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Brief paper

Observer design for linear singular time-delay systems[☆]

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ABSTRACT

This paper investigates the problem of observer design for a general class of linear singular time-delay systems, in which the time delays are involved in the state and its derivatives, the output and the known input (if there exists). The involvement of the delay could be multiple which however is rarely studied in the literature. Sufficient conditions are proposed which guarantees the existence of a Luenberger-like observer for the general system.

1. Introduction

Singular systems (known also as Descriptor systems/Algebraic –Differential systems) enable us to model many physical, biological and economical systems (Campbell, 1982), which has been well discussed in Wang, Zhang, and Yan (2014). For such systems, time delay can be also encountered (Niculescu & Rasvan, 2000). Thus, the singular time-delay systems have been extensively analyzed during last decade. Lot of researches are focused on the stabilization of singular time-delay systems, such as H_∞ control, observer-based control and so on (Gu, Su, Shi, & Chu, 2013).

Concerning the concepts of observability and observer for the singular time-delay systems, a direct adaptation of existing results on observability and observers from regular systems to singular systems is not immediate due to the fact that they involve both differential and algebraic equations. Up to now, most of the existing works are based on the simple case with only one delay in the state, i.e. $E\dot{x}(t) = A_0x(t) + A_1x(t-h)$ where the input could be also involved. For this simple case, a general solution was derived in Wei (2013), based on which a sufficient condition for exact observability in finite time was deduced. When designing an observer for the mentioned simple linear singular time-delay systems, a few results can be found in the literature. In Feng, Zhu, and Cheng (2003), three kinds of observers (functional observer, reduced-order observer and full-order observer) are studied for the above simple case. In Ezzine, Darouach, Ali, and Messaoud (2011), a functional observer for singular time-delay systems with unknown inputs was presented, and the existence condition of such observer and the gain implemented in the design are obtained by solving LMIs. A Luenberger-like observer is proposed in Khadhraoui, Ezzine, Messaoud, and Darouach (2014) for the linear singular time-delay systems with unknown inputs not affected by time delays.

In this paper, we deal with a quite general linear singular time-delay system of the form: $\sum_{i=0}^{l} E_i\dot{x}(t-ih) = \sum_{i=0}^{k} A_ix(t-ih)$, which in fact covers different types of time-delay system (singular or not, with or without neutral term). For such a presentation, there exist lots of applications, such as LC electrical lines (Brayton, 1968) and so on. More concrete applications can be found in Niculescu (2001). For such a general form, observability analysis and observer design become more difficult. Recently, inspired by the well-known Silverman and Molinari algorithm (see Molinari, 1976; Silverman, 1969) to analyze the observability for linear systems (with or without unknown input), a similar and checkable sufficient condition was proposed in Bejarano and Zheng (in press) to analyze the observability for this general singular time-delay systems. As far as we know, the only work on the observer design for this general case is Perdon and Anderlucci (2006),

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where a Luenberger-like observer is proposed by using a virtual discrete time system with the matrices of the originally considered continuous-time system. In this method, a geometric notion of conditioned invariant submodule is introduced for this class of systems over a ring and a design procedure is presented. However, the sufficient condition deduced in that paper is difficult to be checked, since it highly depends on the assumption of the existence of Hurwitz polynomial. *The contributions of this paper are as follows*. Firstly, the class of the studied systems is quite general (we allow multiple delays both in the $x(t)$ and its derivative), which in fact can cover four different classes of systems. As far as we know, there exist some methods to eliminate (or reduce the degree of) the delay, such as Garate-Garcia, Marquez-Martinez, and Moog (2011), Germani, Manes, and Pepe (2001) and Lee, Neftci, and Olbrot (1982). It has been proven in Garate-Garcia et al. (2011) that the elimination or the reduction of delay degree via a bicausal transformation with the same dimension is possible if some conditions on $A(\delta)$ and $B(\delta)$ are satisfied. In other words, the elimination or the reduction of delay degree is not always possible. This issue has been highlighted in Bejarano and Zheng (2014, 2016a,b) and Zheng, Bejarano, Perruquetti, and Richard (2015). Therefore, allowing multiple delays in the state, in the derivative, and in the output is an essential generalization. The second contribution of this paper is to deduce a checkable sufficient condition such that a Luenberger-like observer exists for such a general linear singular time-delay system. Moreover, it provides as well a constructive way to synthesize the proposed observer.

2. Notations and problem statement

In this paper, we consider the following class of linear systems with commensurate delays:

$$\begin{cases} \sum_{i=0}^{k_e} \tilde{E}_i \dot{x}(t-ih) = \sum_{i=0}^{k_a} \tilde{A}_i x(t-ih) \\ \tilde{y}(t) = \sum_{i=0}^{k_c} \tilde{C}_i x(t-ih) \end{cases} \tag{1}$$

where the vector $x(t) \in \mathbb{R}^n$, the system output vector $\tilde{y}(t) \in \mathbb{R}^p$, h represents the basic delay, the initial condition $\varphi(t)$ is a piece-wise continuous function $\varphi(t) : [-kh, 0] \to \mathbb{R}^n (k = \max\{k_a, k_c, k_e\})$; thereby $x(t) = \varphi(t)$ on $[-kh, 0]$. $\tilde{A}_i$, $\tilde{B}_i$ and $\tilde{E}_i$ are matrices of appropriate dimension with entries in $\mathbb{R}$. It is worth noting that the studied system of the form (1) is quite general, and it covers different types of time-delay systems. More precisely:

- if $k_e = 0$ and $\tilde{E}_0 = \mathrm{I}$, then system (1) becomes the classical linear time-delay system of the form $\dot{x} = \sum_{i=0}^{k_a} \tilde{A}_i x(t-ih)$;
- if $k_e = 0$ and the rank of $\tilde{E}_i$ is not full, then system (1) is equivalent to a linear singular time-delay systems of the form $\tilde{E}_0 \dot{x}(t) = \sum_{i=0}^{k_a} \tilde{A}_i x(t-ih)$;
- when $k_e > 0$ and $\tilde{E}_0 = \mathrm{I}$, then system (1) can be written as $\dot{x}(t) = \sum_{i=0}^{k_a} \tilde{A}_i x(t-ih) - \sum_{i=1}^{k_e} \tilde{E}_i \dot{x}(t-ih)$ which is typically a linear time-delay neutral system;
- moreover, if $k_e > 0$ and the rank of $\tilde{E}_0$ is not full, system (1) represents a general linear singular time-delay neutral system of the form $\tilde{E}_0 \dot{x}(t) = \sum_{i=0}^{k_a} \tilde{A}_i x(t-ih) - \sum_{i=1}^{k_e} \tilde{E}_i \dot{x}(t-ih)$.

Nevertheless, the form (1) is so general that it might include as well advanced systems for which the existence of the solution cannot be always guaranteed. In order to exclude such ill-conditioned systems, it is assumed in this paper that system (1) admits at least one solution. It is worth noting that *the existence of a unique solution is not necessary* for the observability analysis and the observer design. Take any system with several solutions as an example, this issue can be easily identified if the output of this system is all the state. In this case, the system is always observable even if several solutions exist.

Remark 1. For the special class of systems mentioned above, there exist some results on observer design in the literature, for example Darouach and Boutayeb (1995), Ezzine et al. (2011) and Hou, Zitek, and Patton (2002), and lots of the existing works consider only one delay in $x(t)$, i.e. $k_e = k_c = 0$ and $k_a = 1$ in (1). However, for a linear system with commensurate delays of the general form (1) covering a large class of linear singular (or not) time-delay neutral (or not) systems, to the best of our knowledge, rare results on observer design have been reported in the literature. Therefore, the problem of designing an observer is still an open problem for the general form (1).

Motivated by this fact, this paper proposes a Luenberger-like observer and sufficient conditions are deduced which guarantee the existence of such an observer.

In the following, for the purpose of simplifying the analysis, let us introduce the delay operator $\delta : x(t) \to x(t-h)$ with $\delta^k x(t) = x(t-kh), k \in \mathbb{N}_0$. Then the following notations will be used in this paper. $\mathbb{R}$ is the field of real numbers. The set of positive integers is denoted by $\mathbb{N}$. I_n for $n \in \mathbb{N}$ means the identity matrix of order n. $\mathbb{R}[\delta]$ is the polynomial ring over the field $\mathbb{R}$. $\mathbb{R}^n[\delta]$ is the $\mathbb{R}[\delta]$-module whose elements are the vectors of dimension n and whose entries are polynomials. By $\mathbb{R}^{q\times s}[\delta]$ we denote the set of matrices of dimension $q \times s$, whose entries are in $\mathbb{R}[\delta]$. For a matrix $M(\delta)$, $\mathrm{rank}_{\mathbb{R}[\delta]} M(\delta)$ means the rank of the matrix $M(\delta)$ over $\mathbb{R}[\delta]$. $M(\delta) \sim N(\delta)$ means the similarity between two polynomial matrices $M(\delta)$ and $N(\delta)$ over $\mathbb{R}[\delta]$, i.e. there exist two unimodular matrices $U_1(\delta)$ and $U_2(\delta)$ over $\mathbb{R}[\delta]$ such that $M(\delta) = U_1(\delta)N(\delta)U_2(\delta)$.

After having introduced the delay operator δ, system (1) may be then represented in the following compact form:

$$\begin{cases} \tilde{E}(\delta)\dot{x}(t) = \tilde{A}(\delta)x(t) \\ \quad \tilde{y}(t) = \tilde{C}(\delta)x(t) \end{cases} \tag{2}$$

where $\tilde{A}(\delta) \in \mathbb{R}^{\bar{n}\times n}[\delta]$, $\tilde{C}(\delta) \in \mathbb{R}^{p\times n}[\delta]$ and $\tilde{E}(\delta) \in \mathbb{R}^{\bar{n}\times n}[\delta]$ are matrices over the polynomial ring $\mathbb{R}[\delta]$, defined as $\tilde{A}(\delta) := \sum_{i=0}^{k_a} \tilde{A}_i\delta^i$, $\tilde{C}(\delta) := \sum_{i=0}^{k_c} \tilde{C}_i\delta^i$ and $\tilde{E}(\delta) := \sum_{i=0}^{k_e} \tilde{E}_i\delta^i$. Without loss of generality, it is thus assumed in this paper that $\bar{n} \le n$, $\mathrm{rank}_{\mathbb{R}[\delta]}\tilde{E}(\delta) = q \le \bar{n}$ and $\mathrm{rank}_{\mathbb{R}[\delta]}\tilde{C}(\delta) = p$.

Remark 2. The above assumptions are not restrictive. $\mathrm{rank}_{\mathbb{R}[\delta]} \tilde{E}(\delta) = q \le \bar{n} \le n$ is due to the fact that the studied system might be singular or neutral. Moreover, if $\mathrm{rank}_{\mathbb{R}[\delta]}\tilde{C}(\delta) = \tilde{p} < p$, we can always eliminate the dependent outputs of (2). More precisely, if $\mathrm{rank}_{\mathbb{R}[\delta]}\tilde{C}(\delta) = \tilde{p} < p$, then there exists a polynomial matrix $U(\delta) \in \mathbb{R}^{\tilde{p}\times p}[\delta]$ such that $U(\delta)\tilde{C}(\delta) \in \mathbb{R}^{\tilde{p}\times n}[\delta]$ is of full row rank. In this case, we can rewrite the output as $U(\delta)\tilde{y} = U(\delta)\tilde{C}(\delta)x$ with $\mathrm{rank}_{\mathbb{R}[\delta]}U(\delta)\tilde{C}(\delta) = \tilde{q}$.

3. Definition, assumptions and preliminary result

For each instant t, the available measurements are only $y(t)$ and its delayed values which can be used to estimate $x(t)$. We cannot in fact utilize the future value of $y(t)$, otherwise it is not causal. Therefore, it is desired to use only the actual and the past information (not the future information) of the measurement to design an observer for time-delay systems due to the requirement of the causality. Thus the following definition of backward observability is given.