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Bipartite and cooperative output synchronizations of linear heterogeneous agents: A unified framework[☆]

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ABSTRACT

This paper investigates cooperative output synchronization and bipartite output synchronization of a group of linear heterogeneous agents in a unified framework. For a structurally balanced signed graph, we prove that the bipartite output synchronization is equivalent to the cooperative output synchronization over an unsigned graph whose adjacency matrix is obtained by taking the absolute value of each entry in the adjacency matrix of the signed graph. We obtain a new H_∞-criterion which is sufficient for both cooperative output synchronization and bipartite output synchronization.

1. Introduction

Cooperative consensus of multi-agent systems has been studied widely in the literature (Olfati-Saber, Fax, & Murray, 2007). One particular interest is the Cooperative Output Synchronization (COS), where the outputs of the agents synchronize to each other or to a reference trajectory. There are several applications for COS like formation control, distributed control of UAVs, sensor networks, *etc.* (Huang & Ye, 2014; Olfati-Saber et al., 2007). However, in a number of contexts such as social networks, marketing or games the interactions among agents are not necessarily cooperative (Altafini, 2013), which are usually described by a signed graph, where positive and negative edge weights denote cooperation and competition among concerned nodes respectively.

One type of agreement over a signed graph is bipartite synchronization, where agents reach an agreement over the modulus of a variable. Bipartite Output Synchronization (BOS) studies output synchronization of the agents in modulus with possibly different signs. There are many engineering applications for BOS like analyzing trustworthiness of the nodes in a network (Ermon, Schenato, & Zampieri, 2009) and anticipating unanimity of the opinions in a decision process in the presence of stubborn agents (Altafini & Lini, 2015).

This paper studies a bipartite output synchronization problem in comparison with Altafini (2013), Valcher and Misra (2014) and Zhang and Chen (2014), which consider a bipartite state synchronization problem. In contrast to other existing works which have restrictive assumptions such as homogeneity of the agents (Valcher & Misra, 2014; Zhang & Chen, 2014), undirected communication graphs (Valcher & Misra, 2014) or first-order dynamics (Altafini, 2013), our framework allows heterogeneity of the agents, and a general directed and time-invariant signed communication graph.

This paper is an extended version of Adib Yaghmaie, Su, and Lewis (2016) and has two main contributions. Firstly, we prove that the H_∞-criterion for the COS problems reported in Adib Yaghmaie, Lewis and Su (2016) and Huang and Ye (2014) can be relaxed for some classes of communication graphs to ensure the existence of solutions for a larger set of problems. Secondly, we prove that the BOS problem is equivalent to the COS problem in the sense that a control solution to one problem induces a control solution to the other problem. In particular, that generalized H_∞ criterion introduced in the first contribution can also be applied to the BOS problem which is more relaxed than the H_∞ criterion reported in Adib Yaghmaie, Su et al. (2016).

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The rest of the paper is organized as follows. In Section 2 we introduce notations and preliminaries. In Section 3 we formulate the BOS and the COS problems, and show that they are equivalent via a novel transformation procedure. In Section 4 we propose a relaxed H_∞ criterion as a sufficient condition to ensure the existence of a solution to the COS problem, which is applicable to the BOS problem as well due to the aforementioned transformation procedure. Simulation results are shown in Section 5, and conclusions are drawn in Section 6.

2. Preliminaries

Let $\mathbb{R}^{n\times m}$ be the set of $n \times m$ real matrices. I_n, $\mathbf{1}_n$ and $\mathbf{0}$ denote the identity matrix of dimension $n \times n$, an n-dimensional column vector of 1, and a matrix of zeros with a compatible dimension, respectively. The Kronecker product of two matrices A and B is denoted as $A \otimes B$. Let $A_i \in \mathbb{R}^{n_i \times m_i}$ for $i = 1, \dots, N$. The operator $\text{Diag}_{1:N}\{A_i\}$ builds a block diagonal matrix with N diagonal blocks, whose ith diagonal block is A_i. The spectrum of matrix A is denoted by $spec(A)$ which is the multiset of its eigenvalues λ_i. The spectral radius of A is denoted as $\rho(A) = \max_{\lambda_i \in spec(A)} |\lambda_i|$. Given $A = [a_{ij}] \in \mathbb{R}^{n\times m}$, let $B := [A]_{n_1:n_2 \times m_1:m_2} \in \mathbb{R}^{(n_2-n_1+1)\times(m_2-m_1+1)}$ be a matrix formed by rows $n_1, \dots, n_2$ and columns $m_1, \dots, m_2$ of A. The cardinality of a set V is denoted by $|V|$. A disjoint union of two sets V^1 and V^2 is denoted by $V^1 \dot{\cup} V^2$. The following definition is used throughout the paper.

Definition 1 (Huang, 2004). A pair of $M_1 = I_p \otimes \beta$, $M_2 = I_p \otimes \tau$ incorporate a *p-copy* internal model of a square matrix A if (β, τ) is controllable and the minimal polynomial of A divides the characteristic polynomial of β. □

By Zaslavsky (1982), a *signed graph* is represented by a touple $\mathcal{G}^s = (V, E, \theta)$, where $V = \{v_0, \dots, v_N\}$ denotes a finite vertex set, $E \subseteq V \times V$ is a directed edge set, and $\theta : E \to \{+1, -1\}$ is a partial edge labeling function, which assigns either a positive or negative sign to each edge. We call $\mathcal{G}^u = (V, E)$ the corresponding *unsigned graph*. A (follower) subgraph of $\mathcal{G}^u$ obtained by removing the (leader) node v_0 can be represented by an $N \times N$ *adjacent* matrix $\mathcal{A}^u = [a^u_{ij}]$, where $a^u_{ij} = 1$ if $(v_j, v_i) \in E$, and $a^u_{ij} = 0$, otherwise. The adjacency of node v_0 and node v_i is denoted by a^u_{i0} and it is defined similarly. The *upper stream neighbor set* of a node $v \in V$ is defined as $N_v = \{v' \in V | (v', v) \in E\}$. The in-degree matrix F of that (follower) subgraph is defined as $F = \text{Diag}_{1:N}\{|N_{v_i}|\}$. The Laplacian of that (follower) subgraph is defined as $L^s = F - \mathcal{A}^s$, where $\mathcal{A}^s := [a^s_{ij} := \theta(v_j, v_i) a^u_{ij}]$ is the signed adjacent matrix. The signed pinning gain from the node v_0 to other nodes is denoted by the matrix $G^s = \text{Diag}_{1:N}\{g^s_i := \theta(v_0, v_i) a^u_{i0}\}$, and $G^u = \text{Diag}_{1:N}\{g^u_i := a^u_{i0}\}$ is the unsigned pinning gain. While the entries of the adjacency matrix $\mathcal{A}^u$ of the unsigned graph $\mathcal{G}^u$ are nonnegative, the entries of the adjacency matrix $\mathcal{A}^s$ of $\mathcal{G}^s$ can attain positive or negative values.

A directed graph is a *directed tree* if every node, except for one node called the root, has an in-degree equal to one, and the root node has its in-degree equal to zero, and in addition, each non-root node is reachable from the root node via a directed path. A directed graph has a *spanning tree* if it contains a directed tree over all nodes. A subgraph $\mathcal{G}^s_k = (V_k, E_k, \theta_k)$, where $V_k \subseteq V$, $E_k \subseteq E$ and θ_k being the restriction of θ over E_k, is called a *strongly connected subgraph* of $\mathcal{G}^s$ if each pair of different nodes $v_{ik}, v_{jk} \in V_k$ are reachable from each other via a directed path in the subgraph. In particular, a subgraph consisting of only one node, which is called a *single-node subgraph*, is always a strongly connected subgraph. $\mathcal{G}^s_k$ is *maximal* if there does not exist another strongly connected subgraph, which contains $\mathcal{G}^s_k$ as a subgraph.

Definition 2 (*Structurally Balanced Graph* Altafini, 2013). A signed graph $\mathcal{G}^s = (V, E, \theta)$ is *structurally balanced* if it admits a bipartition of the nodes, $V = V^1 \dot{\cup} V^2$, such that (i) for all $(v_i, v_j) \in E \cap (V^q \times V^q)$ with $q = 1, 2$, $\theta(v_i, v_j) = 1$; and (ii) for all $v_i \in V^q$, $v_j \in V^r$ with $(v_i, v_j) \in E$, $q, r \in \{1, 2\}, : q \neq r$, $\theta(v_i, v_j) = -1$. □

Let $\mathcal{D}$ be the set of gauge transformations $\mathcal{D} = \{\Sigma = \text{Diag}_{1:N}\{\sigma_i\} | \sigma_i \in \{\pm 1\}\}$. We define the following notations

$$H^s = \underset{1:N}{\text{Diag}}\left\{\frac{1}{|N_{v_i}| + g^u_i}\right\}(F - \mathcal{A}^s + G^u),$$
$$H^u = \underset{1:N}{\text{Diag}}\left\{\frac{1}{|N_{v_i}| + g^u_i}\right\}(F - \mathcal{A}^u + G^u). \tag{1}$$

Lemma 1 (Altafini, 2013; Zhang & Chen, 2014). *Let $\mathcal{G}^s = (V, E, \theta)$ be a signed graph which is structurally balanced with the bipartition $V = V^1 \dot{\cup} V^2$, and $\mathcal{G}^u = (V, E)$ be its unsigned equivalent. Then $\Sigma_1 \mathcal{A}^s \Sigma_1 = \mathcal{A}^u$ and $\Sigma_1 H^s \Sigma_1 = H^u$ if and only if $\Sigma_1 = \text{Diag}_{1:N}\{\sigma_i\} \in \mathcal{D}$, where for all $v_i \in V^q$, $v_j \in V^r$ with $q, r \in \{1, 2\}$, we have $\sigma_i = \sigma_j$ if and only if $q = r$.* □

Lemma 2 (Lewis, Zhang, Hengster-Movric, & Das, 2014). *Let a graph $\mathcal{G} = (V, E)$ contain K maximal strongly connected subgraphs $\mathcal{G}_k = (V_k, E_k)$, $k = 1, \dots, K$. One can reorder the nodes such that the adjacent matrix $\mathcal{A}$ of $\mathcal{G}$ is lower block triangular and its mth diagonal blocks is $\Xi_m \in \{\mathcal{A}_k | 1 \le k \le K\}$, where $\mathcal{A}_k$ is the adjacent matrix of $\mathcal{G}_k$.* □

3. Bipartite and cooperative output synchronization problems

Consider a group of $N + 1$ linear heterogeneous agents consisting of N followers labeled as $i = 1, \dots, N$ and a leader labeled as 0:

$$\dot{x}_i = A_i x_i + B_i u_i, \tag{2}$$
$$y_i = C_i x_i, \qquad z_i = D_i x_i, \quad i = 1, \dots, N \tag{3}$$
$$\dot{x}_0 = A_0 x_0, \qquad y_0 = C_0 x_0, \tag{4}$$

where $x_i \in \mathbb{R}^{n_i}, : y_i \in \mathbb{R}^p$, $u_i \in \mathbb{R}^{m_i}$ and $z_i \in \mathbb{R}^{q_i}$ are the state, the output, the control and the measured output of the agent i ($i = 0, \dots, N$), respectively. We make the following assumption.

Assumption 1. The signed graph $\mathcal{G}^s = (V, E, \theta)$ associated with the multi-agent system is structurally balanced.

Without loss of generality, let $\Sigma_1 = \text{Diag}_{1:N}\{\sigma_i\}$ be the gauge transformation introduced in Lemma 1, where $v_0 \in V^1$, $(\forall v_i \in V^1)\, \sigma_i = 1$, and $(\forall v_j \in V^2)\, \sigma_j = -1$.

Problem 1 (*Bipartite Output Synchronization (BOS) Problem*). Consider a group of $N + 1$ linear heterogeneous agents defined by (2)–(4). Assume that the agents communicate y_i's, over a structurally balanced signed graph $\mathcal{G}^s = (V, E, \theta)$. Design the static matrices $K_{1i} \in \mathbb{R}^{m_i \times q_i}$, $K_{2i} \in \mathbb{R}^{m_i \times n_{\eta_i}}$, $R_i \in \mathbb{R}^{n_{\eta_i} \times n_{\eta_i}}$, $S_i \in \mathbb{R}^{n_{\eta_i} \times p}$ for each $i = 1, \dots, N$, such that

$$u_i = K_{1i} z_i + K_{2i} \eta_i, \qquad \dot{\eta}_i = R_i \eta_i + S_i \delta_i, \quad \eta_i \in \mathbb{R}^{n_{\eta_i}} \tag{5}$$
$$\delta_i = \frac{1}{|N_{v_i}| + g^u_i}\left[\sum_{j=1}^N (a^u_{ij} y_i - a^s_{ij} y_j) + g^u_i y_i - g^s_i y_0\right],$$

render $\lim_{t\to+\infty} e_{bi}(t) = y_i(t) - \sigma_i y_0(t) = 0$. □

In this paper, we transform the BOS problem into another problem called cooperative output synchronization problem, which is defined below.