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Brief paper

Global robust stabilization of nonlinear cascaded systems with integral ISS dynamic uncertainties ☆

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ABSTRACT

The authors consider a global robust asymptotic stabilization problem (GRS) for cascaded systems having dynamic uncertainties that are not necessarily input-to-state stable (ISS). Specifically, a recursive Lyapunov design approach is developed by induction on the system relative degree, providing a smoothly globally stabilizing controller. It is applicable to nonlinear cascaded systems having multiple distinct iISS dynamic uncertainties. The proposed design is constructive and leads to an iISS-Lyapunov characterization in a superposition form for the closed-loop system.

1. Introduction

Since the fundamental notions of ISS and iISS were introduced by Sontag (see Sontag, 2007 for an overview), they have substantially promoted the study of global robust stabilization of nonlinear systems having dynamic uncertainties, see Chen and Huang (2015), Jiang, Mareels, Hill, and Huang (2004), Karafyllis and Jiang (2011), Krstic, Kanellakopoulos, and Kokotovic (1995) and relevant references thereof. For nonlinear systems, it is usually more realistic to perform output (or partial state) feedback control in practical situations due to the lack of full state information as well as the system uncertainties. In view of this fact, an effective strategy is to regard partial state variables of plant dynamics as system *dynamic uncertainties*, boosted in the pioneering work of Praly and Jiang (1993). Then, the stabilization problem can be more tractable by output feedback control when the dynamic uncertainties have certain ISS-like dissipation properties. Coping with various types of dynamic uncertainties for nonlinear systems, the relevant GRS problem has been an active topic.

Toward globally stabilizing control of nonlinear systems in the presence of ISS or iISS dynamic uncertainties, it is of importance to note that some growth condition should be imposed on system nonlinearity. For example, when the system dynamic uncertainties are ISS, the tolerable growth rate of system nonlinearity can be unbounded comparison functions of such dynamic uncertainties. In this case, to handle global stabilization (hence, global output regulation) of nonlinear systems in the presence of ISS dynamic uncertainties, there have come up with various methods, including nonlinear small-gain theorem (Jiang, Teel, & Praly, 1994) based design, see, e.g., Jiang and Mareels (1997), Huang (2004), and Lyapunov's direct method based design, see, e.g., Chen and Huang (2004, 2008). In summary, this case has been extensively studied for many classes of nonlinear systems such as output feedback systems and lower triangular systems.

In the past few years, a number of interesting problems for iISS interconnected systems have been studied, see Angeli, Sontag, and Wang (2000), Arcak, Angeli, and Sontag (2002), Chaillet, Angeli, and Ito (2014a), Dashkovskiy, Rüffer, and Wirth (2010), Ito (2010), Ito and Jiang (2009), Ito, Jiang, Dashkovskiy, and Rüffer (2013) and Rüffer, Kellett, and Weller (2010). Because the iISS property is strictly weaker than the ISS one, the aforementioned tolerable growth rate on system nonlinearity needs to be further restricted to ensure solvability of the stabilization problem. Especially, when

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the dynamic uncertainties are iISS\ISS,[1] the growth rate should be bounded functions of these dynamic uncertainties in some sense. To the best of our knowledge, stabilization of nonlinear systems in the presence of multiple types of iISS dynamic uncertainties was only partially solved in the literature, see, e.g., Jiang et al. (2004) and Xu, Huang, and Jiang (2013). In Jiang et al. (2004), a regulation problem was solved for a special class of nonlinear lower triangular systems, named output feedback systems, having either iISS\ISS or ISS dynamic uncertainties. In Xu et al. (2013), different from Jiang et al. (2004), a global stabilization problem for systems having both iISS\ISS and ISS dynamic uncertainties was further studied, as a byproduct of nonlinear output regulation design, still for output feedback systems.

The main objective of this paper is to explore the GRS problem for nonlinear cascaded systems (in a general lower triangular form) having iISS dynamic uncertainties. The proposed results are applicable to systems having *multiple* iISS\ISS and ISS combined dynamic uncertainties while their tolerable growth rates can be partially bounded and partially unbounded. In contrast to the ISS one of single-type, this circumstance is much more complicated and challenging. Besides its own theoretical interest, a basic yet convincing motivation for this circumstance is its high impact in resolving global robust output regulation in the framework of internal model based design (see Huang, 2004; Wang, Chen, & Xu, under review or Section 3.3 of this paper). The internal model state variables, as a key component of the solution to achieve output regulation, essentially play the role of dynamic uncertainties in stabilization of certain translated augmented systems. In particular, it may result in multiply mixed iISS\ISS and ISS dynamic uncertainties in the problem.

The main contribution of this paper is to develop a new recursion based approach, giving rise to a smooth global stabilization controller as well as construction of iISS-Lyapunov function for the closed-loop system. It partially extends some existing results in Chen and Huang (2015), Karafyllis and Jiang (2011) and Krstic et al. (1995), for nonlinear systems having iISS dynamic uncertainties. Hence, the present study may offer a complement or an alternative to some ISS studies in the existing literature.

Paper Organization. Section 2 formulates the problem and presents several definitions used in this paper. Section 3 shows the main result of this paper. Section 4 gives an illustrative example. Section 5 closes the paper.

Terminology. $\|\cdot\|$ is the Euclidean norm. A function is called (sufficiently) smooth if it is C^k (with k continuous derivatives) for a sufficiently large integer k of any technical requirement. The function $f:\mathbb{R}^n\to\mathbb{R}_+$ is said to be positive definite if, $f(x)>0$ for $x\neq 0$ and $f(0)=0$. The set of continuous and positive definite functions is denoted by $\mathscr{P}$. The set of continuous, bounded and positive definite functions is denoted by $\mathscr{P}_o$. Given $\alpha\in\mathscr{P}$, let $\widehat{\mathscr{O}}(\alpha)$ be the set $\widehat{\mathscr{O}}(\alpha)\triangleq\left\{\gamma\in\mathscr{P}\mid \limsup_{s\to 0^+}\gamma(s)/\alpha(s)<\infty,\text{ and }\limsup_{s\to\infty}\gamma(s)/\alpha(s)<\infty \text{ if } \alpha\in\mathscr{P}_o\right\}$. The function $f:\mathbb{R}_+\to\mathbb{R}_+$ is of class $\mathcal{K}$, i.e., $f\in\mathcal{K}$ if, it is continuous, positive definite, and strictly increasing. $f:\mathbb{R}_+\to\mathbb{R}_+$ is of class $\mathcal{K}_\infty$ if, it is of class $\mathcal{K}$ and unbounded. *Id* denotes the identical $\mathcal{K}_\infty$ function. The set of bounded $\mathcal{K}$ functions is denoted by $\mathcal{K}_o$, i.e., $\mathcal{K}_o=\mathcal{K}\setminus\mathcal{K}_\infty$. For a pair of functions $f_1(\cdot), f_2(\cdot)$ of compatible dimensions, $f_1\circ f_2(\cdot)$ denotes function composition $f_1(f_2(\cdot))$. For any column vectors $x_1,\ldots,x_r$, $\mathrm{col}(x_1,\ldots,x_r)\triangleq[x_1^\mathsf{T},\ldots,x_r^\mathsf{T}]^\mathsf{T}$ and $x_{[i]}\triangleq\mathrm{col}(x_1,\ldots,x_i)$ for $1\le i\le r$. $\{a_i\}_{i=1}^k$ denotes a set of functions or numbers $\{a_1,\ldots,a_k\}$. For a positive integer n, the expression $n!$ denotes the product $n\times(n-1)\times\cdots\times 2\times 1$.

[1] A system is called iISS\ISS if it is iISS but not ISS.

2. Formulation and preliminary

Consider nonlinear cascaded systems having dynamic uncertainties (see, e.g., Jiang & Mareels, 1997) described by

$$\begin{aligned}\dot z_i &= f_i(z_{[i]}, x_{[i]}, \mu(t)),\\ \dot x_i &= x_{i+1}+g_i(z_{[i]}, x_{[i]}, \mu(t)),\quad 1\le i\le n\end{aligned}\tag{1}$$

with control input $x_{n+1}\triangleq u$, measurable state $x=x_{[n]}\in\mathbb{R}^n$, dynamic uncertainty $z=z_{[n]}\in\mathbb{R}^{n_z}$, and static uncertainty $\mu(t)$ varying in a compact set $\mathbb{D}$, i.e., $\mu(t)\in\mathbb{D}$ for all $t\ge 0$. For each $1\le i\le n$, it is assumed that both $f_i(z_{[i]}, x_{[i]}, \mu)$ and $g_i(z_{[i]}, x_{[i]}, \mu)$ are smooth in their arguments, satisfying $f_i(0,0,\mu)=0, g_i(0,0,\mu)=0$ for all μ.

Problem 2.1 (*GRS*). For the system (1), if possible, find a controller of the form

$$x_{n+1}=\kappa(x)+\tilde x_{n+1}\tag{2}$$

for a smooth function $\kappa:\mathbb{R}^n\to\mathbb{R}$, such that the closed-loop system of (1) and (2) is iISS w.r.t. state (z,x) and input $\tilde x_{n+1}$ in the sense of Definition 2.1.

In the rest of this section, two definitions and a useful lemma are presented, relating to iISS systems or dynamical networks.

Consider a general nonlinear system

$$\dot x = f(x,u,\mu(t))\tag{3}$$

where $x\in\mathbb{R}^{n_x}$ is the state, $u\in\mathbb{R}^{n_u}$ the control input, $\mu(t)\in\mathbb{D}$ for all $t\ge 0$, and f a smooth vector field in its arguments, satisfying $f(0,0,\mu)=0$ for all μ.

Definition 2.1. For the system (3), a smooth function $V:\mathbb{R}_+\times\mathbb{R}^{n_x}\to\mathbb{R}_+$ is called a (robustly) iISS-Lyapunov function (w.r.t. state x and input u, robustly on μ) if, it satisfies, along trajectories of (3),

$$\underline{\alpha}(\|x\|)\le V(t,x)\le\bar\alpha(\|x\|),\qquad \dot V\le\chi(\|u\|)-\alpha\circ V(t,x)\tag{4}$$

where $\underline{\alpha},\bar\alpha\in\mathcal{K}_\infty$, $\alpha\in\mathscr{P}$, and $\chi\in\mathcal{K}$. The system (3) is called iISS with a *supply pair* $\{\alpha,\chi\}$ if it has an iISS-Lyapunov function satisfying (4), and α is called the *dissipation gain*.

Remark 2.1. In Definition 2.1, if $\alpha\in\mathcal{K}_\infty$, it is clearly an ISS-Lyapunov function for (3). For the sake of technical simplicity, we write the dissipation term as $-\alpha\circ V(t,x)$ in Definition 2.1 instead of the general one $-\alpha(\|x\|)$; cf. Angeli et al. (2000) and Chaillet et al. (2014a); also see Ito, Dashkovskiy, and Wirth (2012) and Liu, Hill, and Jiang (2011) for the same setup of ISS and iISS characterizations.

Definition 2.2. Consider the system (3) rewritten by the following equations

$$\dot x_i = f_i(x_1,\ldots,x_r,u,\mu),\quad 1\le i\le r\tag{5}$$

with $x=x_{[r]}$ and $x_i\in\mathbb{R}^{n_i}$. Then, the system of the form (5) is called an *iISS network* (w.r.t. state x and input u) with iISS-Lyapunov functions $\{V_i(t,x_i)\}_{i=1}^r$ and an *(interconnection) gain index* p if, for $1\le i\le r$, along trajectories of (5),

$$\begin{aligned}&\underline{\alpha}_i(\|x_i\|)\le V_i(t,x_i)\le\bar\alpha_i(\|x_i\|),\\ &\dot V_i\le\sum_{j\neq i}^{r}\chi_{i,j}\circ V_j(t,x_j)+\varpi_i(\|u\|^2)-\alpha_i\circ V_i(t,x_i)\end{aligned}\tag{6}$$

where $\underline{\alpha}_i,\bar\alpha_i\in\mathcal{K}_\infty$, $\alpha_i,\chi_{i,j}\in\mathscr{P}$, $\varpi_i\in\mathscr{P}\cap\widehat{\mathscr{O}}(Id)$, and moreover, for each pair $(i,j): 1\le i,j\le r, i\neq j$,

$$\chi_{i,j}(s)\le\frac{1}{2^p}\alpha_j(s),\quad \forall s\ge 0.\tag{7}$$