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Brief paper

Time-varying formation control for general linear multi-agent systems with switching directed topologies ☆

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ABSTRACT

Time-varying formation analysis and design problems for multi-agent systems with general linear dynamics and switching directed interaction topologies are investigated. Different from the previous results, the formation in this paper can be defined by specified piecewise continuously differentiable vectors and the switching topologies are directed. Firstly, necessary and sufficient conditions for general linear multi-agent systems with switching directed topologies to achieve time-varying formations are proposed, where a description of the feasible time-varying formation set and approaches to expand the feasible formation set are given. Then an explicit expression of the time-varying formation reference function is derived to describe the macroscopic movement of the whole formation. An approach to assign the motion modes of the formation reference is provided. Moreover, an algorithm consisting of four steps to design the formation protocol is presented. In the case where the given time-varying formation belongs to the feasible formation set, it is proven that by designing the formation protocol using the proposed algorithm, time-varying formation can be achieved by multi-agent systems with general linear dynamics and switching directed topologies if the dwell time is larger than a positive threshold. Finally, numerical simulations are presented to demonstrate the effectiveness of the theoretical results.

1. Introduction

Cooperative control of multi-agent systems has received significant attention from both scientific and engineering communities in recent years. This research field includes consensus control (Ren & Beard, 2005; Zhou & Lin, 2014), rendezvous control (Dong & Huang, 2014; Zavlanos, Tanner, Jadbabaie, & Pappas, 2009), containment control (Ji, Ferrari-Trecate, Egerstedt, & Buffa, 2008; Notarstefano, Egerstedt, & Haque, 2011) and formation control (Navaravong, Kan, She, & Dixon, 2012; Wang, Huang, Wen, & Fan, 2014), etc. As one of the most important research topics, formation control of multi-agent systems has broad range of applications in various areas, such as unmanned aerial vehicles (Dong, Yu, Shi, & Zhong, 2015; Karimoddini, Lin, Chen, & Lee, 2013), mobile robots (Yoo & Kim, 2015; Zheng, Lin, Fu, & Sun, 2015), autonomous underwater vehicles (Leonard et al., 2010; Wang, Yan, & Li, 2012). As a matter of fact, formation control problems have been studied a lot in robotics community during the past decades, and three formation control approaches, namely, leader–follower based approach (Das, Fierro, Kumar, & Ostrowski, 2002), behavior based approach (Balch & Arkin, 1998) and virtual structure based approach (Lewis & Tan, 1997), have been proposed.

One of the main challenges in formation control of multi-agent systems lies in the fact that each agent usually cannot rely on centralized coordination and has to use local information to achieve the desired formation (see the latest survey paper (Oh, Park, & Ahn, 2015) for more details). Ren (2007) proposed a consensus based formation control approaches for second-order multi-agent systems and proved that leader–follower, behavior and virtual structure based approaches can be unified in the framework of consensus based approaches. Consensus or graph based formation control problems for first-order and second-order multi-agent systems were studied in Antonelli, Arrichiello, Caccavale, and Marino (2014), Du, Li, and Lin (2013), Guo, Zavlanos, and Dimarogonas (2014), Guzey, Dierks, and Jagannathan (2015), Hawwary (2015), Liu and Jiang (2013), Lin, Wang, Han, and Fu (2015), Mylvaganam and Astolfi (2015), Tian and Wang (2013),

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Xiao, Wang, Chen, and Gao (2009) and Wang, Xie, and Cao (2014). In some practical applications, the dynamics of each agent can only be described by high-order model. Results on time-invariant formation control of high-order linear multi-agent systems can be found in Fax and Murray (2004), Lafferriere, Williams, Caughman, and Veerman (2005), Ma and Zhang (2012) and Porfiri, Roberson, and Stilwell (2007). Time-varying formation control problems for high-order linear multi-agent systems with fixed and switching undirected interaction topologies were addressed in Dong, Shi, Lu, and Zhong (2014) and Dong, Xi, Lu, and Zhong (2014), respectively. In practice, undirected interaction topologies mean that the communication among agents is bidirected, which may consume twice the communication and energy resources used in the directed links. Directed interaction among agents is more practical. Therefore, it is meaningful to study time-varying formation control problems for general linear multi-agent systems with switching directed interaction topologies. Moreover, the Laplacian matrix for undirected interaction topology is symmetric while the Laplacian matrix for directed interaction topology does not have the symmetric structure, and the eigenvalues of the Laplacian matrix can be complex, which makes the analysis and design much complicated.

Motivated by the facts and challenges stated above, in this paper, time-varying formation analysis and design problems for multi-agent systems with general linear dynamics and switching directed interaction topologies are investigated. Compared with previous results on formation control, the contributions of the current paper are threefold. Firstly, the formation can be time-varying and each agent has general linear dynamics. In Du et al. (2013), Fax and Murray (2004), Lafferriere et al. (2005), Liu and Jiang (2013) Ma and Zhang (2012), Porfiri et al. (2007), Ren (2007), Tian and Wang (2013) and Xiao et al. (2009), the formation is assumed to be time-invariant. Because the time-varying formation will bring the derivative of the formation information to both the analysis and design, the results for time-invariant formations cannot be directly applied to time-varying formations. In Antonelli et al. (2014), Hawwary (2015) and Wang et al. (2014), the formation can be time-varying, but the dynamics of each agent is first-order. Secondly, the interaction topology can be switching and directed, and each possible topology only needs to have a spanning tree. However, the topologies in Dong et al. (2014) is fixed. Although the topologies in Dong et al. (2014) can be switching, each topology is required to be undirected and connected. The common Lyapunov functional approach used in Dong et al. (2014) cannot be applied to solve the switching directed topology problems in the current paper. Thirdly, a description of the feasible time-varying formation set and an explicit expression of the time-varying formation reference function are derived. It is revealed that switching topologies, dynamics of each agent, initial states of all the agents and the time-varying formation have effects on the macroscopic movement of the whole formation.

2. Preliminaries and problem description

In this section, firstly, basic notations, definitions and useful results on graph theory are introduced. Then the problem description is presented.

2.1. Preliminaries

Some notations used in this paper are given as follows. Let 0 and **1** be appropriate zero matrix and column vector of ones with scalar 0 and scalar 1 as special cases. Let I_N represent an identity matrix with dimension N, and $\otimes$ denote the Kronecker product. We use the superscript T to denote the transpose of a matrix.

The interaction topology of the general linear multi-agent system will be described by the directed graph $G = \{Q, E, W\}$, where $Q = \{q_1, q_2, \ldots, q_N\}$ is a node set, $E \subseteq \{(q_i, q_j) : q_i, q_j \in Q\}$ is an edge set, and $W = [w_{ij}] \in \mathbb{R}^{N\times N}$ is a weighted adjacency matrix. An edge in G is denoted by $q_{ij} = (q_i, q_j)$ $(i \neq j)$. The adjacency elements in W satisfy that $w_{ji} > 0$ if and only if $q_{ij} \in E$, and $w_{ij} = 0$ otherwise. Moreover, $w_{ii} = 0$ for all $i \in \{1, 2, \ldots, N\}$. The neighbor set of node q_i is denoted by $N_i = \{q_j \in Q : q_{ji} \in E\}$. Define the in-degree of node q_i as $\deg_{in}(q_i) = \sum_{j=1}^{N} w_{ij}$. Let $D = \operatorname{diag}\{\deg_{in}(q_i), i = 1, 2, \ldots, N\}$ be the degree matrix of G. The Laplacian matrix $L \in \mathbb{R}^{N\times N}$ of G is defined as $L = D - W$.

The directed interaction topology of the multi-agent system is assumed to be switching and there exists an infinite sequence of uniformly bounded non-overlapping time intervals $[t_k, t_{k+1})$ $(k \in \mathbb{N})$, with $t_1 = 0$, $0 < \tau_0 \leqslant t_{k+1} - t_k \leqslant \tau_1$, and $\mathbb{N}$ being the set of natural numbers. The time sequence t_k $(k \in \mathbb{N})$ is called the switching sequence, at which the interaction topology changes. τ_0 is named as the dwell time, during which the interaction topology keeps fixed. Let $\sigma(t) : [0, +\infty) \to \{1, 2, \ldots, p\}$ be a switching signal whose value at time t is the index of the topology. Define $G_{\sigma(t)}$ and $L_{\sigma(t)}$ as the corresponding interaction topology and Laplacian matrix at $\sigma(t)$. Let $N^i_{\sigma(t)}$ be the neighbor set of the ith agent at $\sigma(t)$. If for a given $\sigma(t) \in \{1, 2, \ldots, p\}$, $G_{\sigma(t)}$ has a spanning tree, then the Laplacian matrix $L_{\sigma(t)}$ has the following property.

Lemma 1 *(Ren & Beard, 2005). If $G_{\sigma(t)}$ contains a spanning tree, then zero is a simple eigenvalue of $L_{\sigma(t)}$ with associated right eigenvector* **1***, and all the other $N - 1$ eigenvalues have positive real parts.*

2.2. Problem description

Consider a group of N agents. Suppose that each agent has the general linear dynamics described by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i \in \{1, 2, \ldots, N\}, \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $x_i(t) \in \mathbb{R}^n$ and $u_i(t) \in \mathbb{R}^m$ are the state and the control input of the ith agent, respectively.

Assumption 1. The matrix B is of full column rank, i.e., rank$(B) = m$.

Remark 1. As shown in Li, Ren, Liu, and Fu (2013), Yang, Wang, Hung, and Gani (2006) and Zhang, Feng, Qiu, and Shen (2013), Assumption 1 is standard and mild, which means that the columns of B are independent with each other and there exist no redundant control input components.

Assumption 2. Each possible topology $G_{\sigma(t)}$ contains a spanning tree.

The desired time-varying formation is specified by vector $h(t) = [h_1^T(t), h_2^T(t), \ldots, h_N^T(t)]^T \in \mathbb{R}^{nN}$ with $h_i(t)$ $(i = 1, 2, \ldots, N)$ piecewise continuously differentiable. It should be pointed out that $h(t)$ is only used to characterize the desired time-varying formation rather than providing reference trajectory for each agent to follow (see, e.g., Dong et al., 2015 for more details).

Definition 1. Multi-agent system (1) is said to achieve time-varying formation $h(t)$ if for any given bounded initial states, there exists a vector-valued function $r(t) \in \mathbb{R}^n$ such that $\lim_{t\to\infty}(x_i(t) - h_i(t) - r(t)) = 0$ $(i = 1, 2, \ldots, N)$, where $r(t)$ is called the formation reference function.