



Brief paper

Global attitude estimation using single delayed vector measurement and biased gyro[☆]



Somayeh Bahrami, Mehrzad Namvar

Department of Electrical Engineering, Sharif University of Technology, Tehran, Iran

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ABSTRACT

Time delay in attitude determination systems is unavoidable and it is usually caused by low-quality data sampling, poor sensor synchronization, momentary sensor outage or operational restrictions of sensors. Despite its performance-degrading effect, time delay has been widely neglected in the existing attitude estimation methods. This paper presents a general framework for design of attitude estimators with and without $SO(3)$ manifold restriction by assuming that measurement of a *single* vector measurement is available with *time delay*. Also, measurement of rigid-body angular velocity is considered available with an *unknown bias*. The proposed estimator guarantees *global* and asymptotic convergence of attitude estimate to its true value. The observer gain is calculated by solving a delay-dependent matrix differential equation. Solvability of the differential equation depends on an observability condition related to the reference vectors. To illustrate performance of the proposed observer, we present simulation of a satellite system where the single delayed vector measurement is provided by a magnetometer.

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1. Introduction

The orientation of a rigid body with respect to a known reference frame is called its attitude. The attitude estimation problem has been a wide topic of research in the past decades, see [Crassidis, Markley, and Cheng \(2007\)](#) and the references therein. In most realistic cases, presence of time delay in measurements is inevitable. For examples, it can be induced by low cost on-board sensors, poorly synchronized sensors together with low-rate data sampling, momentary sensor outages and operational restrictions of some attitude sensors. The reader is referred to [Bahrami and Namvar \(2014\)](#) and [Kingston and Beard \(2004\)](#) to see more details about the sources of the measurement delay. It is known that ignoring time delay causes performance degradation of attitude estimators and even leads to divergence due to nonlinearity of rigid-body dynamics and kinematic equations, [Sidi \(1997\)](#).

In [Kingston and Beard \(2004\)](#) a modified extended Kalman filter was used to estimate rigid body attitude in presence of GPS measurement delay. Despite their favorable performance in practice and their popularity ([Crassidis & Markley, 2003](#)), extended

Kalman filters have some drawbacks such as high computational load and unknown convergence properties in case of nonlinear systems, [Martin and Salaun \(2010\)](#). As a result, nonlinear observers have emerged to deal with convergence issues. [Khosravian, Trumpf, Mahony, and Hamel \(2014\)](#) proposed a locally convergent nonlinear attitude estimator on $SO(3)$ for rigid bodies performing high acceleration maneuvers by assuming that measurements of linear velocity and the magnetic field were available with known and constant delays.

A number of continuous-time observers on $SO(3)$ have recently been proposed to achieve almost-global convergence under delay free conditions, [Grip, Fossen, Johansen, and Saberi \(2012\)](#), [Laila, Lovera, and Astolfi \(2011\)](#), [Mahony, Hamel, Trumpf, and Lageman \(2009\)](#), [Martin and Salaun \(2010\)](#), [Sommer, Forbes, Siegart, and Furgale \(2016\)](#), [Tayebi, McGillvray, Roberts, and Moallem \(2007\)](#), [Vasconcelos, Cardeira, Silvestre, Oliveira, and Batista \(2011\)](#), [Zlotnik and Forbes \(2015\)](#). Unfortunately, convergence of $SO(3)$ -based observers is affected by topological restrictions which result in slow convergence for large initial attitude estimation error angles close to π , [Bhat and Bernstein \(2000\)](#). Motivated by practical considerations, globally convergent attitude observers were proposed by [Batista, Silvestre, and Oliveira \(2012\)](#) and [Namvar and Safaei \(2013\)](#) assuming that only a single reference vector measurement was available but without delay. Time delay in vector measurements was considered in [Bahrami and Namvar \(2014\)](#) and global convergence was achieved by a high dimensional nonlinear observer.

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E-mail addresses: bahrami_somayeh@ee.sharif.edu (S. Bahrami), namvar@sharif.ir (M. Namvar).

In the context of attitude control, the effect of unknown time-varying delay was investigated in [Samiei, Butcher, Sanyal, and Paz \(2015\)](#) where locally asymptotic attitude stabilization was achieved. In [Chunodkar and Akella \(2013\)](#), assuming an unknown time-varying delay in the feedback loop, locally exponential attitude regulation was ensured by using complete-type Lyapunov–Krasovskii functionals. The quaternion-based controller used in [Mazenc, Yang, and Akella \(2015\)](#) achieved local attitude stabilization assuming unknown point-wise delay in the input torque. In [Bahrami and Namvar \(2015\)](#), assuming a known delay in the attitude, globally asymptotic attitude tracking was ensured.

In this paper we consider two challenging assumptions. First, we assume that only one delayed vector measurement is available for observer design. This assumption is mostly based on practical considerations in using small satellites, see [Bahrami and Namvar \(2014\)](#), [Batista et al. \(2012\)](#), [Khosravian and Namvar \(2010\)](#), [Laila et al. \(2011\)](#), [Mahony et al. \(2009\)](#), [Namvar and Safaei \(2013\)](#) and the references therein. Second, we assume that gyro measurement is contaminated by an unknown and constant bias. The principal observer has no manifold restriction on $SO(3)$ and hence achieves global convergence. It is also shown how the observer structure can be modified to enforce manifold restriction in cost of achieving almost global convergence. An observability condition is derived to guarantee convergence of the estimator when a single delayed vector measurement is available. Finally, simulation examples illustrate performance of the proposed observer.

Notations. We denote $\|\cdot\|$ as the Euclidean norm of vectors and the induced norm of matrices. The trace of the matrix X is denoted by $\text{tr}(X)$. L_∞ and L_2 denote the space of bounded and square integrable functions.

The following facts are useful in the sequel.

Fact 1. For any matrix $A \in \mathbb{R}^{n \times n}$, the inequality $\text{tr}(A^T A) \leq n\|A\|^2$ is satisfied, [Gene and Golub \(2013\)](#).

Fact 2 ([Gu, 2001](#)). Any matrix A satisfies the inequality $(\int_0^h A(s)^\top ds)(\int_0^h A(s) ds) \leq h \int_0^h A(s)^\top A(s) ds$.

2. Problem formulation

We consider the rigid body kinematics as $\dot{R} = RS(\omega)$, where $R \in SO(3)$ is the rotation matrix of the body frame with respect to the inertial frame. $\omega \in \mathbb{R}^3$ is the angular velocity of the body frame, expressed in the body frame. $S(\cdot)$ is the skew-symmetric matrix with the property $\text{tr}(XS(\omega)X^\top) = 0$. The output of the on-board attitude sensor is a vector measurement, expressed in the body frame, and we denote it by $v_b \in \mathbb{R}^3$. We define the reference vector $v_r \in \mathbb{R}^3$ as the representation of the measured vector v_b in the inertial frame. Obviously, we have $v_b = R^\top v_r$.

In this note we assume that only a single vector measurement and the corresponding reference vector are available with a known time-delay $h \geq 0$. Without loss of generality, we assume that $\|v_r\| = \|v_b\| = 1$. Furthermore, we assume that the measurement of the angular velocity is available with a constant and unknown bias $b \in \mathbb{R}^3$. Our problem is to design an adaptive observer by using $v_b(t-h)$ and $v_r(t-h)$ together with the measurement of the rigid-body angular velocity ω_s , in order to estimate the attitude matrix R , globally and asymptotically.

3. Observer design: bias free case

In this section we assume that the angular velocity is available without bias, $\omega_s = \omega$. Let \hat{R} denote the estimate of R . The proposed

observer is as follows

$$\dot{\hat{R}} = \hat{R}S(\omega) + L(t)\tilde{v}_b^\top(t-h) \quad (1)$$

$$\tilde{v}_b^\top(t-h) = v_r^\top(t-h)\hat{R}(t-h) - v_b^\top(t-h) \quad (2)$$

$$L = -\beta Q^{-1}v_r(t-h) \quad (3)$$

$$\dot{Q} = g_1 v_r(t-h)v_r^\top(t-h) - (\delta_1 + \delta_2)Q \quad (4)$$

with the initial condition $\hat{R}(t) = \hat{R}_0 \in SO(3)$, for all $t \in [-h, 0]$, and $Q(0) = Q_0 \in \mathbb{R}^{3 \times 3} > 0$. $L \in \mathbb{R}^3$ is the observer gain. δ_1 and δ_2 are appropriate positive scalars. Also, positive scalars β , p_1 , p_2 are chosen such that

$$g_1 := 2\beta - h\beta^2(p_1^{-1} + p_2^{-1}) > 0. \quad (5)$$

Remark 1. If the delay in evaluation of v_r is different from the delay in measurement of v_b , we set $h = \max\{h_b, h_r\}$ where h_b and h_r are the delays in v_b and v_r , respectively.

Assumption 1 (Observability). We assume that there exist positive scalars T_v and μ_v such that

$$\int_t^{t+T_v} v_r(s-h)v_r^\top(s-h)ds \geq \mu_v I, \quad \forall t \geq 0. \quad (6)$$

As shown in [Moeini and Namvar \(2016\)](#), [Assumption 1](#) introduces an explicit condition on the reference vectors for guaranteeing observability of the system, and depends on orbit characteristics and type of the used attitude sensors.

Remark 2. A practical way to verify the condition (6) and to compute T_v and μ_v , is to calculate μ_v as a function of T_v by $\mu_v(T_v) = \min_{t \geq 0} \underline{\lambda}(\int_t^{t+T_v} v_r(s-h)v_r^\top(s-h)ds)$ where $\underline{\lambda}(\cdot)$ denotes the smallest eigenvalue of the matrix. Furthermore, it is possible to use the value of T_v that maximizes μ_v , [Khosravian and Namvar \(2010\)](#).

Remark 3. It is straightforward to show, [Namvar and Safaei \(2013\)](#), that under [Assumption 1](#) and condition (5), any solution of (4) is positive definite and satisfies $Q(t) \geq \min\{\underline{\lambda}(Q_0), g_1\mu_v\} e^{-(\delta_1 + \delta_2)T_v} I =: q_m I, \forall t \geq 0$.

Theorem 1. Consider the observer given by (1)–(4). For given value of delay h , assume that the positive scalars p_1 , p_2 , β , δ_1 and δ_2 are chosen based on (5) and also satisfy

$$\delta_2 q_m - 3hp_1\gamma_\omega^2 > 0, \quad \delta_1 q_m^3 - 3hp_2\beta^2 > 0, \quad (7)$$

where q_m is defined by [Remark 3](#), and the positive scalar γ_ω satisfies $\|S(\omega)\| \leq \gamma_\omega$. Then, \hat{R} converges to R , globally and asymptotically. \square

3.1. Selection of the design parameters

The positive design parameters β , p_1 , p_2 , δ_1 and δ_2 must be chosen such that conditions (5) and (7) are satisfied. In the sequel we propose a simplified algorithm for selection of the parameters:

1. Based on the used attitude sensor, compute the reference vector v_r and verify the observability condition (6) to obtain T_v and μ_v .
2. Set $\delta_1 = \delta_2 = (6T_v)^{-1}$.
3. Select $p_1, p_2 > 0$ such that for given value of delay h , the condition $h^2 \leq \min\left(\frac{\mu_v}{3p_1\gamma_\omega^2}, \frac{\mu_v}{3p_2}\right) \frac{1}{p_1^{-1} + p_2^{-1}} \eta_{\max}$ is satisfied, where $\eta_{\max} = (6T_v)^{-1}e^{-1}$.
4. Set $\beta = h^{-1}(p_1^{-1} + p_2^{-1})^{-1}$.

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