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Brief paper

Angular velocity nonlinear observer from vector measurements ☆

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ABSTRACT

This paper proposes a technique to estimate the angular velocity of a rigid body from vector measurements. Compared to the approaches presented in the literature, it does not use attitude information nor rate gyros as inputs. Instead, vector measurements are directly filtered through a nonlinear observer estimating the angular velocity. Convergence is established using a detailed analysis of the linear-time varying dynamics appearing in the estimation error equation. This equation stems from the classic Euler equations and measurement equations. A high gain design allows to establish local uniform exponential convergence. Simulation results are provided to illustrate the method.

1. Introduction

This article considers the question of estimating the angular velocity of a rigid body from embedded sensors. This broad question has applications in various fields of engineering and applied science. Some specific examples are as follows. In aerospace, the deployment phase of spinning satellites starts by a detumbling maneuver during which the angular velocity is controlled in an active way until it reaches a zero value (Bošković, Li, & Mehra, 2000). The control strategy employs an estimation of this variable, in closed-loop. High velocity spinning objects are very common in ballistics. The XM25 air-burst rifle (smart-weapon) fires smart shells which estimate their rotation to determine the traveled distance (so that explosion of the projectile can be activated at any user-defined distance). Finally, the problem of angular velocity estimation can also be found in the emerging field of smart devices for sports such as the on-board football camera (Kitani, Horita, & Hideki, 2012) as it is important for athletes in many sports to train their skills to spin a ball.

In the literature, several types of methods have been proposed to address this question. On the one hand, the straightforward solution is to use a strap-down rate gyro (Titterton & Weston, 2004), which directly provides measurements of the angular velocities. However, rate gyros being relatively fragile and expensive components, prone to drift, other types of solutions are often preferred. Instead, a *two-step approach* is commonly employed. The *first step* is to determine attitude from vector measurements, i.e. on-board measurements of reference vectors being known in a fixed frame. Vector measurements play a central role in the problem of attitude determination as discussed in a recent survey (Crassidis, Markley, & Cheng, 2007). In a nutshell, when two or more independent vectors are measured with vector sensors attached to a rigid body, its attitude can be simply defined as the solution of the classic Wahba problem (Wahba, 1965) which formulates a minimization problem having the rotation matrix from a fixed frame to the body frame as unknown. The *second step* is to reconstruct angular velocities from the attitude. At any instant, full attitude information can be obtained (Bar-Itzhack, 1996; Choukroun, 2003; Shuster, 1978, 1990). In principles, once the attitude is known, angular velocity can be estimated from a time-differentiation. The survey (Bar-Itzhack, 2001) names this approach the *derivative method*. However, noise disturbs this process. To address this issue, introducing *a priori* information in the estimation process is a valuable technique to filter-out noise from the estimates. For this reason, numerous observers using Euler's equations for a rigid body have been proposed to estimate angular velocity (or angular momentum, which is equivalent) from full attitude information (Jorgensen & Gravdahl, 2011; Salcudean, 1991; Sunde, 2005; Thienel & Sanner, 2007). Besides this two-step approach, a more direct solution can be proposed. In this paper, we expose an algorithm that directly uses the vector measurements and reconstructs the angular velocity in a simple manner.

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The contribution of this paper is a nonlinear observer reconstructing the angular velocity of a rotating rigid body from vector measurements *directly*, namely by bypassing the relatively heavy first step of attitude estimation. Variants and extensions of this approach can be found in Magnis (2015), Magnis and Petit (2013) and Magnis and Petit (2015a,b). The proposed method allows one to estimate the angular velocity without any gyroscope. Contrary to the method presented in Oshman and Dellus (2003), it does not employ time differentiation of the measurements.

This paper is organized as follows. In Section 2, we introduce the notations and the problem statement. We analyze the attitude dynamics (rotation and Euler equations) and relate it to the measurements. In Section 3, we define a nonlinear observer with extended state and output injection. To prove its convergence, the error equation is identified as a linear time-varying (LTV) system perturbed by a linear–quadratic term. The dominant part of the LTV dynamics can be shown, by a scaling resulting from a high gain design, to generate an arbitrarily fast exponentially convergent dynamics. In turn, this property reveals instrumental to conclude on the exponential uniform convergence of the error dynamics. Illustrative simulation results are given in Section 4. Conclusions and perspectives are given in Section 5.

2. Notations and problem statement

2.1. Notations

Norms. The Euclidean norms in $\mathbf{R}^3$ and in $\mathbf{R}^9$ are denoted by $|\cdot|$. The induced norm on 9×9 matrices is denoted by $\|\cdot\|$. Namely, $\|M\| = \max_{X\in\mathbf{R}^9,\ |X|=1} |MX|$.

The cross-product matrix associated with a vector $x \in \mathbf{R}^3$ is denoted by $[x_\times]$, i.e. $\forall y \in \mathbf{R}^3,\ \ [x_\times]\, y = x \times y$. Namely,

$$[x_\times] \triangleq \begin{pmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{pmatrix}$$

where x_1, x_2, x_3 are the coordinates of x in the standard basis of $\mathbf{R}^3$.

2.2. Problem statement

Consider a rigid body rotating with respect to an inertial frame $\mathcal{R}_i$. Note R the rotation (orthogonal) matrix representing the linear mapping from $\mathcal{R}_i$ to a body frame $\mathcal{R}_b$ attached to the rigid body, expressed in $\mathcal{R}_i$. R satisfies the differential equation

$$\dot{R} = R\,[\omega_\times] \tag{1}$$

where ω is the *angular velocity* of the rigid body expressed in the body frame. The dynamics of ω itself is governed by the famed Euler's equations (Landau & Lifchitz, 1982)

$$\dot{\omega} = J^{-1}\left(J\omega \times \omega + \tau\right) \tag{2}$$

where $J = \operatorname{diag}(J_1, J_2, J_3)$ is the matrix of inertia[1] and τ is the external torque applied to the rigid body.

Consider two reference vectors $\mathring{a}, \mathring{b}$ expressed in the inertial frame. Then, the expressions of a, b in the body frame at time t are

$$a(t) = R(t)^T\mathring{a}, \qquad b(t) = R(t)^T\mathring{b}. \tag{3}$$

The variables a, b are called *vector measurements*. For implementation, they can be produced by direction sensors such as e.g. accelerometers, magnetometers or Sun sensors to name a few (Magnis & Petit, 2014).

We now formulate some assumptions.

Assumption 1. $\mathring{a}, \mathring{b}$ are constants and linearly independent.

Assumption 2. J and τ are known.

Assumption 3. ω is bounded: $|\omega(t)| \le \omega_{\max}$ at all times.

The problem we address in this paper is the following.

Problem 1. From measurements of the type (3), find an estimate $\hat{\omega}$ of the angular velocity ω appearing in (1), assuming it satisfies (2).

Remark 1. Without loss of generality, we assume $\mathring{a}^T\mathring{b} \ge 0$ (if not, one can simply consider $-\mathring{a}$ instead of $\mathring{a}$). We denote

$$p \triangleq \mathring{a}^T\mathring{b} \ge 0.$$

Assumption 1 implies that p is constant and $p \in [0, 1)$. Note that, for all time t

$$a(t)^T b(t) = \mathring{a}^T R(t) R(t)^T \mathring{b} = \mathring{a}^T\mathring{b} = p.$$

3. Observer definition and analysis of convergence

3.1. Observer definition

From Assumption 1, we have $\frac{d}{dt}\mathring{a} = 0$. Hence, the time derivative of the measurement a is

$$\dot{a} = \dot{R}^T\mathring{a} = -[\omega_\times]\,R^T\mathring{a} = a \times \omega \tag{4}$$

and the same holds for $\dot{b} = b \times \omega$. To solve Problem 1, the main idea of the paper is to consider the reconstruction of the extended 9-dimensional state X by its estimate $\hat{X}$

$$X = \begin{pmatrix} a^T & b^T & \omega^T \end{pmatrix}^T, \qquad \hat{X} = \begin{pmatrix} \hat{a}^T & \hat{b}^T & \hat{\omega}^T \end{pmatrix}^T.$$

The state is governed by

$$\dot{X} = \begin{pmatrix} a \times \omega \\ b \times \omega \\ E(\omega) + J^{-1}\tau \end{pmatrix} \tag{5}$$

and the following observer is proposed

$$\dot{\hat{X}} = \begin{pmatrix} a \times \hat{\omega} - \alpha k(\hat{a} - a) \\ b \times \hat{\omega} - \alpha k(\hat{b} - b) \\ E(\hat{\omega}) + J^{-1}\tau + k^2\left(a \times \hat{a} + b \times \hat{b}\right) \end{pmatrix} \tag{6}$$

where $\alpha \in (0, 2\sqrt{1-p})$ and $k > 0$ are constant (tuning) parameters. Denote

$$\tilde{X} \triangleq X - \hat{X} \triangleq \begin{pmatrix} \tilde{a}^T & \tilde{b}^T & \tilde{\omega}^T \end{pmatrix}^T \tag{7}$$

the error state. We have

$$\dot{\tilde{X}} = \begin{pmatrix} -\alpha kI & 0 & [a_\times] \\ 0 & -\alpha kI & [b_\times] \\ k^2[a_\times] & k^2[b_\times] & 0 \end{pmatrix}\tilde{X} + \begin{pmatrix} 0 \\ 0 \\ E(\omega) - E(\hat{\omega}) \end{pmatrix}. \tag{8}$$

In Section 3.4 we will exhibit, for each value $0 < \alpha < (2\sqrt{1-p})$, a threshold value k^*such that for $k > k^*$, $\tilde{X}$ converges locally uniformly exponentially to zero.

[1] Without loss of generality, we consider that the axes of $\mathcal{R}_b$ are aligned with the principal axes of inertia of the rigid body.