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Brief paper

Minimum upper-bound filter of Markovian jump linear systems with generalized unknown disturbances[☆]

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ABSTRACT

This paper presents the estimation problem of Markovian jump linear systems (MJLSs) with generalized unknown disturbances (GUDs). There exist multiple uncertainties including Markovian switching parameters and GUDs, along with traditional random noises. Here, the state transition of MJLS is treated as the jump from one vertex to another on a fixed polyhedron whose vertex represents a mode. Since the transition is dependent on stochastic Markovian switching parameter, a more general polytopic system with stochastic weights is considered and the corresponding upper-bound filter (UBF) is derived. Then, the MJLS with GUDs is transformed into a special case of the considered polytopic system by letting the corresponding stochastic weight as the binary value constructed by Markovian switching parameters and hence the recursive UBF is obtained. The parameters in the derived UBF are further optimized in pursuit of the minimum upper bounds of estimation error covariances. The simulation via maneuvering target tracking shows the effectiveness of the proposed filter.

1. Introduction

Markovian jump linear systems (MJLSs) have wide and successful applications in many fields, for example, target tracking (Boers & Driessen, 2005; Li & Jilkov, 2005), fault-tolerant control (Li, Gao, Shi, & Zhao, 2014; Liu, Ho, & Shi, 2015), process control (Xiong, Lam, Gao, & Ho, 2005), and signal processing (Johnston & Krishnamurthy, 2001). In general, the estimation issue concerning MJLSs is in the framework of the multiple model (MM) and there are three generations of MM estimators with the last two generations for the MJLSs (Lan, Li, & Mu, 2011; Li & Jilkov, 2005). However, all these MM methods need Gaussian assumptions on both process noises and measurement noises. In some situations, such Gaussian assumption does not always hold. For example, in maneuvering target tracking, the significant attitude changes bring out significant variations of radar reflections, leading to high-tailed and non-Gaussian (also called "glint") measurement noises (Bilik & Tabrikian, 2010). Meanwhile, the recursive calculation of the first two moments of the interested vector sometimes is enough in practice. For instance, target tracking focuses on estimating the state including position and velocity (the first moment) and covariance (the second moment). In other words, calculating the conditional state distribution given measurements is not only too computation-intensive but also may not be necessary in the view of desirable balance between estimation accuracy and computation burden. Thus, it motivates the development of linear minimum-mean-square-error (LMMSE) estimator for the MJLS.

In Costa (1994), the LMMSE estimator for the MJLS was derived from geometric augment based on directly estimating $x_k 1_{\{\Theta_k=i\}}$ instead of the state x_k, where $1_{\{\Theta_k=i\}}$ is an indicator being 1 if $\Theta_k = i$ or 0 otherwise; Θ_k is the state of Markov chain. And its error covariance can converge to the unique positive-semidefinite solution of an Nn-dimensional algebraic Riccati equation under the conditions of mean square stability of the MJLS and the ergodicity of the associated Markov chain, where n is the dimension of the state vector and N is the number of states of the Markov chain. Furthermore, a time-invariant (a fixed-gain matrix) LMMSE estimator was derived for MJLSs (Costa, 2002). By the fact that roundoff errors in solving the above Riccati equation may cause the loss of the symmetry and positive-semidefinition, an

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array implementation with the better numerical robustness was developed (Terra, Ishihara, & Junior, 2007). Recently, the LMMSE estimations of MJLSs are also extended to the cases of stochastic coefficient matrices (Yang, Liang, Pan, Qin, & Yang, 2014) and randomly delayed measurements (Yang, Liang, Yang, Qin, & Pan, 2014).

However, all these methods for the MJLSs, including the MM methods and the LMMSE methods, never consider the presence of unknown disturbances (UDs). In fact, the UDs exist in many actual applications (Qin, Liang, Yang, Wang, & Yang, 2014). In target tracking, the sensor bias and deception jamming existing in sensor measurements can be modeled as the UDs to the nominal model (Greco, Gini, & Farina, 2008; Qin et al., 2014). In process control, the faults or failures can also be represented by UDs to the fault-free model in fault detection and isolation (Li et al., 2014; Liu et al., 2015).

By the fact that many actual applications in the complex environment always face multiple unpredictable disturbances/ uncertainties, this paper formulates the estimation problem of MJLSs with generalized unknown disturbances (GUDs) in measurements. Here, the upper-bound filter (UBF) for the more general polytopic system with stochastic weights (regarded as stochastic parameters) and GUDs is first derived by constructing the upper bounds of covariances of estimation errors. Then, the MJLS with GUDs is transformed into a special case of above polytopic system via parameter substitution and the UBF is obtained recursively through calculating the correlated relationship about Markovian switching parameters. Furthermore, the optimal parameters are derived in pursuit of the minimum upper bounds. The simulation about the maneuvering target tracking validates the proposed filter.

Throughout this paper, I and O are the identity matrix and zero matrix with proper dimensions, respectively. $(\cdot)$ denotes the same content as that in the previous parenthesis and $[\cdot]_{i,j}$ represents the (i,j)th sub-block of the corresponding matrix. $E(\cdot)$ and 'col' represent the mathematical expectation operator and column vector, respectively. For any two square matrices A and B, $A \geq B (A > B)$ means $A - B$ is positive semi-definite (positive definite). The symbol ':=' means definition and '$\otimes$' refers to the Kronecker product. An indicator function $1_{\{\Theta_k=j\}}$ will be 1 if $\Theta_k = j$ or 0 otherwise.

The rest of this paper is organized as follows. The problem formulation is presented in Section 2. The UBF and the MUBF are given in Section 3. A simulation about maneuvering target tracking is presented in Section 4 to testify the proposed method. The conclusion is finally made. All proofs are presented in Appendix.

2. Problem formulation

In maneuvering target tracking in the electronic countermeasures (ECMs) environment, the target maneuvering motion is usually described by Markovian switching of multiple models (Li & Jilkov, 2005) while the sensor bias, deception jamming and linearization approximation lead to the time-varying GUDs in measurements (Qin et al., 2014). In other words, there coexist the Markovian switching parameter and GUDs, which motivates us to formulate the following discrete-time MJLS with GUDs in measurements:

$$x_{k+1} = F_{\Theta_k} x_k + G_{\Theta_k} w_k, \tag{1}$$

$$z_k = H_{\Theta_k} x_k + A_k \delta_k + D_{\Theta_k} v_k, \tag{2}$$

where x_k and z_k represent the system state and measurement, respectively. $\{\Theta_k\}$ is the state of Markov chain with finite state space $\{1, \ldots, M\}$ and transition probability matrix P_t with its (i,j)th element being $p_{ij} := P\{\Theta_{k+1} = j | \Theta_k = i\}$. $\pi_{j,k} := P(\Theta_k = j)$ represents the jth mode probability at instant k. F_{Θ_k}, G_{Θ_k}, H_{Θ_k}, A_k and D_{Θ_k} are known matrices with proper dimensions. w_k and v_k are zero-mean and white noises with covariances Q_k and R_k, respectively, and independent of the initial state x_0 satisfying $E(x_0 1_{\{\Theta_0=i\}}) = \psi_{i,0}$ and $E(x_0 x_0^T 1_{\{\Theta_0=i\}}) = V_{i,0}$. Here, $\{w_k\}$, $\{v_k\}$ and $\{\Theta_k\}$ are independent mutually, and δ_k satisfies

$$\begin{cases} E\left\{\delta_k w_{l-1}^T\right\} = 0 \\ E\left\{\delta_k v_l^T\right\} = 0 \end{cases} \quad (\forall\, l \geq k). \tag{3}$$

As shown in Qin et al. (2014), δ_k represents a more general uncertainty (i.e. GUD): an arbitrary linear weighted sum of f_{1k}, f_{2k} and q_k, where f_{1k} representing a class of UD with dynamic property is a linear time-varying function of $\bar{W}^{k-1}$, $\bar{V}^k$, and $\bar{\Delta}^{k-1}$ with $\bar{W}^{k-1} := \left[w_0^T, \ldots, w_{k-1}^T\right]^T$, $\bar{V}^k := \left[v_1^T, \ldots, v_k^T\right]^T$ and $\bar{\Delta}^{k-1} := \left[\delta_1^T, \ldots, \delta_{k-1}^T\right]^T$; f_{2k} representing deterministic UD is an arbitrary deterministic time-varying function; and q_k representing random UD is white noise independent of $\bar{W}^{k-1}$, $\bar{V}^k$, and $\bar{\Delta}^{k-1}$.

Due to the presence of δ_k, the routine of orthogonality principle for designing the LMMSE estimator will not work unless δ_k or its effect on estimation error covariance can be identified. In fact, such identification requires certain conditions which may be hardly satisfied:

- to identify the value of the GUD, the precondition is that the dimension of δ_k should be less than the rank of measurement matrix. Otherwise, the GUD has to be further constrained being piece-wise constant;
- to obtain the optimal filter gain $K = P_{xz} P_{zz}^{-1}$, we need to estimate the cross-covariance P_{xz} between the predicted state and measurement and the innovation covariance P_{zz}. However, it is an intractable task due to the unknown relationship among δ_k and δ_t, w_{t-1} or v_t for $t < k$.

It is highly demanded to develop a new filter with the looser design condition in pursuit of the best result in the worst situation.

Definition 1 (*Definition of the UBF*)**.** A filter is called the UBF, i.e.,

$$\left\{\hat{\xi}_{k+1|k+1}, \Phi^*_{k+1|k}, S^*_{k+1}, \Phi^*_{k+1|k+1}\right\} = \mathrm{UBF}\left\{z_{k+1}, \hat{\xi}_{k|k}, \Phi^*_{k|k}\right\}$$

if there exists a sequence of positive-definite matrices $\Phi^*_{k+1|k}$, S^*_{k+1} and $\Phi^*_{k+1|k+1}$ that satisfy

$$\Phi^*_{k+1|k} \geq \Phi_{k+1|k} := E[(\xi_{k+1} - \hat{\xi}_{k+1|k})(\cdot)^T], \tag{4}$$

$$S^*_{k+1} \geq S_{k+1} := E[(z_{k+1} - \hat{z}_{k+1|k})(\cdot)^T], \tag{5}$$

$$\Phi^*_{k+1|k+1} \geq \Phi_{k+1|k+1} := E[(\xi_{k+1} - \hat{\xi}_{k+1|k+1})(\cdot)^T], \tag{6}$$

where the geometry augmentation $\xi_k := \mathrm{col}\{x_k 1_{\{\Theta_k=i\}}, i = 1, \ldots, M\}$; $\hat{\xi}_{k|k}$ and $\hat{\xi}_{k+1|k}$ are the estimate and prediction of ξ_k given $Z_{1:k} := \{z_1, \ldots, z_k\}$, respectively; $\hat{z}_{k+1|k}$ is the measurement prediction; $\Phi_{k+1|k}$, S_{k+1} and $\Phi_{k+1|k+1}$ are the covariances of the prediction error, innovation and estimate error, respectively; $\Phi^*_{k+1|k}$, S^*_{k+1} and $\Phi^*_{k+1|k+1}$ are the corresponding upper bounds to be determined.

Our aim is to construct the upper bounds with one or more free parameters to guarantee the upper-boundness in (4)–(6) and further optimize parameters for the minimum upper bounds. As shown later, the adaptive estimation problem with GUDs is transformed into the online constrained parameter optimization.

Remark 2.1. The common idea of both the UBF and robust H_∞ filters (Xiong & Lam, 2006; Zhang, 2009; Zhang & Boukas, 2009; Zhao & Zeng, 2010) is to obtain the best accuracy in the worst case. However, the considered estimation problem cannot be solved by the H_∞ filter because: