Automatica 75 (2017) 144–153

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Continuous-time model predictive control of under-actuated spacecraft with bounded control torques[☆]

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ARTICLE INFO

Article history:
Received 21 January 2016
Received in revised form
8 August 2016
Accepted 14 August 2016

Keywords:
Spacecraft
Model predictive control (MPC)
Stabilization
Nonholonomic systems
Control constraints

ABSTRACT

The stabilization problem of rigid spacecraft is essential for space explorations and operations. This paper studies the model predictive stabilization problem of a class of underactuated rigid spacecrafts with two bounded control torques. A novel model predictive control (MPC) algorithm is designed by making use of the homogeneity of the system dynamics. In addition, a local homogeneous Lyapunov function is constructed based on which the approach to designing the terminal set and other parameters are developed. Finally, the conditions for ensuring algorithm feasibility and closed-loop stability are provided. We show that under the given conditions, the designed MPC algorithm is feasible, and the closed-loop system is asymptotically stable. Simulation and comparison studies verify that the developed results are effective and valid, and the designed controller fulfills the constraint satisfaction and achieves much faster convergence rate in comparison with conventional continuous time-varying controllers.

1. Introduction

The stabilization problem of underactuated spacecraft (i.e., normally with two control torques) is a very challenging issue due to the nonholonomic constraints. The study can be traced back to Byrnes and Isidori (1991), where it has been shown that there does not exist a (local) static state feedback controller to stabilize the spacecraft with two control torques, because it violates the Brockett necessary condition (Brockett, 1982).

Due to the design challenges, much attention has been directed to studying the stabilization problem of spacecraft of two control torques. In the literature, several strategies have been developed to solve this stabilization problem, including the time-varying continuous feedback controllers (Morin & Samson, 1997; Morin, Samson, Pomet, & Jiang, 1995), the time-invariant discontinuous controllers (Casagrandea, Astolfi, & Parisini, 2008), the hybrid control strategy (Teel & Sanfelice, 2008) and the "Quadratic Immersion"-based approach (Carravetta, 2014). In some special cases such as spin-axis stabilization, when there are no requirements on stabilizing the angular velocities, the stabilization problem of full states can be reduced to that of partial states and may be solved by linear feedback controllers (Tsiotras, 1997). However, the above mentioned results with few exceptions (e.g., Tsiotras, 1996; Tsiotras & Luo, 2000), neither consider the optimal control performance nor practical control constraints, though these two factors are particularly important in practical implementations.

The references in Gui and Vukovich (2015) and Tsiotras and Luo (1997, 2000) investigate the control problem of underactuated spacecraft with bounded control inputs. In Tsiotras and Luo (1997), the stabilization and tracking problem of underactuated axisymmetric spacecraft with two control inputs is studied, and large control inputs are avoided by designing the controllers that divide the state space into two regions, and drive the state away from the region where the large control inputs are generated. The idea is further extended into Tsiotras and Luo (2000), where a priori specified control bound is given and needs to be satisfied. Recently, an adaptive spin-axis stabilization strategy of a symmetric spacecraft with two bounded torques is proposed in Gui and Vukovich (2015), where a saturated proportional–derivative controller with an adaptive scheme is developed to handle saturation and uncertainties simultaneously. Note that the aforementioned results are

☆ This work was supported in part by National Natural Science Foundation of China (NSFC) under Grants 61473225, 61502395, 51579210; the Fundamental Research Funds for the Central Universities under grant 3102016AXXX012; the Scientific Research Foundation for the Excellent Returned Overseas Scholars, Shaanxi Province. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Riccardo Scattolini under the direction of Editor Ian R. Petersen.

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http://dx.doi.org/10.1016/j.automatica.2016.09.024
0005-1098/

focused on the axisymmetric spacecraft and only consider kinematics, which is quite different from this paper in which we will study the asymmetric underactuated spacecraft and consider complete system model including kinematics and dynamics.

It is well known that the model predictive control (MPC), also known as receding horizon control, can provide almost optimal control performance and well respect system constraints, which has been widely implemented in the industry. This motivates the current study to solve the model predictive stabilization problem of spacecraft with two bounded control torques. Note that the model predictive stabilization problem for such underactuated systems is nontrivial, as the well-developed MPC strategies (Chen & Allgöwer, 1998; Li & Shi, 2014a,b; Michalska & Mayne, 1993) are not valid and cannot be directly applied. This is because the design of these MPCs (Chen & Allgöwer, 1998; Li & Shi, 2014a,b; Michalska & Mayne, 1993) heavily relies on a hypothesis that there exists a state feedback controller for corresponding linearized systems, which does not hold for the underactuated spacecraft dynamics. A general MPC framework is proposed in Fontes (2001) which mainly depends on the stability condition SC5, however no way has been provided to construct such a condition, and it is generally very hard to find such a condition for underactuated spacecraft.

In the literature, most of the MPC strategies for spacecraft control are focused on fully actuated models and/or linearized ones. For example, a fixed-point MPC strategy with quadratic cost functions is developed for tracking attitude set points of a full-actuated spacecraft in Guiggiani, Kolmanovsky, Patrinos, and Bemporad (2014); an explicit model predictive control algorithm is proposed for a linearized spacecraft model in Hegrenæ, Gravdahl, and Tøndel (2005); a nonlinear MPC algorithm is designed for attitude control of full-actuated spacecraft on SO(3) in Kalabic, Gupta, Di Cairano, Bloch, and Kolmanovsky (2014), where the Lie group variational integrator based discrete-time model is adopted. It is worth noting that for spacecraft with two control torques, an MPC-based stabilization strategy has been designed in Marchand and Alamir (2003), where a numeric approximation of the system trajectory is utilized by Chebyshev's polynomial basis, but the theoretical properties of the designed algorithm are not studied. The theoretical results of MPC strategies for underactuated underwater vehicles are developed in Li, Xie, and Yan (2016) and Li and Yan (2016), but they cannot be directly applied for underactuated spacecraft. Therefore, it is very necessary to investigate the model predictive stabilization problem of underactuated spacecraft with practical constraints, and further provide a rigorous analysis to uncover theoretical properties of the designed MPC algorithm.

In this study, we will study the MPC-based stabilization problem of rigid spacecraft with two control torques and practical control input constraints, and develop a systematic parameter-design approach to ensure the theoretical properties of the designed MPC algorithm. The main contributions of this paper are three-fold:

- A novel model predictive stabilization algorithm is designed for rigid spacecraft with two bounded control torques. The novelty of the designed algorithm lies in that the design of cost function makes full use of the homogeneity of the system dynamics. Different from the conventional MPCs (Chen & Allgöwer, 1998; Li & Shi, 2014a,b; Li & Yan, 2015; Michalska & Mayne, 1993), where the cost functions use the Euclidean norm of the system states and control inputs, the stage cost function is built on the homogeneous norm of partial system states, and the terminal cost function is time-varying and captures the properties of system dynamics. In addition, the terminal constraint is imposed in terms of the time-varying cost function unlike the conventional ones in which a terminal set is constructed by using the time-invariant terminal cost function (Chen & Allgöwer, 1998; Li & Shi, 2014a,b; Michalska & Mayne, 1993).
- An approach to designing parameters for the MPC algorithm is developed. Due to the fact that the terminal cost is time-varying and parameter-dependent, we develop conditions to construct a local stable set for the closed-loop system by using the time-varying control law in Morin and Samson (1997), and show that the terminal cost function is qualified as a local homogeneous Lyapunov function for such closed-loop system with appropriate parameters. Furthermore, an optimization problem which combines the control input constraints and the local stable set is formulated for designing the terminal constraint.
- The theoretical conditions on ensuring algorithm feasibility and closed-loop stability are established. Under the way of parameter design (i.e., Algorithm 2), we prove that the designed MPC algorithm is iteratively feasible, and the closed-loop system is asymptotically stable and the control input constraints are satisfied. These theoretical results provide a useful tool for algorithm design and analysis in practical implementations.

Notations: We use the symbol $\mathbb{R}$ and $\mathbb{R}_+$ to denote the real space and real space with positive numbers, respectively. We adopt "T" to represent the matrix transposition, and $\mathrm{col}\{x_1, x_2, \dots, x_n\}$ to denote the column operation as $[x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \dots, x_n^{\mathrm{T}}]^{\mathrm{T}}$ for column vectors $x_1, x_2, \dots, x_n$. For a column vector $x \in \mathbb{R}^n$, its Euclidean norm (and its absolute value for $n = 1$) is denoted by $|x|$. For a vector $x \in \mathbb{R}^3$, its skew operation is written as $S(x) = [0, x_3, -x_2; -x_3, 0, x_1; x_2, -x_1, 0]$. Given a matrix $P \in \mathbb{R}^{n\times n}$, we use $P > 0\,(P \geqslant 0)$ to denote its positive definiteness (semi-positive definiteness). For a symmetric matrix, we use $*$ to represent its symmetric elements with respect to its main diagonal. For two sets $\mathcal{U} \subseteq \mathbb{R}^n$ and $\mathcal{W} \subseteq \mathbb{R}^n$ with $\mathcal{U} \subseteq \mathcal{W}$, define $\mathcal{W} \setminus \mathcal{U} = \{x \in \mathbb{R}^n | x \in \mathcal{W}, \notin \mathcal{U}\}$. We say that a function f belongs to the class $\mathcal{C}^k$, if its derivatives up to the order of k exist and are continuous.

2. Problem formulation and preliminaries

2.1. Problem statement

Consider a rigid spacecraft with two control torques as:

$$\begin{aligned} \dot{R} &= S(\omega)R, \\ J\dot{\omega} &= S(\omega)J\omega + B\tau, \end{aligned} \tag{1}$$

where $R \in SO(3)$ is the rotation matrix from the body frame $\mathcal{B}$ to the inertial frame $\mathcal{I}$, $\omega \in \mathbb{R}^3$ is the angular velocity in the frame $\mathcal{I}$, $J = \mathrm{diag}(j_1, j_2, j_3)$ is the principal moments of inertia, $\tau = \mathrm{col}(\tau_1, \tau_2, 0)$ with τ_i, $i = 1, 2$, are torques driving the spacecraft, and $B = I_3$. Note that there are only two control torques, making the system in (1) nonholonomic. The control torques are bounded by

$$\tau_i \in \Gamma_i, \quad i = 1, 2, \tag{2}$$

where $\Gamma_i \subseteq \mathbb{R}$, contains zero as its interior point.

Taking $x = \mathrm{col}(x_1, x_2, x_3) = -k\tan\theta$ as in Morin and Samson (1997) and Morin et al. (1995), where $\vec{k}$ is the unit vector around which R is rotated, $\theta \in (-\pi, \pi)$ is the rotated angle, and k is coordinate of $\vec{k}$ in the frame $\mathcal{I}$, the system in (1) can be transformed into the following form (Morin & Samson, 1997; Morin et al., 1995)

$$\begin{aligned} \dot{x} &= \frac{1}{2}(\omega + S(\omega)x + (x^{\mathrm{T}}\omega)x), \\ \dot{\omega}_1 &= c_1\omega_2\omega_3 + u_1, \\ \dot{\omega}_2 &= c_2\omega_1\omega_3 + u_2, \\ \dot{\omega}_3 &= c_3\omega_1\omega_2, \end{aligned} \tag{3}$$