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Optimal dynamic formation control of multi-agent systems in constrained environments☆

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ABSTRACT

We address the optimal dynamic formation problem in mobile leader–follower networks where an optimal formation is generated to maximize a given objective function while continuously preserving connectivity. We show that in a convex mission space, the connectivity constraints can be satisfied by any feasible solution to a mixed integer nonlinear optimization problem. For the class of optimal formation problems where the objective is to maximize coverage, we show that the optimal formation is a tree which can be efficiently constructed without solving a MINLP. In a mission space constrained by obstacles, we separate the formation process into intervals with no obstacles detected and intervals where one or more obstacles are detected. In the latter case, we propose a minimum-effort reconfiguration approach for the formation which still optimizes the objective function while avoiding the obstacles and ensuring connectivity. We include simulation results illustrating this dynamic formation process.

1. Introduction

The multi-agent system framework consists of a team of autonomous agents cooperating to carry out complex tasks within a given environment that is potentially highly dynamic, hazardous, and even adversarial. The overall objective of the system may be time-varying and combines exploration, data collection, and tracking to define a "mission", see Cao, Yu, Ren, and Chen (2013), Cassandras and Li (2005), Choi, Oh, and Horowitz (2009) and Shamma (2008). In many cases, mobile agents are required to establish and maintain a certain spatial configuration, leading to a variety of *formation control* problems. These problems are generally approached in two ways: in the leader–follower setting, an agent is designated as a team leader moving on some given trajectory with the remaining agents tracking this trajectory while maintaining the formation; in the leaderless setting the formation must be maintained without any such benefit. Examples of formation control problems may be found in Desai, Kumar, and Ostrowski (1999), Ji and Egerstedt (2007), Wang and Xin (2013), Yamaguchi and Arai (1994) and references therein. In robotics, this is a well-studied problem; for instance in Yamaguchi and Arai (1994), a desired shape for a networked strongly connected group of robots is achieved by designing a quadratic spread potential field on a relative distance space. In Desai et al. (1999), a leader and several followers move in an area with obstacles which necessitate the transition from an initial formation shape to a desired new shape; however, the actual choice of formations for a particular mission is not addressed in Desai et al. (1999), an issue which is central to our approach in this paper. In Ji and Egerstedt (2007) the authors consider the problem of preserving connectivity when the nodes have limited sensing and communication ranges; this is accomplished through a control law based on the gradient of an edge-tension function. More recently, in Wang and Xin (2013), the goal is to integrate formation control with trajectory tracking and obstacle avoidance using an optimal control framework.

In this paper, we take a different viewpoint of formations. Since agent teams are typically assigned a mission, there is an objective (or cost) function associated with the team's operation which depends on the spatial configuration (formation) of the team. Therefore, we view a formation as the result of an optimization problem which the agent team solves in either centralized or distributed manner. We adopt a leader–follower approach, whereby the leader moves according to a trajectory that only he/she controls. During the mission, the formation is preserved or

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must adapt if the mission (hence the objective function) changes or if the composition of the team is altered (by additions or subtractions of agents) or if the team encounters obstacles which must be avoided. In the latter case in particular, we expect that the team adapts to a new formation which still seeks to optimize an objective function so as to continue the team's mission by attaining the best possible performance. The problem is complicated by the fact that such adaptation must take place in real time. Thus, if the optimization problem determining the optimal formation is computationally demanding, we must seek a fast and efficient control approach which yields possibly sub-optimal formations, but guarantees that the initial connectivity attained is preserved. Obviously, once obstacles are cleared, the team is expected to return to its nominal optimal formation.

Although the optimal dynamic formation control framework proposed here is not limited by the choice of tasks assigned to the team, we will focus on the dynamic coverage control problem because its static version is well studied and amenable to efficient distributed optimization methods; see Breitenmoser, Schwager, Metzger, Siegwart, and Rus (2010), Caicedo-Nuez and Zefran (2008a); Caicedo-Nunez and Zefran (2008b), Cassandras and Li (2005), Cortes, Martinez, Karatas, and Bullo (2004) and Zhong and Cassandras (2011), while also presenting the challenge of being generally non-convex and sensitive to the agent locations during the execution of a mission. The local optimality issue, which depends on the choice of objective function, is addressed in Gusrialdi, Dirza, Hatanaka, and Fujita (2013), Schwager, Bullo, Skelly, and Rus (2008) and Sun, Cassandras, and Gokbayrak (2014), while the problem of connectivity preservation in view of limited communication ranges is considered in Ji and Egerstedt (2007) and Zhong and Cassandras (2011).

The contribution of this paper is to formulate an optimization problem which jointly seeks to position agents in a two-dimensional mission space so as to optimize a given objective function while at the same time ensuring that the leader and remaining agents maintain a connected graph dictated by minimum distances between agents, thus resulting in an *optimal* formation. The minimum distances may capture limited communication ranges as well as constraints such as maintaining desired relative proximity between agents. We show that the solution to this problem guarantees such connectivity. For the class of optimal coverage control problems, we show that an optimal formation is a *tree* whose construction is much more computationally efficient than that of a general connected graph. The formation becomes *dynamic* as soon as the leader starts moving along a trajectory which may either be known to all agents in advance or determined only by the leader. Thus, it is the team's responsibility to maintain an optimal formation. We show that this is relatively simple as long as no obstacles are encountered. When one or more obstacles are encountered (i.e., they come within the sensing range of one or more agents), then we propose a scheme for adapting with minimal effort to a sequence of new formations which maintain connectivity while still seeking to optimize the original team objective.

The paper is organized as follows. In Section 2, we formulate a general optimal formation control problem and, for a convex feasible space, derive a mixed integer nonlinear optimization problem whose solution is shown to ensure connectivity while maintaining an optimal formation. In Section 3, we focus on optimal coverage control problems, prove that a tree is an optimal formation, and propose an algorithm to construct such a tree in a convex mission space. In Section 4, we address the optimal formation problem in a mission space with obstacles. We propose an algorithm to first obtain a connected formation and then optimize it while maintaining connectivity. Simulation results are included in Section 5.

2. Optimal formation problem

Consider a set of $N+1$ agents with a leader labeled 0 and N followers labeled 1 through N in a mission space $\Omega \in \mathbb{R}^2$. Agent i is located at $s_i(t) \in \mathbb{R}^2$ and let $\mathbf{s}(t) = (s_0(t), \ldots, s_N(t))$ be the full agent location vector at t. The leader follows a predefined trajectory $s_0(t)$ over $t \in [0, T]$ which is generally not known in advance by the remaining agents. We model the agent team as a directed graph $\mathscr{G}(\mathbf{s}) = (\mathscr{N}, \mathscr{E}, \mathbf{s})$, where $\mathscr{N} = \{0, 1, \ldots, N\}$ is the set of agent indices and let $\mathscr{N}_F = \{1, \ldots, N\} \subset \mathscr{N}$ be the set of follower indices. In this model, the set of edges $\mathscr{E} = \{(i,j) : i,j \in \mathscr{N}\}$ contains all possible agent pairs for which constraints may be imposed.

In performing a mission, let $H(\mathbf{s}(t))$ be an objective function dependent on the agent locations $\mathbf{s}(t)$. If the locations are unconstrained, the problem is posed as $\max_{\mathbf{s}(t)\in\Omega} H(\mathbf{s}(t))$ subject to dynamics that may characterize the motion of each agent. If t is fixed, then this is a nonlinear parametric optimization problem over the mission space Ω (Zhong & Cassandras, 2011). If, in addition, agents are required to satisfy some constraints relative to each other's position, then a *formation* is defined as a graph that satisfies these constraints. We then introduce a Boolean variable $c(s_i, s_j)$ to indicate whether two agents satisfy these constraints:

$$c(s_i, s_j) = \begin{cases} 1 & \text{all constraints are satisfied} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

and if $c(s_i, s_j) = 1$ we say that agents i and j are *connected*. A loop-free path from the leader to agent i, $\pi_i = \{0, \ldots, a, b, \ldots, i\}$, is defined as an ordered set where neighboring agents are connected such that $c(s_a, s_b) = 1$. Let Π_i be the set of all possible paths from i connected to the leader. The graph $\mathscr{G}(\mathbf{s})$ is connected if $\Pi_i \neq \emptyset$ for all $i \in \mathscr{N}_F$. We can now formulate an optimal formation problem with connectivity preservation as follows, for any fixed $t \in [0, T]$:

$$\begin{aligned} & \max_{\mathbf{s}(t)\in\Omega} H(\mathbf{s}(t)) \\ \text{s.t.}\quad & s_i(t) \in F \subseteq \Omega,\ i \in \mathscr{N}_F \\ & s_0(t) \text{ is given} \\ & \mathscr{G}(\mathbf{s}(t)) \text{ is connected.} \end{aligned} \tag{2}$$

For the sake of generality, we impose the constraint $s_i(t) \in F \subseteq \Omega$ for all follower agents to capture the possibility that a formation is constrained. The *feasible space* F can be convex (e.g., followers may be required to be located on one side of the leader relative to a line in Ω that goes through $s_0(t)$) or non-convex (e.g., followers may be forbidden to enter polygonal regions, possibly physical obstacles, and F is the set Ω excluding all interior points of these regions). The solution to this problem is an *optimal formation* at time t and is denoted by $\mathscr{G}^*(\mathbf{s}(t))$. Given a time interval $[t_1, t_2]$, the formation is *maintained* in $[t_1, t_2]$ if $s_i(t) - s_i(t_1) = s_0(t) - s_0(t_1)$ holds for all $t \in [t_1, t_2]$, $i \in \mathscr{N}_F$; otherwise, it is a new formation. Fig. 1 shows an example of optimal dynamic formation control in a mission space with obstacles. Clearly, this is a challenging problem. To begin with, the last constraint in (2) is imprecise and may be different in a convex or non-convex feasible space. In addition, the computational complexity of obtaining a solution may be manageable in determining an initial formation but becomes infeasible if a new formation $\mathscr{G}^*(\mathbf{s}(t))$ is required during the real-time execution of a mission. We first propose a general approach to solve this problem in a convex feasible space for arbitrary $H(\mathbf{s}(t))$. In the next section, we will limit ourselves to the class of optimal coverage problems in both convex and non-convex feasible spaces and show how to take advantage of the specific structure of $H(\mathbf{s}(t))$ in such cases.

In a convex feasible space, the simplest connection constraints are of the form $d_{ij}(t) \equiv \|s_i(t) - s_j(t)\| \leq C_{ij}$ for some pair (i,j), $i,j \in \{0, 1, \ldots, N\}$, where $C_{ij} > 0$ is a given scalar. This