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Brief paper

State estimation of linear systems in the presence of sporadic measurements☆

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ABSTRACT

This paper deals with the state estimation of linear time-invariant systems for which measurements of the output are available sporadically. To solve the considered problem, we provide an observer with jumps triggered by incoming measurements, which is studied in a hybrid systems framework. Specifically, the resulting system is written in estimation error coordinates and augmented with a timer variable that triggers the event of new measurements arriving. Then, the observer design is performed to achieve global exponential stability (GES) of a closed set including the points for which the state of the plant and its estimate coincide. Furthermore, a computationally tractable design procedure for the proposed observer is presented. Finally, the effectiveness of the proposed methods is demonstrated in two numerical examples.

1. Introduction

Recent technological advances have enabled the control of dynamical systems using data that is transmitted over communication networks. When the networks are not fully reliable, data can get lost or can only be available intermittently (Hespanha, Naghshtabrizi, & Xu, 2007; Hristu-Varsakelis & Levine, 2005; Walsh, Hong, & Bushnell, 2002). In settings where the controller and the system to control are connected through a network, the classical estimation paradigms of accessing the output of the plant continuously (Luenberger, 1971) or discretely at desired time instances (Chen, Francis, & Hagiwarac, 1998) do not apply and new approaches are required. The limitations of these paradigms have lead to recent efforts in the literature focusing on the impact of intermittent availability of resources in control and estimation. Such works include Xu and Hespanha (2005) where state estimation algorithms for networked control systems under the presence of both uncontrolled and controlled sampling are proposed, Postoyan and Nešić (2012) where a general framework for state estimation for nonlinear networked systems is considered, and Dačić and Nešić (2008) where the design, via linear matrix inequalities, of state observer–protocol pairs for linear networked systems is presented (in the presence of periodic measurements).

This paper is concerned with the modeling and design of an observer to exponentially estimate the state of a linear time-invariant plant in the presence of sporadically available measurements. As the classical paradigm of continuously measuring the output does not apply to our setting, a suitable observation scheme is needed. To overcome this problem, one may adopt an emulation approach, that is, design an observer while ignoring the lack of continuous information on the measured output, and then replace the measured output by a suitable estimate generated by the most recent measured value. This approach is adopted, for example, in Postoyan and Nešić (2012). Another possibility relies on the design of a robust (with respect to bounded sampling time variations) discrete-time observer for the discretized version of the plant. Such an approach can be built upon the results presented, *e.g.*, in Halimi, Millerioux, and Daafouz (2013). In this paper, we pursue a different approach. Specifically, building from the idea proposed in Andrieu, Nadri, Serres, and Vivalda (2013) and Raff and Allgöwer (2007) and assuming that the plant input is known, we consider an open-loop observer along with a suitable event-triggered update law to reset

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instantaneously the state of the observer. Essentially, the proposed observer has a state that undergoes a jump whenever a new measurement is available. Since the evolution of the proposed observer exhibits both continuous-time behavior and instantaneous changes (not necessarily periodic), we provide a hybrid model capturing the dynamics of the observer interconnected to the plant. A unique feature of this model is that it is not deterministic; in fact, a set-valued update law is proposed to capture all possible events within a given bounded range. Then, using Lyapunov theory for hybrid systems, we propose a condition that guarantees global exponential stability of a set of points in which the estimation error is zero as well as robustness with respect to bounded perturbations (in an input-to-state stability sense; see Cai & Teel, 2009 and Sontag, 1989). The proposed approach based on hybrid modeling allows us to effectively exploit the properties of the time domain of the solutions to the resulting hybrid system, in particular, the persistence of jumps. This feature not only provides a better understanding of the system behavior but also enables us to construct a Lyapunov function to certify global exponential stability and characterize the effect of measurement noise via input-to-state stability.

As a second step, our condition guaranteeing global exponential stability is exploited to derive a design algorithm for the proposed observer. To this end, the said condition is first rewritten as a parametric linear matrix inequality. Then, by means of a novel polytopic embedding technique, the obtained parametric linear matrix inequality is turned into a finite number of linear matrix inequalities, whose solution defines suitable parameters for the proposed observer. This result provides a constructive design procedure, which is computationally tractable.

The paper is organized as follows. Section 2 presents the system under consideration, the state estimation problem we solve, and the hybrid modeling of the proposed observer. Section 3 is dedicated to the main results. Section 4 is devoted to numerical issues about the observer synthesis and provides a design algorithm based on linear matrix inequalities for the observer gain. Finally, Section 5 shows the effectiveness of the results presented in two numerical examples.

Notation: The set $\mathbb{N}_0$ is the set of the positive integers including zero, $\mathbb{N}$ is the set of the positive integers, and $\mathbb{R}_{\geq 0}$ represents the set of the nonnegative real scalars. For every complex number ω, $\mathfrak{Re}(\omega)$ and $\mathfrak{Im}(\omega)$ stand respectively for the real and the imaginary part of ω. For a matrix $A \in \mathbb{R}^{n\times m}$, A' denotes the transpose of A, $\|A\|$ denotes the induced 2-norm, and $\mathrm{He}(A) = A + A'$. For two symmetric matrices, A and B, $A > B$ means that $A - B$ is positive definite. In partitioned symmetric matrices, the symbol $\star$ stands for symmetric blocks. For a vector $x \in \mathbb{R}^n$, $\|x\|$ denotes the Euclidean norm. Given two vectors x, y, we denote $(x, y) = [x' \ y']'$. Given a set X, $\mathrm{co}\,X$ represents the convex hull of X. Given a vector $x \in \mathbb{R}^n$ and a closed set $\mathcal{A} \subset \mathbb{R}^n$, $|x|_{\mathcal{A}} = \inf_{y\in\mathcal{A}} \|x - y\|$. For any function $z : \mathbb{R} \to \mathbb{R}^n$, we denote $z(t^+) := \lim_{s\to t^+} z(s)$. A function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is said to belong to class $\mathcal{K}$ if it is continuous, zero at zero, and strictly increasing. A function $\beta : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is said to belong to class $\mathcal{KL}$ if it is nondecreasing in its first argument, nonincreasing in its second argument, and $\lim_{s\to 0^+} \beta(s, t) = \lim_{t\to\infty} \beta(s, t) = 0$. Given a function $f : X \to \mathbb{R}$, ess. sup stands for its essential supremum. For a given real interval $\mathcal{I}$, and a real matrix $A \in \mathbb{R}^{n\times n}$, we denote $e^{A\mathcal{I}} := \{Y \in \mathbb{R}^{n\times n} : \exists v \in \mathcal{I} \text{ such that } Y = e^{Av}\}$.

2. Problem statement

2.1. System description

We consider continuous-time linear time-invariant systems of the form

$$\begin{aligned} \dot z &= Az + Bu \\ y &= Mz \end{aligned} \tag{1}$$

where $z \in \mathbb{R}^n$, $y \in \mathbb{R}^q$, and $u \in \mathbb{R}^p$ are, respectively, the state, the measured output, and the input of the system, while A, B and M are constant matrices of appropriate dimensions. We assume that the input u belongs to the class of measurable and locally bounded functions $u : [0, \infty) \to \mathbb{R}^p$. Our goal is to design an observer providing an estimate $\hat z$ of the state z with sporadic measurements of y; namely, when the output y is available only at some time instances t_k, $k \in \mathbb{N}$, not known *a priori* (a similar setup is considered in Postoyan & Nešić, 2012). We assume that the sequence $\{t_k\}_{k=1}^{\infty}$ is strictly increasing and unbounded, and that for such a sequence there exist two positive real scalars $T_1 \leq T_2$ such that

$$\begin{aligned} &0 \leq t_1 \leq T_2 \\ &T_1 \leq t_{k+1} - t_k \leq T_2 \quad \forall k \in \mathbb{N}. \end{aligned} \tag{2}$$

As also pointed out in Hetel, Daafouz, Tarbouriech, and Prieur (2013), the lower bound T_1 in condition (2) prevents the existence of accumulation points in the sequence $\{t_k\}_{k=1}^{\infty}$, and, hence, avoids the existence of Zeno behaviors, which are typically undesired in practice. In fact, T_1 defines a strictly positive minimum time in between two consecutive transmissions. Furthermore, T_2 defines maximum time in between two consecutive transmissions.

Since the information on the output y is available in an impulsive fashion, motivated by Andrieu et al. (2013) and Raff and Allgöwer (2007), to solve the considered estimation problem we design an observer with jumps in its state following the law

$$\begin{cases} \dot{\hat z}(t) = A\hat z(t) + Bu(t) & \forall t \neq t_k,\ k \in \mathbb{N} \\ \hat z(t^+) = \hat z(t) + L(y(t) - M\hat z(t)) & \forall t = t_k,\ k \in \mathbb{N} \end{cases} \tag{3}$$

where L is a real matrix of appropriate dimensions to be designed. Note that, in between events, the observer runs in "open-loop" in the sense that no information of the output is used.

Along the lines of Sanfelice and Praly (2012), the state estimation problem is formulated as a set stabilization problem. Namely, our goal is to design the matrix L such that the set wherein the plant state z and its estimate $\hat z$ coincide is globally exponentially stable for the plant (1) interconnected with the observer in (3). At this stage, as usual in estimation problems, we define the estimation error as

$$\varepsilon := z - \hat z. \tag{4}$$

Thus, since at times t_k the plant state is unchanged, the error dynamics are given by the following dynamical system with jumps:

$$\begin{cases} \dot\varepsilon(t) = A\varepsilon(t) & \forall t \neq t_k,\ k \in \mathbb{N} \\ \varepsilon(t^+) = (\mathbf{I} - LM)\varepsilon(t) & \forall t = t_k,\ k \in \mathbb{N}. \end{cases} \tag{5}$$

Due to the linearity of system (1), the estimation error dynamics and the dynamics of z are decoupled. Then, for the purpose of estimation, one can effectively only consider system (5).

2.2. Hybrid modeling

The fact that the observer experiences jumps when a new measurement is available and evolves according to a differential equation in between updates suggests that the updating process of the error dynamics can be described via a hybrid system. Due to this, we represent the whole system composed by the plant (1), the observer (3), and the logic triggering jumps as a hybrid system (see Li & Sanfelice, 2013 where a similar approach is adopted to model a finite-time convergent observer).

To utilize this hybrid systems approach, a model of the hidden time-driven mechanism triggering the jumps of the observer is required. To this end, in a similar manner as in Carnevale and Teel (2007), we augment the state of the system with an auxiliary timer variable τ that keeps track of the duration of flows and triggers