Automatica 80 (2017) 295–299

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Generalized non-autonomous metric optimization for area coverage problems with mobile autonomous agents ☆

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ARTICLE INFO

Article history:
Received 26 June 2015
Received in revised form
14 October 2016
Accepted 13 February 2017

Keywords:
Agents
Asymptotic stability
Co-operative control
Optimization problems
State feedback

ABSTRACT

Motivated by area coverage optimization problems with time-varying risk densities, in this paper we propose a decentralized control law for a team of autonomous mobile agents in a 2-D area such that their asymptotic configurations optimize a generalized non-autonomous coverage metric. We emphasize that the generalized non-autonomous coverage metric explicitly depends on a nonuniform time-varying measurable scalar field that is defined by the trajectories of a set of mobile targets (distinct from the agents). The time-varying density that we consider here is not directly controllable by agents. We show that under certain conditions on the density defined on a closed bounded region of operation, the agents configure themselves asymptotically to optimize a related generalized non-autonomous coverage metric. A set of simulations illustrates the proposed control.

1. Introduction

Recent technological advancements of networked mobile agents have received a thorough attention due to their promising applications in military and civilian domains, such as harbor patrolling (Miah, Nguyen, Bourque, & Spinello, 2014; Simetti & Cresta, 2007), perimeter surveillance (Pimenta et al., 2013; Zhang, Fricke, & Garg, 2013), search and rescue missions (Hu, Xie, Lum, & Xu, 2013), and cooperative estimation (Spinello & Stilwell, 2014), among others. In this context, an important class of applications involves area coverage, where a team of mobile agents spatially configure themselves over an area of interest to maximize a coverage metric that typically encodes agents' performance and a risk density that weights points in the area. Area coverage optimization with non-autonomous coverage metric typically emerges from time-varying risk densities, associated with uncontrollable events in the area to be covered. This scenario models, among others, a situation when an environment under observation is influenced by the sudden entrance of mobile targets (or events) that influence the risk function. Here we show that under certain conditions on the nonuniform measurable risk field, a decentralized state-feedback control law spatially configures a team of autonomous mobile agents to optimize a generalized non-autonomous coverage metric asymptotically, which mimics the maximization of detecting a certain event in an area, for instance.

Developing motion control algorithms for networked mobile agents has attracted considerable attention due to their capabilities to address, in part, various classes of problems in the field of multi-agent systems, such as area coverage (Caicedo-Nunez & Zefran, 2008; Cortes, Martinez, & Bullo, 2005; Cortes, Martinez, Karatas, & Bullo, 2004; Kantaros, Thanou, & Tzes, 2015; Leonard & Olshevsky, 2013; Miah, Nguyen, Bourque, & Spinello, 2015; Pimenta, Kumar, Mesquita, & Pereira, 2008; Zhong & Cassandras, 2011), locational optimization (Guruprasad & Ghose, 2013), target tracking (Yang, Freeman, & Lynch, 2008); and environmental tracking and monitoring (Porfiri, Roberson, & Stilwell, 2007). The work in Cortes, Martinez, Karatas, and Bullo (2002), Lekien and Leonard (2009) and Lee, Diaz-Mercado, and Egerstedt (2015) have addressed nonuniform coverage control problems with time-varying density. Cortes et al. (2002) considered a class of time-varying density functions such that the coverage metric is conserved, and therefore the resulting non-autonomous coverage would be a negative definite Lyapunov function along their

☆ The material in this paper was partially presented at the 2016 ASME Dynamic Systems and Control Conference, October 12–14, 2016, Minneapolis, MN, USA. This paper was recommended for publication in revised form by Associate Editor Dimos V. Dimarogonas under the direction of Editor Christos G. Cassandras.

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http://dx.doi.org/10.1016/j.automatica.2017.02.044
0005-1098/

proposed time-varying feedback law. Lee et al. (2015) addressed several issues related to coverage control problems with time-varying density functions and showed promising results with a real-time implementation using Khepera III robots operating on the ground. They proposed a motion control algorithm whose trajectories are optimal with respect to non-autonomous coverage metrics under the hypothesis that at the initial condition the agents are overlapped with Voronoi centroids. In general, there is no guarantee of optimality if this hypothesis is violated. In Zhong and Cassandras (2011) the authors have combined area coverage and data collection by proposing a task based algorithm, in which agents use area coverage control to determine optimal spatial locations, and trajectory control to maximize data collection quality whenever data sources are detected in the domain.

The current work advances previously published work (Miah et al., 2015) in proposing a methodology that takes into account a time-varying density to optimize a related generalized non-autonomous coverage metric. The main contribution of this paper is two-fold. First, we introduce a state-feedback control law for a group of autonomous agents such that a generalized time-varying coverage metric is optimized. Second, under a certain condition on the time-varying density function that depends on the trajectories of a set of mobile targets, we prove the asymptotic convergence of agents' states while optimizing the coverage metric. Trajectories of moving targets are encoded in the coverage metric, so that area coverage and target tracking are coupled through the optimal motion control algorithm.

2. System model and problem formulation

We restrict to a 2-dimensional area coverage problem with respect to a generalized non-autonomous coverage metric defined over the area, which is represented by a closed bounded convex set $\Omega \subset \mathbb{R}^2$. The non-autonomous coverage metric relies on agents' individual area coverage performances, and therefore the operating area is naturally partitioned among agents. In this work, we employ the geometric Voronoi tessellation (Okabe, Boots, Sugihara, & Chiu, 2000) as it is ubiquitous in cooperative strategies, coordination tasks, and the interaction of robotic networks with a physical environment.

We consider a group of n homogeneous mobile agents where the motion of the ith, $i \in \{1, 2, \dots, n\} \equiv \mathcal{I}$, agent is described by a simple integrator as $\dot{\mathbf{p}}^{[i]}(t) = \mathbf{u}^{[i]}(t)$, where $\mathbf{p}^{[i]}(t) = \left[x^{[i]}(t),\ y^{[i]}(t)\right]^T$ is the 2-D position and $\mathbf{u}^{[i]} = \left[u_x^{[i]}(t),\ u_y^{[i]}(t)\right]^T$ is the velocity vector at time $t \geq 0$. The state of the agents' therefore collectively evolves as

$$\dot{\mathbf{p}}(t) = \mathbf{u}(t), \tag{1}$$

where $\mathbf{p}(t) = \left[\mathbf{p}^{[1]}(t), \dots, \mathbf{p}^{[n]}(t)\right]^T \in \mathbb{R}^{2n}$ is the state of agents' group at time t and $\mathbf{u}(t) = \left[\mathbf{u}^{[1]}(t), \dots, \mathbf{u}^{[n]}(t)\right]^T \in \mathbb{R}^{2n}$ the corresponding velocity vector.

A time varying density defined in Ω weights each point with a measure of risk. In area protection problems, the risk quantifies the relative importance of different regions in Ω, dictating how to distribute resources to protect the area. In this work we consider a time varying risk density function affected by the motion of m homogeneous mobile targets with 2D positions $\mathbf{s}^{[j]}(t)$ and 2D velocities $\mathbf{v}^{[j]}$, respectively, for jth, $j = 1, \dots, m$ target. We assume that the risk density ϕ is C^2 in Ω, and it is comprised of a time invariant part which can be considered as a priori independent of the targets, and of a time varying part associated to the motion of the targets as $\phi(\mathbf{q}, t) = \bar{\phi}(\mathbf{q}) + \sum_{j=1}^{m} \phi_j(\mathbf{q}, t)$, where $\mathbf{q} \in \Omega$ is a point in the area, $\bar{\phi}(\mathbf{q}) > 0$ represents the time-invariant density in the absence of any target, and $\phi_j(\mathbf{q}, t)$ is given by

$$\phi_j(\mathbf{q}, t) = \exp\left(-\left\|\mathbf{q} - \mathbf{s}^{[j]}(t)\right\|^2 / \left(2\sigma^2\right)\right) \tag{2}$$

where we have adopted Gaussian functions centered at targets' positions, to reflect the assumption that each target contribution to the risk distribution ϕ is maximal at its own position, and it decreases with the relative distance from it. The choice of $\sigma > 0$ determines how narrow is the distribution of ϕ_j around $\mathbf{s}^{[j]}$. The non-autonomous coverage control feedback proposed in this work applies to C^2 time varying density function, not necessarily of the class just introduced.

For optimal spatial placements and area coverage, agents partition the area to be covered using Voronoi tessellations, see Okabe et al. (2000), where the optimality has to be intended as local since the objective function is in general nonconvex (Schwager, Rus, & Slotine, 2011). Following Guruprasad and Ghose (2013), the area Ω is partitioned in terms of Voronoi cells $\mathcal{V}(\mathbf{p}) = (\mathcal{V}_1(\mathbf{p}), \dots, \mathcal{V}_n(\mathbf{p}))$, where the ith agent operates in the ith Voronoi cell, $\mathcal{V}_i(\mathbf{p})$, defined by

$$\mathcal{V}_i(\mathbf{p}) = \left\{\mathbf{q} \in \Omega : f(r_i) \geq f(r_j),\ \forall j \in \mathcal{I} \setminus \{i\}\right\}, \tag{3}$$

$\forall i \in \mathcal{I}$, $r_i = \|\mathbf{q} - \mathbf{p}^{[i]}\|$ is the Euclidean distance between the point $\mathbf{q} \in \Omega$ and the ith agent position, and $f(\cdot)$ is the agent's sensor performance function, which is differentiable in its argument due to the selection of the performance function f the Voronoi partition is the optimal tessellation of the workspace (Bullo, Cortes, & Martinez, 2009, Proposition 2.1). Therefore, the generators of the Voronoi partition are the states $(\mathbf{p}^{[1]}, \dots, \mathbf{p}^{[n]})$. For simplicity, $\mathcal{V}_i(\mathbf{p})$ will be denoted by $\mathcal{V}_i$ throughout the paper. Intuitively, $\mathcal{V}_i$ represents an area where each point is better sensed by the ith agent than to all other agents. The mass and the centroid of the ith Voronoi cell with respect to the density ϕ are respectively defined as $M_i(\mathcal{V}_i, t) = \int_{\mathcal{V}_i} \phi(\mathbf{q}, t)\mathrm{d}\Omega$ and $\mathbf{C}_i(\mathcal{V}_i, t) = \frac{1}{M^i(\mathcal{V}_i, t)} \int_{\mathcal{V}_i} \mathbf{q}\phi(\mathbf{q}, t)\mathrm{d}\Omega$.

Consider a time-varying map $\phi : \Omega \times \mathbb{R}^+ \to \mathbb{R}^+$ representing a density function representing the likelihood that some events take place over Ω at time t. This leads to a nonuniform time varying distribution of agents, where more (less) agents are deployed with higher (lower) values of the measure $\phi(\mathbf{q}, t)$, $\forall \mathbf{q} \in \Omega$. Furthermore, we assume that the sensing performance function $f(r_i)$ is Lebesgue measurable, homogeneous, and that it is strictly decreasing with respect to the Euclidean distance r_i (Okabe et al., 2000). Motivated by the locational optimization problem (Okabe et al., 2000), we define the total coverage metric as

$$H(\mathbf{p}, \mathcal{V}, t) = \sum_{i=1}^{n} \int_{\mathcal{V}_i} f(r_i)\phi(\mathbf{q}, t)\mathrm{d}\Omega. \tag{4}$$

The model (4) encodes how rich the coverage in Ω is. In other words, higher H implies that the corresponding distribution of agents achieves better coverage of the area Ω (Bullo et al., 2009, Sec. 2.3.1). Hence, the problem can be stated as follows: Given a time-varying density function which is strictly positive and twice differentiable in Ω, find a distribution of agents such that the coverage H is maximized, *i.e.*,

$$\max_{\mathbf{p}} H(\mathbf{p}, \mathcal{V}, t), \quad \text{subject to (1) as } t \to \infty. \tag{5}$$

As established in Liu et al. (2009, Remark 1), smoothness of f with respect to the argument and $C^2(\Omega)$ risk density ϕ ensure that the coverage metric is C^2 with respect to the generators of the Voronoi tessellation, that are the agents' states.

3. Main results

When the density ϕ is time-invariant, *i.e.*, $\phi(\mathbf{q}, t) = \phi(\mathbf{q})$, geometric center laws (see Lloyd, 1982; Okabe et al., 2000, for details) on planar vehicles (agents) provide a well-established