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Brief paper

A novel analysis on the efficiency of hierarchy among leader-following systems[☆]

Jinliang Shao[a,1], Jiahu Qin[b], Adrian N. Bishop[c,d], Ting-Zhu Huang[a], Wei Xing Zheng[e]

[a] School of Mathematical Sciences, University of Electronic Science and Technology of China, Chengdu 610054, PR China
[b] Department of Automation, University of Science and Technology of China, Hefei 230027, PR China
[c] University of Technology Sydney (UTS), Australia
[d] Data61 (CSIRO) Canberra Research Lab, Australia
[e] School of Computing, Engineering and Mathematics, Western Sydney University, Penrith, NSW 2751, Australia

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ABSTRACT

In a recent *NATURE* paper, Nagy et al. find a well-defined hierarchy among the individuals of the pigeon flock, which may lead to a rapid decision making in the directional choice dynamics of the flock. Motivated by this interesting discovery, we present a novel analysis on the efficiency of the hierarchical topology among the leader-following systems in this paper. To this end, we first propose a measurement of the convergence rate of leader-following consensus, and then connect the convergence rates with the communication topologies of leader-following systems. It is proved that the hierarchical network organization can achieve the best performance in terms of convergence rates. It is also established that the connections between the leader and the followers have effective impacts on increasing the convergence rates. Extensive numerical results are provided to show the effectiveness of our conclusions.

1. Introduction

Collective behaviors of various animal species such as schooling fish, swarming ants and flocking birds have attracted considerable attention and interest of the researchers from different areas. It is found that the complex group decision-making may emerge from the self-organized motion based on simple local rules of interaction among the individuals and many interesting results have been reported in Ballerini et al. (2008), Couzin, Jens, Franks, and Levin (2005), Nagy, Ákos, Biro, and Vicsek (2010), Jadbabaie, Lin, and Morse (2003), and Vicsek et al. (1995), just to name a few. A central problem on group decision-making is figuring out how the navigational information transfers among the individuals to make sure that all the group members can change their direction of motion in a very short time. Group decision-making usually involves some form of leadership, which has been widely found in a variety of animals (Dyer, Johansson, Helbing, Couzin, & Krause, 2009). The bird flock and schooling fish as shown in Fig. 1 provide us with an intuitive sense of leadership, where the whole formation formed by the birds and fish are all hierarchical structures like pyramids.

Nagy et al. (2010) record the movement of homing pigeons flying in flocks up to 10 individuals with the help of high-resolution lightweight GPS devices, and find a well-defined hierarchy among the individuals in the directional choice dynamics of the flock to guarantee the rapid decision making of the flocks. They conclude that the hierarchical structure may lead to a rapid consensus of the flight directions for the individuals, which is stated as "the hierarchical organization of group flight may be more efficient than an egalitarian one, at least for those flock sizes that permit regular pairwise interactions among group members, during which leader–follower relationships are consistently manifested". In this paper, we use the continuous-time multi-agent system model (Jadbabaie et al., 2003; Ren & Beard, 2005) as a framework to give a theoretical analysis on the hierarchical structure of leader-following systems.

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E-mail addresses: jinliangshao@126.com (J. Shao), jhqin@ustc.edu.cn (J. Qin), adrian.bishop@data61.csiro.au (A.N. Bishop), tingzhuhuang@126.com (T.-Z. Huang), w.zheng@uws.edu.au (W.X. Zheng).

[1] Corresponding author.

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(a) Bird flock.

(b) Schooling fish.

Fig. 1. Bird flock and schooling fish in nature (Adopted from http://www.flickr.com).

Inspired by the collective behaviors ubiquitous in nature, the collaboration control of multi-agent system has become a research hotspot for its wide industrial applications such as multi-robot system coordination (Fan, Feng, Wang, & Qiu, 2011), formation control (Hu & Cao, 2015; Lin, Francis, & Maggiore, 2005), consensus/synchronization (Cao, Ren, & Egerstedt, 2012; Hu, Cao, Hu, & Guo, 2014; Hu, Cao, Yu, & Hayat, 2014; Lin, Francis, & Maggiore, 2007; Qin & Gao, 2012; Qin & Yu, 2013; Qin, Zheng, & Gao, 2011; Song, Liu, Cao, & Yu, 2013; Xiao & Wang, 2008; Xiong, Hayat, & Cao, 2015; Yu & Wang, 2010; Yu, Ren, Zheng, Chen, & Lü, 2013), and sensor networks (Shen, Wang, & Hung, 2010). It has been widely accepted that the distributed multi-agent system can be employed to model the collective behaviors of animals (Couzin et al., 2005; Jadbabaie et al., 2003; Vicsek et al., 1995; Vicsek & Zafeiris, 2012).

Essentially speaking, the efficiency problem for the bird flock as discussed in Nagy et al. (2010) is nothing but the convergence rate problem in multi-agent coordination community. To better specify the efficiency of the whole system, it is imperative to study the convergence rate of the leader-following consensus problem. Convergence rate analysis is a fundamental issue in multi-agent consensus problem. For leaderless consensus with directed topologies, based on the assumption that the topology is strongly connected, Olfati-Saber and Murray (2004) point out that the second smallest eigenvalue λ_2 of the Laplacian matrix of the mirror graph of the topology graph can represent the convergence rate of average consensus on multi-agent system. They conclude that a network with dense interconnections solves an agreement problem faster than a connected but sparse network. To speed up average consensus of multi-agent system with undirected topology, an iterative greedy-type algorithm to maximize λ_2 is proposed by Kim and Mesbahi (2006). Olfati-Saber (2005) shows that the algebraic connectivity can be increased significantly for certain small-world networks generated by random rewiring procedure, and hierarchical decomposition of the topology graph is introduced by Epstein, Lynch, Johansson, and Murray (2008).

For leader-following consensus with directed topology, Shi, Sou, Sandberg, and Johansson (2014) consider an informed-agents selection problem of leader-following consensus. Under the assumption that the topology of follower agents is strongly connected, the upper and lower bounds for the convergence speed are established by the maximal distance from the leader to the followers. A discrete-time leader-following consensus protocol over a numerosity-constrained network is defined in Abaid and Porfiri (2012), and a closed form expression for the convergence rate is established. In fact, if the topology among the follower agents is assumed to be undirected or strongly connected, then the followers can influence each other, namely, the follower agents may have the same status in the flock rather than the hierarchical structure. Thus, the results derived in the above literatures cannot be used to analyze the consensus problem with the hierarchical topologies. Under hierarchical leadership, Shen (2006) studies the emergent behavior of Cucker–Smale flocking model and presents the convergence rates for coherent group patterns in different scenarios. However, to study the efficiency of the hierarchical organization, it is necessary to figure out the relationship between the convergence speed and the topologies of the leader-following system. Based on the assumption that there exists a directed path from the leader to every follower agent, which is the weakest topology condition for the convergence of the leader-following problem (Ren & Beard, 2005), we first establish a measurement for convergence speed of leader-following consensus, and then prove that the hierarchical structure can achieve the best performance in the current paper, which coincides with the conclusion from the experiments in Nagy et al. (2010).

The rest of the paper is organized as follows. The continuous-time leader-following model and some necessary preliminaries are given in Section 2. We also present a measurement of the convergence speed for leader-following system in this section. In Section 3, the relationship between the topology graph and the convergence speed is studied based on M-matrix theory, and three numerical examples are given to illustrate the effectiveness of our results. Finally, the conclusion is drawn in Section 4.

2. Model description and preliminaries

Consider a system consisting of n follower agents which are labeled by $1, 2, \ldots, n$ and a leader 0, denoted by $\mathscr{V} = \{0, 1, 2, \ldots, n\}$. $\mathscr{G}_F = (\mathscr{V}_F, \mathscr{E}_F)$ denotes the interaction topology of information exchange between n followers by a directed graph, where $\mathscr{V}_F = \{1, 2, \ldots, n\}$ is the set of vertices representing n follower agents and $\mathscr{E}_F \subseteq \mathscr{V}_F \times \mathscr{V}_F$ is the set of edges of the graph. The index set of neighbors of vertex i is denoted by $\mathscr{N}_i = \{j \in \mathscr{V} : (j, i) \in \mathscr{E}_F\}$. An edge in a directed graph $\mathscr{G}_F$ is denoted by (i, j), representing that agent j can directly receive information from agent i, i is called the parent node and j the child node. The weighted adjacency matrix of graph $\mathscr{G}_F$ is denoted by $A = (a_{ij}) \in \mathbb{R}^{n\times n}$ with nonnegative entries, where $a_{ij} > 0$ if $(j, i) \in \mathscr{E}_F$. The self-loops of the nodes are forbidden in this paper. Denote $\Delta = \mathrm{diag}(\delta_1, \delta_2, \ldots, \delta_n)$ with $\delta_i = \sum_{j=1, j\neq i}^{n} a_{ij}$. The Laplacian matrix associated with A is defined as $L = (l_{ij}) \in \mathbb{R}^{n\times n} = \Delta - A$. For a given matrix $A \in \mathbb{R}^{n\times n}$, $\Lambda(A) = \{\lambda_i(A), i = 1, 2, \ldots, n\}$ denotes the spectrum of A, and $\rho(A) = \max(|\lambda_i(A)|)$ denotes its spectral radius. For $S_1, S_2 \subseteq \mathscr{V}$, a submatrix of A specified by the index subsets S_1 and S_2 is denoted by $A[S_1, S_2] = (a_{ij})_{i\in S_1, j\in S_2}$. Specifically, the principal submatrix $A[S, S]$ is also denoted by $A[S]$, $S \subseteq \mathscr{V}$. I denotes the identity matrix with compatible dimension. For $A = (a_{ij}), B = (b_{ij}) \in \mathbb{R}^{n\times n}$, $A \geq B$ means $a_{ij} \geq b_{ij}$ for all i, j. $A \geq 0$ denotes that A is a nonnegative matrix.

$\mathscr{G} = (\mathscr{V}, \mathscr{E})$ denotes the graph that consists of graph $\mathscr{G}_F$, the leader agent 0 and directed edges from 0 to some followers. Define the leader adjacency matrix associated with graph $\mathscr{G}$ as a diagonal matrix $B = \mathrm{diag}\{b_1, \ldots, b_n\}$, where the weight $b_i > 0$ if there is a directed edge from 0 to i (i.e., i is connected to the leader) and $b_i = 0$ otherwise. It is obvious that $d_i = b_i + \delta_i$ is the weighted in-degree of agent i in $\mathscr{G}$, then we denote the in-degree matrix by a diagonal matrix $D = \mathrm{diag}(d_1, d_2, \ldots, d_n) = B + \Delta$, and $d_M = \max_{i\in\mathscr{V}_F}\{d_i\}$, $d_m = \min_{i\in\mathscr{V}_F}\{d_i\}$. A path in a digraph $\mathscr{G}$ is a sequence $i_0, \ldots, i_m \in \mathscr{V}$ of distinct vertices such that (i_{j-1}, i_j) is an