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Family of controllers for attitude synchronization on the sphere ☆

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ABSTRACT

In this paper we study a family of controllers that guarantees attitude synchronization for a network of agents in the unit sphere domain, i.e., S^2. We propose distributed continuous controllers for elements whose dynamics are controllable, i.e., control with torque as command, and which can be implemented by each individual agent without the need of a common global orientation frame among the network, i.e., it requires only local information that can be measured by each individual agent from its own orientation frame. The controllers are constructed as functions of distance functions in S^2, and we provide conditions on those distance functions that guarantee that *i)* a synchronized network of agents is locally asymptotically stable for an arbitrary connected network graph; *ii)* a synchronized network is asymptotically achieved for almost all initial conditions in a tree network graph. When performing synchronization along a principal axis, we propose controllers that do not require full torque, but rather torque orthogonal to that principal axis; while for synchronization along other axes, the proposed controllers require full torque. We also study the equilibria configurations that come with specific types of network graphs. The proposed strategies can be used in attitude synchronization of swarms of under actuated rigid bodies, such as satellites.

1. Introduction

Decentralized control in a multi-agent environment has been a topic of active research for the last decade, with applications in large scale robotic systems. Attitude synchronization in satellite formations is one of those applications (Lawton & Beard, 2002), where the control goal is to guarantee that a network of fully actuated rigid bodies acquires a common attitude. Coordination of underwater vehicles in ocean exploration missions can also be casted as an attitude synchronization problem (Leonard et al., 2007).

In the literature of attitude synchronization, different solutions for consensus in the special orthogonal group are found (Bondhus, Pettersen, & Gravdahl, 2005; Cai & Huang, 2014; Dimarogonas, Tsiotras, & Kyriakopoulos, 2009; Krogstad & Gravdahl, 2006; Lawton & Beard, 2002; Nair & Leonard, 2007; Sarlette, Sepulchre, & Leonard, 2009; Song, Thunberg, Hu, & Hong, 2015; Thunberg, Song, Montijano, Hong, & Hu, 2014), which focus on *complete* attitude synchronization. In this paper, we focus on *incomplete* attitude synchronization, which has not received the same attention: in this scenario, each rigid body has a main direction and the global objective is to guarantee alignment of all rigid bodies' main directions; the space orthogonal to each main direction can be left free of actuation or controlled to accomplish some other goals. Complete attitude synchronization requires more measurements when compared to incomplete attitude synchronization, and it might be the case that a rigid body is not fully actuated but rather only actuated in the space orthogonal to a specific direction, in which case incomplete attitude synchronization is still feasible. *Incomplete* attitude synchronization is also denoted synchronization on the sphere in Dörfler and Bullo (2014), Li and Spong (2014), Moshtagh and Jadbabaie (2007), Olfati-Saber (2006), Paley (2009) and Sarlette, Tuna, Blondel, and Sepulchre (2008), where the focus has been on kinematic or point mass dynamic agents, i.e., dynamical agents without moment of inertia.

In Dimarogonas et al. (2009), attitude control in a leader–follower network of rigid bodies has been studied, with the special orthogonal group being parametrized with Modified Rodrigues Parameters. The proposed solution guarantees attitude synchronization for connected graphs, but it requires all rigid

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bodies to be aware of a common and global orientation frame. In Bondhus et al. (2005) and Krogstad and Gravdahl (2006), a controller for a single-leader single-follower network is proposed that guarantees global attitude synchronization at the cost of introducing a discontinuity in the control laws. In Cai and Huang (2014), attitude synchronization in a leader–follower network is accomplished by designing a non-linear distributed observer for the leader. In Chung, Ahsun, and Slotine (2009) and Chung, Bandyopadhyay, Chang, and Hadaegh (2013), a combination of a tracking input and a synchronization input is used; the tracking input adds robustness if connectivity is lost and it is designed in the spirit of leader-following, where the leader is a virtual one and it encapsulates a desired trajectory; however, this strategy requires all agents to be aware of a common and global reference frame. In another line of work, in Nair and Leonard (2007) and Sarlette et al. (2009), attitude synchronization is accomplished without the need of a common orientation frame among agents. Additionally, in Sarlette et al. (2009), a controller for switching and directed network topologies is proposed, and local stability of consensus in connected graphs is guaranteed, provided that the control gain is sufficiently high. In Lawton and Beard (2002), attitude synchronization is accomplished with controllers based on behavior based approaches and for a bidirectional ring topology. The special orthogonal group is parametrized with quaternions, and the proposed strategy also requires a common attitude frame among agents. In Mayhew, Sanfelice, Sheng, Arcak, and Teel (2012), a quaternion based controller is proposed that guarantees a synchronized network of rigid bodies is a global equilibrium configuration, provided that the network graph is acyclic. This comes at the cost of having to design discontinuous (hybrid) controllers. A discrete time protocol for complete synchronization of kinematic agents is found in Tron, Afsari, and Vidal (2012). The authors introduce the notion of *reshaping function*, and a similar concept is presented in this manuscript. The protocol provides almost global convergence to a synchronized configuration, which relies on proving that all other equilibria configurations, apart from the equilibria configuration where agents are synchronized, are unstable. In Thunberg et al. (2014), controllers for complete attitude synchronization and for switching topologies are proposed, but this is accomplished at the kinematic level, i.e., by controlling the agents' angular velocity (rather than their torque). This work is extended in Song et al. (2015) by providing controllers at the torque level, and similarly to Lawton and Beard (2002), stability properties rely on high gain controllers.

In Moshtagh and Jadbabaie (2007) and Olfati-Saber (2006) and incomplete synchronization of kinematic agents on the sphere is studied, with a constant edge weight function for all edges. In particular, in Moshtagh and Jadbabaie (2007), incomplete synchronization is used for accomplishing a flocking behavior, where a group of agents moves in a common direction. In Paley (2009), dynamic agents, which move at constant speed on a sphere, are controlled by a state feedback control law that steers their velocity vector so as to force the agents to attain a collective circular motion; since the agents are mass points, the effect of the moment of inertia is not studied. In Li and Spong (2014), dynamic point mass agents, constrained to move on a sphere, are controlled to form patterns on the sphere, by constructing attractive and repelling forces; in the absence of repelling forces, synchronization is achieved. Also, the closed-loop dynamics of these agents are invariant to rotations, or symmetry preserving, as those in Moshtagh and Jadbabaie (2007) and Olfati-Saber (2006) in the sense that two trajectories, whose initial condition – composed of position and velocity – differs only on a rotation, are the same at each time instant apart from the previous rotation. In our framework this property does not hold, since our dynamic agents have a moment of inertia, unlike the agents in Li and Spong (2014), Moshtagh and Jadbabaie (2007) and Olfati-Saber (2006), which is another novelty of the paper in hand.

We propose a distributed control strategy for synchronization of elements in the unit sphere domain. The controllers for accomplishing synchronization are constructed as functions of distance functions (or *reshaping functions* as denoted in Tron et al., 2012), and, in order to exploit results from graph theory, we impose a condition on those distance functions that will restrict them to be invariant to rotations of their arguments. As a consequence, the proposed controllers can be implemented by each agent without the need of a common orientation frame. We restrict the proposed controllers to be continuous, which means that a synchronized network of agents cannot be a global equilibrium configuration, since S^2 is a non-contractible set (Liberzon, 2003). Our main contributions lie in proposing for the first time a controller that does not require full torque when performing synchronization along a principal axis, but rather torque orthogonal to that axis; in finding conditions on the distance functions that guarantee that a synchronized network is locally asymptotically stable for arbitrary connected network graphs, and that guarantee that a synchronized network is achieved for almost all initial conditions in a tree graph; in providing explicit domains of attraction for the network to converge to a synchronized network; and in characterizing the equilibria configurations for some general, yet specific, types of network graphs. A preliminary version of this work was submitted to the 2015 IEEE Conference on Decision and Control (Pereira & Dimarogonas, 2015). With respect to this preliminary version, this paper presents significantly more details on the derivation of the main theorems and provides additional results. In particular, the concept of cone has been modified, with a clearer intuitive interpretation; the proof for the proposition that supports the result on local stability of the synchronized network has been simplified; further details on the condition imposed on the distance functions are provided; additional examples on possible distance functions, and their properties, are presented; and supplementary simulations are provided which further illustrate the theoretical results. The remainder of this paper is structured as follows. In Section 3, the problem statement is described; in Section 4, the proposed solution is presented; in Sections 5 and 6, convergence to a synchronized network is discussed for tree and arbitrary graphs, respectively; and, in Section 7, simulations are presented that illustrate the theoretical results.

2. Notation

$\mathbf{0}_n \in \mathbb{R}^n$ and $\mathbf{1}_n \in \mathbb{R}^n$ denote the zero column vector and the column vector with all components equal to 1, respectively; when the subscript n is omitted, the dimension n is assumed to be of appropriate size. $\mathrm{I}_n \in \mathbb{R}^{n\times n}$ stands for the identity matrix, and we omit its subscript when $n = 3$. The matrix $S(\cdot) \in \mathbb{R}^{3\times 3}$ is a skew-symmetric matrix and it satisfies $S(\mathbf{a})\,\mathbf{b} = \mathbf{a}\times\mathbf{b}$, for any $\mathbf{a},\mathbf{b} \in \mathbb{R}^3$. The map $\Pi : \{\mathbf{x}\in\mathbb{R}^3 : \mathbf{x}^T\mathbf{x} = 1\} \mapsto \mathbb{R}^{3\times 3}$, defined as $\Pi(\mathbf{x}) = \mathrm{I} - \mathbf{x}\mathbf{x}^T$, yields a matrix that represents the orthogonal projection operator onto the subspace perpendicular to $\mathbf{x}$. We denote the Kronecker product between $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{s\times t}$ by $A\otimes B \in \mathbb{R}^{ms\times nt}$. Given $A_1,\ldots,A_n \in \mathbb{R}^{m\times m}$, for some $n, m\in\mathbb{N}$, we denote $A = A_1\oplus\cdots\oplus A_n \in \mathbb{R}^{nm\times nm}$ (direct sum of matrices) as the block diagonal matrix with block diagonal entries A_1 to A_n. Given $\mathbf{a},\mathbf{b}\in\mathbb{R}^n$, $\mathbf{a} = \pm\mathbf{b} \Leftrightarrow \mathbf{a} = \mathbf{b} \vee \mathbf{a} = -\mathbf{b}$; additionally, we say $\mathbf{a}\neq\mathbf{0}$ and $\mathbf{b}\neq\mathbf{0}$ have the same direction if there exists $\lambda\in\mathbb{R}$ such that $\mathbf{b} = \lambda\mathbf{a}$. We say a function $f : \Omega_1 \mapsto \Omega_2$ is of class C^n, or equivalently $f \in C^n(\Omega_1,\Omega_2)$, if its first $n+1$ derivatives (i.e., $f^{(0)}, f^{(1)},\ldots,f^{(n)}$) exist and are continuous on Ω_1. Finally, given a set H, we use the notation $|H|$ for the cardinality of H.