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Brief paper

Finite-time stabilization of switched stochastic nonlinear systems with mixed odd and even powers ☆

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ABSTRACT

This paper investigates the finite-time stabilization of a class of switched stochastic nonlinear systems in p-normal form, where the power orders of the system are dependent upon the switching signal and the system structure is in non-triangular form. Compared with the existing results, some power orders of the system are allowed to be even. Under suitable assumptions, a state feedback control law with state-dependent switching is designed by using the convex combination method and the adding a power integrator technique. It is shown that the resulting closed-loop system is finite-time stable in probability. Simulation results of a continuously stirred tank reactor (CSTR) system are provided to show the effectiveness and applicability of the proposed method.

1. Introduction

Switched systems, which are used to model many physical or man-made systems displaying switching features, have been extensively studied in past years. The main concerns in the study of switched systems are the issues of stability and stabilization (Liberzon, 2003; Long & Zhao, 2011, 2012, 2014; Ma, Liu, Zhao, Wang, & Zong, 2015; Ma & Zhao, 2010; Sun & Wang, 2013; Wu, 2009). It was shown in Liberzon (2003) that a switched system might become unstable, even if all subsystems are stable. Conversely, it may be possible to stabilize a switched system by means of a suitable switching law, even if all subsystems are unstable. Therefore, how to design an appropriate switching law to achieve stability is of great importance. Switched stochastic systems, as a special kind of switched systems, play essential roles in modeling numerous physical and engineering dynamics with stochastic disturbances. The stability of stochastic differential equations with Markovian switching was studied in Mao (1999), and the controller design for such systems was tackled in Wu, Xie, Shi, and Xia (2009) and Wu, Yang, and Shi (2010). The stability of switched stochastic nonlinear systems was addressed in Wu, Cui, Shi, and Karimi (2013), Zhai, Kang, Zhao, and Zhao (2012) and Zhao, Feng, and Kang (2012). The stabilization of switched stochastic nonlinear systems in strict-feedback form was studied in Hou, Fu, and Duan (2013).

On the other hand, finite-time stability of nonlinear systems has been one of the most important research topics due to its important significance in theory and practice. It was shown in Bhat and Bernstein (1998) that finite-time stable systems might enjoy not only faster convergence but also better robustness and disturbance rejection properties. For some representative work on this topic, to name a few, we refer readers to Bhat and Bernstein (2000), Moulay and Perruquetti (2008) and Yang, Jiang, and Zhao (2015). Recently, Yin, Khoo, Man, and Yu (2011) extended the concept of finite-time stability to stochastic nonlinear systems. For the deterministic case, the finite-time stabilization has been developed (Ding, Li, & Zheng, 2012; Hong, 2002; Huang, Lin, & Yang, 2005; Li & Qian, 2006; Nersesov & Haddad, 2008; Shen & Huang, 2012; Zhang, Feng, & Sun, 2012). Ai, Zhai, and Fei (2013), Khoo, Yin, Man, and Yu (2013) and Yin and Khoo (2015) addressed finite-time stabilization for some stochastic nonlinear systems. Finite-time state feedback controller was constructed for high-order stochastic nonlinear systems in strict-feedback form in Wang and Zhu (2015). Finite-time output feedback stabilization of a class of high-order stochastic nonlinear systems was investigated in Zhai (2014).

It is worth pointing out that for stochastic nonlinear systems, the power orders are all required to be positive odd numbers and the system structure is in triangular form in the aforementioned literature. Questions may arise: for switched stochastic nonlinear

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systems, if one or more power orders are positive even, can we still achieve stabilization or even finite-time stabilization? Furthermore, what if the system structure is not in triangular form? If possible, under what conditions can we design such controllers? The purpose of this paper is to address these questions. To this end, we first propose a finite-time stability criterion for switched stochastic nonlinear systems with state-dependent switching. It should be pointed out that none of individual subsystems are required to be finite-time stable in probability. With the help of the proposed criterion, we investigate the finite-time stabilization for a class of switched stochastic nonlinear systems with mixed odd and even power orders, where the power orders of the system are dependent upon the switching signal. By constructing a state-dependent switching signal and applying the adding a power integrator technique, we design a finite-time state feedback control law to stabilize the system.

The remainder of this paper is organized as follows. In Section 2, the problem formulation and some preliminary results are given. The finite-time stabilization result is developed in Section 3. Section 4 provides an example to illustrate the proposed method. The paper is concluded in Section 5.

Notations. *R_+ denotes the set of all nonnegative real numbers. R^n denotes the real n-dimensional space. $R^{m\times n}$ denotes the space of $m\times n$ matrices with real entries. For a given vector or matrix X, X^T denotes its transpose. $Tr\{X\}$ represents its trace when X is a square matrix. $|\cdot|$ denotes the Euclidean norm. C^i denotes the set of all functions with continuous ith partial derivatives; $E\{x\}$ denotes the expectation of x. I_A is the indicator function of A, i.e., $I_A(x)$ is 1 or 0 accordingly as $x\in A$ or $x\notin A$. κ denotes the set of all functions, $R_+\to R_+$, which are continuous, strictly increasing and vanishing at zero; κ_∞ denotes the set of all functions which are of class κ and unbounded. $a\wedge b$ means the minimum of a and b.*

2. Problem formulation and preliminaries

Considers the following stochastic nonlinear system:

$$\begin{aligned} dx_i &= (h_{i,\sigma(t)}(\bar{x}_i)x_{i+1}^{p_{i,\sigma(t)}} + f_{i,\sigma(t)}(x))dt + g^T_{i,\sigma(t)}(x)dw,\\ &\quad i=1,\dots,n-1,\\ dx_n &= (h_{n,\sigma(t)}(\bar{x}_n)u_{\sigma(t)}^{p_{n,\sigma(t)}} + f_{n,\sigma(t)}(x))dt + g^T_{n,\sigma(t)}(x)dw, \end{aligned} \tag{1}$$

where $x=(x_1,\dots,x_n)^T$ is the system state, $\bar{x}_i=(x_1,\dots,x_i)^T$. w is a q-dimensional standard Wiener process defined on a probability space $(\Omega,F,\{F_t\}_{t\ge t_0},P)$ with Ω being a sample space, F being a σ − field, $\{F_t\}_{t\ge t_0}$ being a filtration and P being a probability measure. $\sigma(t):[t_0,+\infty)\to \underline{N}=\{1,2,\dots,N\}$ is a piecewise right continuous function, called the switching signal, N is the number of subsystems, and t_0 is the initial instant. For each $k\in\underline{N}$, $u_k\in R$ is the control input, $p_{i,k}, i=1,\dots,n$, are the power orders which are positive integers, and $h_{i,k}(\bar{x}_i), i=1,\dots,n$, are smooth functions. The drift terms $f_{i,k}(x)$ and the diffusion terms $g_{i,k}(x), i=1,\dots,n, k\in\underline{N}$, are Borel measurable and satisfy $f_{i,k}(0)=0$ and $g_{i,k}(0)=0$. The state of the system does not jump at each switching instant.

The following assumptions are made on system (1).

Assumption 1. For each $i\in\{1,2,\dots,n\}$, there exists at least one power order p_{i,k_i}, $k_i\in\underline{N}$, which is a positive odd integer.

Assumption 2. There exist a set of smooth functions $h_{1,k_1}(\cdot)>0, h_{2,k_2}(\cdot)>0,\dots,h_{n,k_n}(\cdot)>0$, $k_i\in\underline{N}$, $i=1,2,\dots,n$.

Assumption 3. There exist $q_i\ge 1, i=1,\dots,n+1$, $\tau\in(0,\ 1)$ being a ratio of odd integers, smooth functions $\rho_{fi}(\bar{x}_i)\ge 0$, $\rho_{gi}(\bar{x}_i)\ge 0$, $i=1,2,\dots,n$, and $a_k(x), k\in\underline{N}$, satisfying $0<\lambda_k(x_1)\le a_k(x)<1$ and $\sum_{k=1}^N a_k(x)=1$ with $\lambda_k(x_1)$ being smooth functions, such that

$$\begin{aligned} &\left|\sum_{k=1,k\ne k_i}^N a_k(x)h_{i,k}(\bar{x}_i)x_{i+1}^{p_{i,k}} + \sum_{k=1}^N a_k(x)f_{i,k}(x)\right|\\ &\quad\le \sum_{j=0}^{p_{i,k_i}-1}\sum_{l=1}^{i}\rho_{fi}(\bar{x}_i)|x_{j+1}|^j|x_l|^{\frac{p_{i,k_i}q_l}{q_{i+1}}},\\ &\quad i=1,\dots,n-1, \end{aligned} \tag{2}$$

$$\begin{aligned} &\left|\sum_{k=1,k\ne k_n}^N a_k(x)h_{n,k}(\bar{x}_n)u_k^{p_{n,k}} + \sum_{k=1}^N\sum_{k=1}^N a_k(x)f_{n,k}(x)\right|\\ &\quad\le\left(\sum_{k=1}^N\sum_{j=0}^{p_{n,k_n}-1}\sum_{l=1}^{n}\rho_{fn}(\bar{x}_n)|u_k|^j|x_l|^{\frac{p_{n,k_n}q_l}{q_{n+1}}}\right), \end{aligned} \tag{3}$$

$$\begin{aligned} &\left|\sum_{k=1}^N a_k(x)g_{i,k}(x)g^T_{j,k}(x)\right|\\ &\quad\le\sum_{l=1}^{i}\rho_{gi}(\bar{x}_i)|x_l|^{\frac{(q_{i+1}+p_{i,k_i})q_l}{q_{i+1}}},\quad i\ge j,\ i,j=1,\dots,n, \end{aligned} \tag{4}$$

$$q_1=1,\ 1+\frac{p_{i,k_i}}{q_{i+1}}=\tau+\frac{1}{q_i},\quad i=1,\dots,n. \tag{5}$$

Remark 1. In Assumption 1, we relax some restrictions on power orders in stochastic nonlinear systems considered in the existing literature (Li, Jing, & Zhang, 2011, 2012; Wang & Zhu, 2015; Xie, Duan, & Yu, 2011; Xie & Tian, 2009; Zhai, 2014). Some power orders in system (1) are allowed to be even. Assumption 2 is milder than the general assumption in Hou et al. (2013) with $h_{i,k}(\cdot)=1$. Similar assumptions can be found in Long and Zhao (2011).

Remark 2. From $q_i\ge 1$, $p_{i,k_i}\ge 1$ and (5), we can obtain $\frac{q_j}{q_{i+1}}\le 1$ and $\frac{p_{i,k_i}q_j}{q_{i+1}}=\frac{q_j}{q_i}-q_j(1-\tau)<1, j=1,\dots,i$, which indicates that the restrictions on the nonlinearities $f_{i,k}(x)$ in this paper are relaxed than those in Long and Zhao (2011). Moreover, due to the existences of stochastic factors which were not considered in Long and Zhao (2011), condition (4) is imposed on the nonlinearities $g_{i,k}(x), i=0,1,\dots,n, k\in\underline{N}$.

For the purpose of this paper, we will present some preliminary results related to the finite-time stability for a switched stochastic nonlinear system of the form:

$$dx(t)=f_{\sigma(t)}(x(t))dt+g^T_{\sigma(t)}(x(t))dw(t), \tag{6}$$

where $x(\cdot)\in R^n$ is the system state, and $w(\cdot)$ is a q-dimensional standard Wiener process defined on a probability space $(\Omega,F,\{F_t\}_{t\ge t_0},P)$. $\sigma(t):[t_0,+\infty)\to\underline{N}=\{1,2,\dots,N\}$ a piecewise right continuous function, called the switching signal, N is the number of subsystems, and t_0 is the initial instant. The functions $f_k:R^n\to R^n$ and $g_k:R^n\to R^{q\times n}$, $k\in\underline{N}$, are continuous and satisfy $f_k(0)=0$ and $g_k(0)=0$. The state of the system does not jump at each switching instant.

In the following, we present the existence condition concerning the solution of (6) that can be written as:

$$x(t)=x(t_0)+\int_{t_0}^t f_{\sigma(t)}(x(s))ds+\int_{t_0}^t g^T_{\sigma(t)}(x(s))dw(s).$$