements* collected by the robots. There has been considerable effort in the analysis of estimation and coverage separately. Historically, classical identification techniques are based on parametric estimation paradigms, like ML and PEM (Ljung, 1999). However, these techniques often require persistent excitation to ensure convergence of the parameter (Choi, Oh, & Horowitz, 2009) and may be unsatisfactory when tested on experimental data (Pillonetto, Chiuso, & De Nicolao, 2011). To overcome these issues techniques, grouped under the *nonparametric learning* framework, have been recently developed. The main idea is to ex-

☆ This work is supported by the University of Padova, Progetto di Ateneo # CPDA147754/14 "New statistical learning approach for multi-agents adaptive estimation and coverage control". The material in this paper was partially presented at the 14th annual European Control Conference, July 15–17, 2015, Linz, Austria (Carron, Todescato, Carli, Schenato, & Pillonetto, 2015). This paper was recommended for publication in revised form by Associate Editor Michael M. Zavlanos under the direction of Editor Christos G. Cassandras.

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http://dx.doi.org/10.1016/j.automatica.2017.02.045

ploit black box models to estimate a function from examples collected on input locations drawn from a fixed probability sensory function (Pillonetto et al., 2011; Poggio & Girosi, 1990). The drawback is that the computational complexity grows unbounded as the cube of the number of collected samples. Thus, efficient approaches (Xu, Choi, Dass, & Maiti, 2015) based on, e.g., suitable measurements truncation (Xu, Choi, Dass, & Maiti, 2012; Xu, Choi, & Oh, 2011) or Gaussian Markov random fields (Xu, Choi, Dass, & Maiti, 2013), have been proposed.

Classical approaches to the coverage problem assume the sensory function to be perfectly known in advance. In this spirit, works (Cortés & Bullo, 2005; Cortes, Martinez, Karatas, & Bullo, 2004; Durham, Carli, Frasca, & Bullo, 2012) exploit the concept of Centroidal Voronoi partitioning and present solutions based on gradient descent strategies. In Leonard and Olshevsky (2011) a policy for the coverage of a 1-D environment is presented. In Davison, Schwemmer, and Leonard (2012) a limited number of noise-free samples are considered yet no convergence results are presented. The work (Davison, Leonard, Olshevsky, & Schwemmer, 2015) extends (Davison et al., 2012; Leonard & Olshevsky, 2011) by proving convergence in probability to the optimal configuration. A distributed solution in the presence of known time-varying functions is given in Lee, Diaz-Mercado, and Egerstedt (2015). A different line of research deals with adaptive/optimal sampling strategies to enhance estimation accuracy (Xu & Choi, 2011). In particular, in Xu et al. (2011) is proposed a distributed efficient solution, where each robot independently estimates the function of interest based on a truncated subset of its own measurements and those gathered by its neighbors. In Xu et al. (2013) instead, each robot is in charge of monitoring a fixed area of interest, thus not requiring any exchange of measurements between robots.

Some results to the coupled problem, i.e. when both coverage and estimation are considered, have appeared recently. In Choi, Lee, and Oh (2008) the authors exploit Kalman filtering to perform Gaussian estimation. The final objective is to perform estimation and maximum seeking of a function of interest by means of a coordinated group of robots. They propose a two stage algorithm in which, first, based on information on the posterior variance, the robots are spread throughout the space in order to achieve a good estimate of the sensory function; once achieved a predefined threshold, the robots are driven towards the maximum of the estimated function. However, no convergence results during the estimation phase are shown. In Choi and Horowitz (2010) the authors propose a strategy to drive a formation of robots towards the coverage of an area of interest characterized by an unknown probability density function of event appearance. The result builds on learning diffeomorphic functions with kernels. However, it applies only to one dimensional environments and, if needed, it does not provide any estimate of the function of interest. In Schwager, Rus, and Slotine (2009) the authors propose an algorithm for simultaneous distributed consensus-like parametric estimation from noise-free measurements and optimal coverage based on centroidal Voronoi partitioning. However, to prove estimation convergence an infinite amount of noise-free measurements are assumed to be collected in finite time.

In this work, of which a preliminary version can be found in Carron et al. (2015), we analyze the problem of *simultaneous* estimation and coverage. The main contribution is twofold: the first is to consider a strategy that smoothly moves from estimation to coverage at the benefit of better transient behavior as compared to traditional approaches. The second contribution is to exploit the better estimation performance of non-parametric Gaussian regression as compared to parametric approaches while being able to bound its computational complexity. More specifically, we consider two different communication architectures to address the problem both in a centralized as well as in a distributed framework, namely *client–server* and *peer-to-peer* (p2p), respectively. In the client–server architecture (even referred to as one-to-base station communication (Pater, Frasca, Durham, Carli, & Bullo, 2016)) the robots can communicate with a server/base station. In the p2p architecture robots are allowed to communicate with neighboring peers by means of a *gossip* protocol. The goal is to perform *nonparametric* estimation of an unknown sensory distribution function from noisy samples while driving the robots to optimally cover an area of interest. This is achieved via a probabilistic control strategy which allows the robots to seamless transition between estimation and coverage. Differently from the standard approach (Choi et al., 2008), our control never completely switches from the estimation to the coverage phase but always trade-offs between them in order to achieve the best solution in terms of estimation and coverage. This (i) let us prove convergence in probability of the estimated function to the true one. As so, we obtain a final coverage configuration which is arbitrarily close to a partitioning obtained with the exact prior knowledge of the sensory function. Moreover, (ii) the strategy, compared to threshold-based approaches, e.g., in the same spirit of the algorithm proposed in Choi et al. (2008), can lead to smaller average coverage time. To alleviate the computational burden needed to implement the nonparametric estimation procedure, we also propose an alternative algorithm, based on a spatial discretization, to trade-off between accuracy on the estimated map and computational requirements.

The remainder of the paper is as follows. Section 2, recalls the necessary preliminaries. Section 3 contains the problem at hand. Sections 4–6 present the server-based algorithm, its efficient version and the distributed solution with their convergence analysis, respectively. Section 7 presents compelling simulations to test the proposed solution against other possible strategies as well as in the presence of practical limitations. Section 8 concludes the paper. All the proofs can be found in Appendix.

2. Preliminaries

2.1. Voronoi partitions

Let $\mathscr{X} \subset \mathbb{R}^2$ be compact and convex. Let $\mu : \mathscr{X} \to \mathbb{R}_{>0}$ be a distribution sensory function defined over $\mathscr{X}$. Within the context of this paper, a *partition* of $\mathscr{X}$ is a collection of N convex polygons $\mathscr{P} = (\mathscr{P}_1, \ldots, \mathscr{P}_N)$ with disjoint interiors whose union is $\mathscr{X}$. Given the list of N distinct points in $\mathscr{X}$, $\mathbf{x} = (x_1, \ldots, x_N)$, we define the *Voronoi partition* $\mathscr{W}(\mathbf{x}) = (\mathscr{W}_1(\mathbf{x}), \ldots, \mathscr{W}_N(\mathbf{x}))$ generated by $\mathbf{x}$ as

$$\mathscr{W}_i(\mathbf{x}) = \left\{ q \in \mathscr{X} \mid \|q - x_i\| \leq \|q - x_j\|, \ \forall j \neq i \right\} \tag{1}$$

$\|\cdot\|$ being the Euclidean norm, which can be shown to be convex (Du, Faber, & Gunzburger, 1999). Given a partition $\mathscr{P} = (\mathscr{P}_1, \ldots, \mathscr{P}_N)$, for each region $\mathscr{P}_i$, $i \in \{1, \ldots, N\}$, we define its *centroid* with respect to the sensory function μ as

$$c_i(\mathscr{P}_i) = \left(\int_{\mathscr{P}_i} \mu(q) dq \right)^{-1} \int_{\mathscr{P}_i} q \mu(q) dq.$$

We denote with $\mathbf{c}(\mathscr{P}) = (c_1(\mathscr{P}_1), \ldots, c_N(\mathscr{P}_N))$ the vector of regions centroids. A partition $\mathscr{P} = (\mathscr{P}_1, \ldots, \mathscr{P}_N)$ is said to be a *Centroidal Voronoi partition* of the pair $(\mathscr{X}, \mu)$ if $\mathscr{P} = \mathscr{W}(\mathbf{c}(\mathscr{P}))$, i.e., $\mathscr{P}$ coincides with the Voronoi partition generated by $\mathbf{c}(\mathscr{P})$. Given a partition $\mathscr{P} = (\mathscr{P}_1, \ldots, \mathscr{P}_N)$, a sensory function μ and a set of pints $\mathbf{x} = (x_1, \ldots, x_N)$, we introduce the *cost function* $H(\mathscr{P}, \mathbf{x}, \mu)$ defined as

$$H(\mathscr{P}, \mathbf{x}, \mu) = \sum_{i=1}^{N} \int_{\mathscr{P}_i} \|q - x_i\|^2 \mu(q) dq. \tag{2}$$