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Brief paper

Adaptive disturbance attenuation for generalized high-order uncertain nonlinear systems☆

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ABSTRACT

This paper investigates the problem of adaptive disturbance attenuation for a class of generalized high-order uncertain nonlinear systems. The control strategy is on the basis of continuous domination and delicate adaptive technique, and it can cope with serious coexistence among uncertainties, including time-varying control coefficients which have unknown upper and lower bounds, nonlinear parameters and external disturbances. Adaptive state-feedback controller is one-dimensional irrespective of the number of unknown parameters, and its performance is evaluated in terms of L_2-L_{2p} gain.

1. Introduction

It is well known that high-order nonlinear systems have uncontrollable linearization around the origin, so its stabilization has been viewed as one of the most challenging issues. Fortunately, many results have been obtained with the help of adding a power integrator method and homogeneous domination idea, such as Fu, Ma, and Chai (2015), Li, Xie, and Zhang (2011), Lin and Qian (2002a,b), Liu (2014), Lv, Sun, and Xie (2015), Polendo and Qian (2007), Qian and Lin (2001), Sun, Li, and Yang (2016), Sun and Liu (2009), Sun and Liu (2015), Sun, Xue, and Zhang (2015) and Zhang, Liu, Baron, and Boukas (2011) to name just a few.

On the other hand, practical control systems are always corrupted by various types of unknown disturbances, and one topic in control design is to attenuate their influence on the output as much as possible, since it is hard to realize exact disturbance decoupling. What is worse, uncertainties also have a potential tendency to deteriorate system performance or even destabilize control systems, so their effects have to be taken into consideration. Discarding parameter uncertainties, the topic has been solved partly, see Ito and Jiang (2004), Lin, Qian, and Huang (2003), Marino and Tomei (1999), Willems (1981) and references therein. Specifically, in light of internal stability and a feedback domination design, Lin et al. (2003) and Willems (1981) solved it for linear systems and nonlinear systems by providing necessary and sufficient geometric conditions, respectively. Marino and Tomei (1999) discussed the problems of input-to-state stability with respect to disturbance inputs and almost disturbance decoupling output tracking for strict feedback nonlinear systems, and (Ito & Jiang, 2004) presented an approach to output feedback stabilization with L_2 gain disturbance attenuation in the presence of zero dynamics. Furthermore, Marino and Tomei (2000) made an interesting exploration for a class of nonlinearly parameterized systems, and (Shang & Liu, 2014) permitted the existence of more uncertainties including unknown parameters and unmeasurable states. In comparison, there is little progress on almost disturbance decoupling of high-order nonlinear systems, because it is really difficult to construct state observer and Lyapunov function satisfying assumptions of internal stability in a complex environment. Fortunately, the paper (Qian & Lin, 2000) formulated a well posed almost disturbance decoupling problem for the first time, and illustrated how to utilize the adding a power integrator method to construct a smooth state-feedback control law, while there are no uncertainties in the systems. Therefore, one may propose a natural and interesting question: *How large uncertainties will be allowed to construct a feedback controller for high-order nonlinear systems in the presence of external disturbances*? It is worth emphasizing that the affirmative

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solution to above question is a troublesome task, which can be seen from two aspects. (i) The first difficulty is the identification of uncertainties. Undiscovered parts of the systems can be composed of unmeasurable states, unknown parameters and unclear structures, except for possible disturbance. This paper puts a foothold in dealing with unknown parameters. For expanding the scope of control strategy as large as possible, the systems possess unknown control coefficients and permit unknown parameters to enter the state equations nonlinearly. In order to suppress uncertainties simultaneously, we introduce an appropriate nonlinear function, and use transformation skill combined with adaptive technique to alleviate their effects. (ii) The second difficulty is the simplification of the controller. Its remarkable feature is that the order of dynamic compensator is equal to one, which simplifies the procedure of control design and stability analysis of the closed-loop systems. Under relaxed conditions, the designed adaptive controller guarantees stabilization properties when external disturbance is absent, and attenuates the influence of the disturbance on the output with an arbitrary degree of accuracy in terms of L_2–L_{2p} gain.

We adopt the following notations throughout this paper. $\mathbb{R}^+$ denotes the set of all non-negative real numbers, and $\mathbb{R}^n$ denotes Euclidean space with dimension n. $\mathbb{R}^{\geq 1}_{odd} \triangleq \{\frac{p}{q}|p$ and q are positive odd integers, and $p \geq q\}$. For a real vector $x = [x_1, \ldots, x_n]^T \in \mathbb{R}^n$, $\bar{x}_i \triangleq [x_1, \ldots, x_i]^T \in \mathbb{R}^i$, $i = 1, \ldots, n$, especially $\bar{x}_n = x$, and the norm $\|x\|$ of $x \in \mathbb{R}^n$ is defined by $\|x\| = \sqrt{\sum_{i=1}^n x_i^2}$. The space L_p with $1 \leq p \leq \infty$ is defined as the set of all piecewise continuous functions $x : [0, \infty) \to \mathbb{R}^n$ such that $\|x\|_{L_p} = (\int_0^\infty \|x(t)\|^p dt)^{1/p} < \infty$, $\|x\|_{L_\infty} = \sup_{t\geq 0} \|x(t)\| < \infty$. For a continuously differentiable function $V : \mathbb{R}^n \to \mathbb{R}^+$, it is positive definite if $V(x) \geq 0$ and $V(x) = 0$ if and only if $x = 0$; it is radially unbounded if $V(x) \to \infty$, $\|x\| \to \infty$. The arguments of functions are sometimes simplified, a function $f(x(t))$ can be denoted by $f(x)$, $f(\cdot)$ or f.

2. Problem formulation

We consider the following uncertain nonlinear systems

$$\begin{cases} \dot{x}_i = d_i(t, x, u, \theta)x_{i+1}^{p_i} + f_i(t, x, u, \theta) + g_i(t, x, u, \theta)\omega, \\ \dot{x}_n = d_n(t, x, u, \theta)u^{p_n} + f_n(t, x, u, \theta) + g_n(t, x, u, \theta)\omega, \\ y = h(x_1), \end{cases} \tag{1}$$

where $i = 1, \ldots, n-1$, $x \in \mathbb{R}^n$, $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are system state, control input and system output, respectively. Initial condition is $x(0) = x_0$, and $u \triangleq x_{n+1}$. $\omega : \mathbb{R}^+ \to \mathbb{R}^s$ is a continuous time-varying disturbance signal satisfying $\omega \in L_2$, and $\theta \in \mathbb{R}^m$ represents an unknown parameter vector which can be time-invariable or time-varying. For each $i = 1, \ldots, n$, $p_i \in \mathbb{R}^{\geq 1}_{odd}$ is named by the power of the systems, and $f_i(\cdot)$, $g_i(\cdot)$ and $d_i(\cdot)$ are continuous nonlinear functions, while $h(x_1)$ is a continuously differentiable function with $h(0) = 0$. We clarify the definition of high-order as follows, in fact, it is said to be a high-order nonlinear system, if there is at least one power $p_i > 1$. For example, $\dot{x}_1 = x_2^3 + x_1^3$, $\dot{x}_2 = u$ is a high-order nonlinear system since $p_1 = 3 > 1$, $p_2 = 1$, but $\dot{x}_1 = x_2 + x_1^3$, $\dot{x}_2 = u$ is not a high-order one since $p_1 = p_2 = 1$.

The objective of this paper is to solve the so-called ADA problem, that is,

Adaptive Disturbance Attenuation (ADA): For system (1), find a continuous adaptive controller

$$\begin{cases} u(t) = u(x(t), \hat{\Theta}(t)), & u(0, \hat{\Theta}(t)) = 0, \\ \dot{\hat{\Theta}}(t) = \tau(x(t), \hat{\Theta}(t)), & \tau(0, \hat{\Theta}(t)) = 0, \end{cases} \tag{2}$$

where $\hat{\Theta}(t)$ is on-line estimate of unknown parameter Θ depending on θ, such that closed-loop systems composed of (1) and (2) satisfy the following features.

(i) When $\omega(t) = 0$, states of the closed-loop systems are globally uniformly bounded on the interval $[0, \infty)$, and $\lim_{t\to\infty} x(t) = 0$.
(ii) When $\omega(t) \in L_2$, for any pre-given small real number $\varepsilon > 0$, there holds $\int_0^t |y(s)|^{2p_1} ds \leq \varepsilon^2 \int_0^t \|\omega(s)\|^2 ds + \delta(x(0), \hat{\Theta}(0))$, $\forall t \in [0, \infty]$, where $\delta(\cdot)$ is nonnegative and rests with initial states of the closed-loop systems.

The following assumptions are needed.

Assumption 1. For each $i = 1, \ldots, n$, there is $0 < a_i\lambda_i(\bar{x}_i) \leq |d_i(\cdot)| \leq \mu_i(\bar{x}_{i+1}, \theta)$, where a_i is an unknown constant, λ_i is a positive smooth function, and μ_i is a continuous function.

Assumption 2. For each $i = 1, \ldots, n$, there exist nonnegative continuous functions $f_{il}(\bar{x}_i, \theta)$ with $f_{il}(0, \theta) = 0$, such that $|f_i(\cdot)| \leq \sum_{l=1}^{j_i} f_{il}(\bar{x}_i, \theta)|x_{i+1}|^{q_{il}}$, where j_i's are finite positive integers, and q_{il}'s are real numbers satisfying $0 \leq q_{i1} < q_{i2} < \cdots < q_{ij_i} < p_i$.

Assumption 3. For each $i = 1, \ldots, n$, there exists a nonnegative and continuously differentiable function $\varphi_i(\bar{x}_i, \theta)$ with $\varphi_i(0, \theta) = 0$, such that $\|g_i(\cdot)\| \leq \varphi_i(\bar{x}_i, \theta)$.

In this paper, we call differential equations of the form (1) generalized high-order uncertain nonlinear systems, one reason lies in the general form of the systems in the sense that the unknowns come from uncertain control coefficients, unclear parameters and unpredictable disturbances. Another one is that the systems can be viewed as a generalization of the strict feedback nonlinear systems (Huang, Wen, Wang, & Song, 2016; Khalil, 2002; Krstić, Kanellakopoulos, & Kokotović, 1995; Marino & Tomei, 2000; Shang & Liu, 2014; Zhu, Wen, Su, & Liu, 2014), and these systems govern many physical processes such as synchronous motor and an aircraft wing rock. In what follows we explain the necessity of Assumptions 1–3 and exhibit how to enlarge the scope of the nonlinear systems through a remark.

Remark 1. Assumption 1 indicates that $d_i(\cdot)$ is strictly either positive or negative. Without loss of generality, we just consider the case of $d_i > 0$ in subsequent control design. Compared with assumptions in Lin and Qian (2002b) and Sun and Liu (2009), the paper relaxes upper bound μ_i of $|d_i|$ to a function of $x_1, \ldots, x_{i+1}, \theta$, except for the existence of unknown lower bound of $|d_i|$, hence more delicate manipulation technique should be introduced to achieve the desired control objective. Some complex deductions can change Assumption 2 into $|f_i| \leq \frac{d_i}{2}|x_{i+1}^{p_i}| + f_i^*(\bar{x}_i, \theta)\sum_{j=1}^i |x_j|$, but not vice versa, where f_i^* is a positive smooth function. This inequality is frequently used in the literature, such as Lin and Qian (2002a,b), Sun and Liu (2009) and Fu et al. (2015). Assumption 3 is somewhat weaker than those in Shang and Liu (2014) and Qian and Lin (2000), due to the coupling of ω and θ. □

Remark 2. Even if $\omega = 0$, the adaptive stabilization of system (1) is not trivial. In the following, some comparisons are presented to explain the differences from the related papers and the difficulties encountered in this paper. (i) In Sun, Xue et al. (2015), the control coefficients are identical and known, that is, $d_1 = \cdots = d_n = 1$. (ii) According to Assumption 1 in Sun et al. (2016), although the control coefficients are allowed to be time-varying, they have to be identical and lower bounded by an unknown positive constant. (iii) Assumption 3.1 in Lin and Qian (2002a) shows that the control coefficients can be nonidentical but must be lower bounded by some known positive smooth functions. As can be seen from Assumption 1, this paper successfully overcomes aforementioned restrictions. However, the lower bounds and upper bounds of the control coefficients are uncertain due to the presence of unknown a_i and θ, so one must seek for an effective strategy to trade off their effects, in order to guarantee that the dynamic order of the adaptive controller is minimum. □