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Economic model predictive control with extended horizon ☆

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ABSTRACT

In this work, we consider economic model predictive control (EMPC) with extended horizon based on an auxiliary controller. The extension of the horizon is realized by employing a terminal cost which characterizes the economic performance of the auxiliary controller over a finite terminal horizon. The proposed EMPC design is easy to construct and computationally efficient. We analyze the stability and performance of the proposed EMPC design with special attention paid to the impact of the terminal horizon. It is shown that for strictly dissipative systems satisfying mild assumptions, a finite terminal horizon is sufficient to guarantee the convergence and performance of the EMPC to be approximately upper-bounded by that of the auxiliary controller.

1. Introduction

Model predictive control (MPC), or receding horizon control, refers to a control methodology that approximates the solution of a constrained infinite-horizon optimal control problem by solving finite-horizon optimal control problems in a receding horizon fashion. Existing efforts to achieve closed-loop stability of conventional nonlinear model predictive control (NMPC) can be divided into two categories. One is to resort to the design of MPC by employing a point-wise terminal constraint (Mayne & Michalska, 1990; Nicolao, Magni, & Scattolini, 1996), terminal cost (with terminal region constraint) (Chen & Allgöwer, 1998; Jadbabaie & Hauser, 2005; Limón, Alamo, Salas, & Camacho, 2006), or Lyapunov-based constraint (Mhaskar, El-Farra, & Christofides, 2006). The other one is to rely on the inherent stability of conventional MPC by adopting a sufficiently large optimization horizon (Grimm, Messina, Tuna, & Teel, 2005; Grüne & Rantzer, 2008; Primbs & Nevistić, 2000). In general, the approaches under the first category have readily provable nominal stability but tend to be conservative. Either the optimality or the size of the feasibility region has to be compromised in order to explicitly handle the nonlinearity of the system. On the other hand, while the design-free MPCs under the second category are capable of achieving near-optimal solutions, they could be computationally prohibitive with a large control horizon.

A methodology that provides an ideal trade-off between the two categories is to extend the prediction horizon of NMPC based on an auxiliary controller or control law. In this way, the prediction horizon of the optimization problem can be increased without significantly increasing the computational effort. It is shown in Alamir and Bornard (1995) that a finite prediction horizon is sufficient to guarantee stability. In Magni, De Nicolao, and Scattolini (2001), a locally optimal linear controller is utilized to extend the prediction horizon of the NMPC design. The appealing features of this approach are that enlargement of the stability region and local optimality can be achieved without relying on a large control horizon. It is also worth mentioning that the separation between the control horizon and prediction horizon is not new. The concept arises along the early versions of MPC and is well embraced in industrial MPC (Qin & Badgwell, 2003).

In recent years, a new form of MPC, which is referred to as the economic model predictive control (EMPC) has attracted considerable academic attention. In EMPC, the quadratic-type cost functions used in conventional MPC are replaced with general economic cost functions that are not necessarily positive-definite with respect to the optimal steady state. Consequently, standard stability analysis techniques to use the value function of conventional MPC as a Lyapunov function is no longer viable. In fact, steady-state operation may not even be the economically optimal operation for EMPC. It has been realized that dissipativity plays an important role in characterizing the optimality of steady-state operation as well as establishing the stability of EMPC (Angeli, Amrit, & Rawlings, 2012; Muller, Angeli, & Allgower,

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2015; Müller, Grüne, & Allgöwer, 2015). Different EMPC designs have been proposed which stem from the conventional NMPC designs. For example, EMPC with point-wise terminal constraint (Diehl, Amrit, & Rawlings, 2011), EMPC with terminal cost (Amrit, Rawlings, & Angeli, 2011; Müller, Angeli, & Allgöwer, 2014), and EMPC with Lyapunov-based constraint (Ellis & Christofides, 2014; Heidarinejad, Liu, & Christofides, 2012). These EMPC designs also suffer the shortcomings of the conventional NMPC designs under the first category—they could be overly conservative or difficult to design. In another line of research (Grüne, 2013; Grüne & Stieler, 2014), EMPC without terminal conditions is studied. This line of research reveals some intrinsic properties of EMPC. It is shown that under certain controllability and dissipativity conditions, near-optimal performance can be achieved if a sufficiently large control horizon is used. However, a large control horizon could make online implementations of the EMPC impractical.

Due to these considerations, it is natural to also consider extending the prediction horizon of EMPC based on an auxiliary controller. An attempt was made in our previous work (Liu, Zhang, & Liu, 2015). In Liu et al. (2015), a terminal cost was employed which characterizes the economic performance of the system under a stabilizing controller over a finite time window referred to as the terminal horizon. The proposed EMPC is shown to be very computationally efficient and capable of achieving near-optimal asymptotic performance. However, stability and transient performance of the proposed EMPC are not addressed in Liu et al. (2015). It is conceivable that for general nonlinear systems, the system state does not necessarily converge to the optimal steady state under the proposed EMPC scheme with a finite terminal horizon. In the present work, we systematically discuss the stability and performance of EMPC with extended horizon based on an auxiliary controller. We show that for systems strictly dissipative with respect to the stage cost, a finite terminal horizon is sufficient to guarantee the convergence and performance of the EMPC to be approximately upper-bounded by that of the auxiliary controller. The results explain the computational efficiency of the EMPC framework and provide insights into the terminal cost design of EMPC.

The rest of this work is organized as follows: The system and EMPC formulation are set up in Section 2. Section 3 addresses the stability and convergence of the EMPC design. Practical stability is established for strictly dissipative systems satisfying mild assumptions. Under stronger conditions, the shrinkage of the region which the system state is eventually driven into is shown to be exponential with respect to the increase of the terminal horizon. For a special type of systems satisfying a further condition on the storage function (including conventional MPC with quadratic cost), exponential stability can be achieved. Interestingly, the same result for this type of systems may not be achieved by an EMPC without terminal condition. Section 4 discusses the asymptotic and transient performance of the EMPC design. Results on the asymptotic performance for general nonlinear systems are provided first. Stronger results on the transient performance for strictly dissipative systems are derived subsequently, based on different stability conditions from Section 3. In Section 5, two numerical examples are used to verify our analysis. Finally, we conclude our results in Section 6.

2. Problem setup

2.1. Notation

Throughout this work, the operator $|\cdot|$ denotes the Euclidean norm of a scalar or a vector. The symbol '$\backslash$' denotes set subtraction such that $\mathbb{A}\setminus\mathbb{B} := \{x \in \mathbb{A}, x \notin \mathbb{B}\}$. The symbol $\mathcal{B}_r(x_s)$ denotes the open ball centered at x_s with radius r such that $\mathcal{B}_r(x_s) := \{x : |x - x_s| < r\}$. A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and satisfies $\alpha(0) = 0$. A class $\mathcal{K}$ function α is called a class $\mathcal{K}_\infty$ function if α is unbounded. A continuous function $\sigma : [0, \infty) \to [0, a)$ is said to belong to class $\mathcal{L}$ if it is strictly decreasing and satisfies $\lim_{x\to\infty}\sigma(x) = 0$. A continuous function $\beta : [0, a)\times[0, \infty) \to [0, \infty)$ is said to belong to class $\mathcal{KL}$ if for each fixed r, $\beta(r, s)$ belongs to class $\mathcal{L}$, and for each fixed s, $\beta(r, s)$ belongs to class $\mathcal{K}$.

2.2. System description

We consider a class of nonlinear systems which can be described by the following discrete state-space model:

$$x(k+1) = f(x(k), u(k)) \tag{1}$$

where $x \in \mathbb{R}^{n_x}$ denotes the state vector and $u \in \mathbb{R}^{n_u}$ denotes the control input vector. The system state and input are subject to the constraints $x \in \mathbb{X}$ and $u \in \mathbb{U}$ respectively, where $\mathbb{X} \subset \mathbb{R}^{n_x}$ and $\mathbb{U} \subset \mathbb{R}^{n_u}$ are compact sets. We assume that there exists an optimal steady state (x_s, u_s) that uniquely solves the following steady-state optimization problem:

$$\begin{aligned}(x_s, u_s) = \quad & \arg\min_{x,u}\ l(x, u)\\ \text{s.t.}\quad & x = f(x, u)\\ & x \in \mathbb{X}\\ & u \in \mathbb{U}\end{aligned} \tag{2}$$

where $l(x, u) : \mathbb{X}\times\mathbb{U} \to \mathbb{R}$ is the economic stage cost function.

2.3. EMPC based on an auxiliary controller

It is assumed that there exists an auxiliary explicit controller $u = h(x)$ which renders x_s asymptotically stable with $u_s = h(x_s)$ while satisfying the input constraint for all $x \in \mathbb{X}_f$, where $\mathbb{X}_f \subseteq \mathbb{X}$ is a compact set containing x_s in its interior. It is also assumed that the region $\mathbb{X}_f$ is forward invariant under the controller $u = h(x)$. Namely, $f(x, h(x)) \in \mathbb{X}_f$ holds for all $x \in \mathbb{X}_f$. We use $x_h(k, x)$ to denote the closed-loop state trajectory under the controller h at time instant k with the initial state $x_h(0, x) = x$. The above assumptions imply that there exists a class $\mathcal{KL}$ function β_x such that:

$$\begin{aligned}|x_h(k, x) - x_s| &\le \beta_x(|x - x_s|, k)\\ x_h(k, x) &\in \mathbb{X}_f\\ h(x_h(k, x)) &\in \mathbb{U}\end{aligned} \tag{3}$$

for all $k \ge 0$ and $x \in \mathbb{X}_f$.

Our EMPC design takes advantage of the auxiliary controller $h(x)$ to extend the prediction horizon. Specifically, this is implemented by employing the following terminal cost $V_f(x, N_h)$, which characterizes the economic performance of the controller $h(x)$ for N_h steps with the initial state $x \in \mathbb{X}_f$:

$$V_f(x, N_h) = \sum_{k=0}^{N_h-1} l(x_h(k, x), h(x_h(k, x))).$$

At a time instant n, our EMPC design is formulated as the following optimization problem $\mathcal{P}(n)$:

$$\min_{u(0),\ldots,u(N-1)} \sum_{k=0}^{N-1} l(\tilde{x}(k), u(k)) + V_f(\tilde{x}(N), N_h) \tag{4a}$$

$$\text{s.t. } \tilde{x}(k+1) = f(\tilde{x}(k), u(k)), \quad k = 0, \ldots, N-1 \tag{4b}$$

$$\tilde{x}(0) = x(n) \tag{4c}$$

$$\tilde{x}(k) \in \mathbb{X}, \quad k = 0, \ldots, N-1 \tag{4d}$$

$$u(k) \in \mathbb{U}, \quad k = 0, \ldots, N-1 \tag{4e}$$

$$\tilde{x}(N) \in \mathbb{X}_f \tag{4f}$$

where $\tilde{x}(k)$ denotes the predicted state trajectory, $x(n)$ is the state measurement at time instant n. The optimal solution to the above