Automatica 73 (2016) 231–236

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Approximate discretization of regular descriptor (singular) systems with impulsive mode[☆]

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ARTICLE INFO

Article history:
Received 2 July 2015
Received in revised form
12 May 2016
Accepted 22 June 2016

Keywords:
Descriptor systems
Singular systems
Discretization
Impulsive mode

ABSTRACT

Descriptor systems can describe algebraic constraints and impulsive behaviour of systems and hence represent more general and broad category of complex dynamical systems. The impulsive behaviour of descriptor systems must be eliminated as it can cause degradation and saturation in performance or even can damage the system. To eliminate impulsive modes using discrete-time controllers, discrete-time models of descriptor systems are required. This paper deals with this problem and proposes a novel discretization technique for regular descriptor systems with impulsive mode when the input is applied through a zero-order-hold device. Using the Kronecker canonical form, the system model is decomposed into exponential, static and impulsive modes. Exact discrete-time models of exponential and static modes are obtained but an approximate discrete-time model for impulsive mode is proposed using limiting value functions because exact discretization is not possible as digital devices cannot realize infinite magnitude in zero time. An upper bound on error introduced by discretization is also derived. In addition, a simulation example is presented to show the efficacy of the proposed technique.

1. Introduction

During the mathematical description of the dynamical behaviour of many complex systems, there comes the situation that dynamic and static nature must be considered. Therefore, mathematical models of these systems are represented by both differential and algebraic equations. These systems are called descriptor systems (also known as singular, degenerate, generalized, differential–algebraic, semi-state or implicit system) and appear in many constrained mechanical, electrical, economical, large delay, interconnected, neutral delay systems, aircraft dynamics, image modelling, robotics, chemical, thermal and diffusion processes. Descriptor systems represent more general and broad mathematical framework of complex dynamical systems. Therefore modelling, simulation and control of these systems is a major area of research in control theory.

Descriptor systems have three modes: an exponential mode representing the dynamic nature of the system, static mode describing the static behaviour of the system and an impulsive mode which is due to derivative nature of the system. Exponential and static modes exist in standard state space systems but impulsive modes are the unique behaviour of descriptor systems. Impulsive behaviour may cause degradation in performance and saturation in state response of control systems or even may destroy the system; therefore, it is necessary to eliminate impulsive modes. There exist many continuous-time controllers, such as state feedback, output feedback, derivative state feedback, derivative output feedback, etc., to eliminate impulsive modes but discrete-time controllers do not exist for this purpose. As microcontrollers, microprocessors and digital computers are becoming very common for the dynamic control of physical systems due to increased flexibility, decision making and logic capability. Therefore, it is important to design a digital control law for the descriptor system to eliminate impulsive modes and for other control purposes. For this purpose, we must have discrete-time model of the descriptor system with impulsive mode.

Descriptor systems with non-singular descriptor matrix or with singular descriptor matrix having only exponential and static

[☆] The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Yoshio Ebihara under the direction of Editor Richard Middleton.

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http://dx.doi.org/10.1016/j.automatica.2016.07.017
0005-1098/

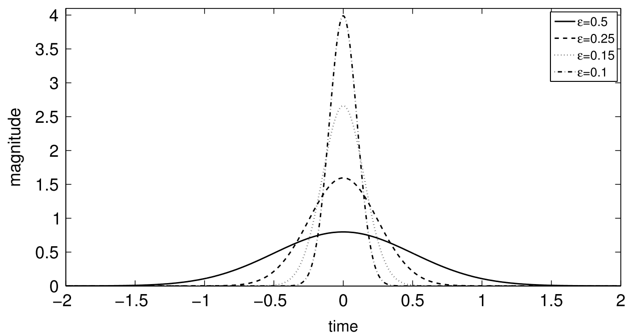

Fig. 1. Gaussian definition of Dirac delta function.

modes can be exactly discretized, their models are derived in Dong (2014), Dong, Mao, Tian, and Wang (2013), Dong and Xiao (2014) and Kawai and Hori (2011). These models, known as delta operator models, are exact in the sense that their responses approach to those of the continuous-time ones when the sampling period approaches zero. Regular descriptor systems under consistent initial conditions do not have impulsive mode, their discrete-time models when an input signal is applied through a zero-order-hold device or a first-order-hold device are derived in Karageorgos, Pantelous, and Kalogeropoulos (2010), Karampetakis (2004) and Karampetakis and Karamichalis (2014) and using the Euler approximation in Rachid (1995). It is noted that none of the aforementioned techniques considered discretization of descriptor systems with impulsive modes. To the author's best knowledge, the only work in this regard is Kawai and Hori (2012) where the discretization of regular descriptor systems with impulsive modes is considered; however, it is an approximate model and picks up the impulsive mode when the sampling period is very small. Also, when the sampling period approaches zero, the discrete-time exponential mode approaches to continuous-time mode but non-exponential mode does not. Therefore, it is of interest to develop a better discretization technique for descriptor systems with impulsive modes.

This paper presents a novel discretization technique for regular descriptor systems with impulsive modes. The given system is decomposed into exponential, static and impulsive modes using the Kronecker canonical form. Exact discrete-time model of exponential and static modes is obtained but an approximate model for impulsive modes is derived by defining limiting value functions such as Gaussian, Lorentzian or Dirichlet. An exact discrete-time model of impulse function is not possible because digital devices cannot realize infinite magnitude in zero time. The effectiveness and superiority of the proposed technique over the existing one is demonstrated using a numerical example.

The paper is organized as follows: Section 1 contains an introduction to the problem, which is followed by problem formulation and preliminaries in Section 2. The proposed discretization technique is given in Section 3. In Section 4, error bound is derived. Section 5 presents a discretization example and its simulation. At the end, a conclusion is drawn in Section 6.

2. Problem formulation and preliminaries

Consider a descriptor system of the form

$$\boldsymbol{E}\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \tag{1}$$

where $\boldsymbol{x}(t) \in R^n$ is the state, $\boldsymbol{u}(t) \in R^r$ is the input, $\boldsymbol{E}, \boldsymbol{A} \in R^{n\times n}$ and $\boldsymbol{B} \in R^{n\times r}$ are the coefficient matrices of the system. We consider the case when dynamical order of the system is less than the system order i.e. $\text{rank}(\boldsymbol{E}) < n$, which results in the singularity of matrix $\boldsymbol{E}$, and the matrix pencil $\boldsymbol{P}_{(\boldsymbol{E},\boldsymbol{A})}(s) = s\boldsymbol{E} - \boldsymbol{A}$ is regular.

As the system in (1) is regular, there always exist two non-singular matrices $\boldsymbol{P}$ and $\boldsymbol{Q}$ such that it is equivalent to the following Kronecker canonical form (Gerdin, 2004; Van Dooren, 1979)

$$\dot{\boldsymbol{x}}_1 = \boldsymbol{A}_1\boldsymbol{x}_1 + \boldsymbol{B}_1\boldsymbol{u} \tag{2}$$

$$\boldsymbol{N}\dot{\boldsymbol{x}}_2 = \boldsymbol{x}_2 + \boldsymbol{B}_2\boldsymbol{u} \tag{3}$$

where $\tilde{\boldsymbol{x}} = \boldsymbol{P}^{-1}\boldsymbol{x} = \begin{bmatrix}\boldsymbol{x}_1^T & \boldsymbol{x}_2^T\end{bmatrix}^T$, $\boldsymbol{QB} = \begin{bmatrix}\boldsymbol{B}_1^T & \boldsymbol{B}_2^T\end{bmatrix}^T$, $\boldsymbol{QEP} = \text{diag}(\boldsymbol{I}_{n_1}, \boldsymbol{N})$, $\boldsymbol{QAP} = \text{diag}(\boldsymbol{A}_1, \boldsymbol{I}_{n_2})$ and $\boldsymbol{N}$ is a nilpotent matrix with nilpotent index h. Also, $\boldsymbol{N} = \text{diag}(\boldsymbol{N}_1, \boldsymbol{N}_2, \ldots, \boldsymbol{N}_l)$ where $\boldsymbol{N}_i$ being a Jordon block with zero diagonal entries. Eqs. (2) and (3) represent the slow (exponential) and fast (static and impulsive) modes, respectively.

Eq. (2) has the following solution with $\boldsymbol{x}_{10}$ as initial condition and $\boldsymbol{u}(t)$ as input, see Åström and Wittenmark (2013)

$$\boldsymbol{x}_1(t) = e^{\boldsymbol{A}_1 t}\boldsymbol{x}_{10} + \int_0^t e^{\boldsymbol{A}_1(t-\tau)}\boldsymbol{B}_1\boldsymbol{u}(\tau)d\tau. \tag{4}$$

It's exact discrete-time model is

$$\boldsymbol{x}_{1,k+1} = \boldsymbol{G}\boldsymbol{x}_{1,k} + \boldsymbol{H}\boldsymbol{u}_k \tag{5}$$

where subscript k represents the kth sampling instant, T is the sampling period, $\boldsymbol{G} = e^{\boldsymbol{A}_1 T}$ and $\boldsymbol{H} = \left(\int_0^T e^{\boldsymbol{A}_1\lambda}d\lambda\right)\boldsymbol{B}_1$. Eq. (3) has the following solution (Duan, 2010)

$$\begin{aligned}\boldsymbol{x}_2(t) = &-\sum_{i=0}^{h-1}\boldsymbol{N}^{i+1}\delta^{(i)}(t)\boldsymbol{x}_{20} - \sum_{i=0}^{h-1}\boldsymbol{N}^i\boldsymbol{B}_2\boldsymbol{u}^{(i)}(t)\\ &-\sum_{i=0}^{h-1}\boldsymbol{N}^i\boldsymbol{B}_2\sum_{j=1}^{i}\delta^{(i-j)}(t)\boldsymbol{u}^{(j-1)}(0)\end{aligned} \tag{6}$$

where $\boldsymbol{x}_{20}$ is the initial condition and $\delta(t)$ is the Dirac delta function. $\delta^{(i)}$ and $\boldsymbol{u}^{(i)}$ are the ith derivatives of $\delta(t)$ and $\boldsymbol{u}(t)$; respectively. The Dirac delta function, $\{\delta(t) = \infty \text{ for } t = 0 \text{ and } 0 \text{ for } t \neq 0\}$ represents the impulsive behaviour of the system and can be described as a limiting function (Economou, 2006). The following definitions exist

$$\delta(t) = \begin{cases}\lim\limits_{\epsilon\to 0}\dfrac{\epsilon}{\pi(t^2+\epsilon^2)}, & \text{Lorentzian}\\ \lim\limits_{\epsilon\to 0}\dfrac{1}{\epsilon\sqrt{2\pi}}\exp\left(-\dfrac{t^2}{2\epsilon^2}\right), & \text{Gaussian}\\ \lim\limits_{\epsilon\to 0}\dfrac{\sin\left(\frac{t}{\epsilon}\right)}{\pi t}, & \text{Dirichlet}\end{cases} \tag{7}$$

e.g. plot of Gaussian function is normal distribution across zero centre for different values of ϵ. By choosing smaller value of ϵ, approximation of $\delta(t)$ improves (see Fig. 1).

3. Discretization

In this section, discretization of regular descriptor system is proposed. During discretization process input $\boldsymbol{u}(t)$ is applied through a zero-order-hold i.e. $\boldsymbol{u}(t) = \boldsymbol{u}(kT) = \boldsymbol{u}_k$ for $\forall t \in [kT, (k+1)T)$ and $k \geq 0$.

Theorem 1. *An approximate discrete-time model for the descriptor system in (1) is given by*

$$\boldsymbol{x}_{k+1} = \boldsymbol{P}\boldsymbol{\Phi}\boldsymbol{P}^{-1}\boldsymbol{x}_k + \boldsymbol{P}\boldsymbol{\Gamma}\boldsymbol{u}_k + \boldsymbol{PC}\boldsymbol{x}_0 + \boldsymbol{PD}\boldsymbol{u}_0 \tag{8}$$

where

$$\begin{aligned}\boldsymbol{\Phi} &= \text{diag}(\boldsymbol{G}, \boldsymbol{I}), \quad \boldsymbol{\Gamma} = \begin{pmatrix}\boldsymbol{H}^T & \boldsymbol{B}_2^T\boldsymbol{Y}^T\end{pmatrix}^T\\ \boldsymbol{C} &= \text{diag}(\boldsymbol{0}, \boldsymbol{W}), \quad \boldsymbol{D} = \begin{pmatrix}\boldsymbol{0} & \boldsymbol{B}_2^T\boldsymbol{Z}^T\end{pmatrix}^T\end{aligned} \tag{9}$$

and $\boldsymbol{P}$ is the transformation matrix defined above. $\boldsymbol{G}$, $\boldsymbol{H}$, $\boldsymbol{W}$, $\boldsymbol{Y}$ and $\boldsymbol{Z}$ are given in Eqs. (5), (14), (15) and (18).