Automatica 75 (2017) 109–118

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Stochastic link activation for distributed filtering under sensor power constraint☆

Wen Yang [a], Chao Yang [a], Hongbo Shi [a], Ling Shi [b], Guanrong Chen [c]

[a] Key Laboratory of Advanced Control and Optimization for Chemical Processes(East China University of Science and Technology), Ministry of Education, China

[b] Department of Electronic and Computer Engineering, Hong Kong University of Science and Technology, Hong Kong, China

[c] Department of Electronic Engineering, City University of Hong Kong, Hong Kong, China

ARTICLE INFO

Article history:
Received 12 July 2015
Received in revised form
8 May 2016
Accepted 10 August 2016

Keywords:
Distributed filtering
Sensor scheduling
Consensus
Convex optimization

ABSTRACT

We consider the problem of link activation for distributed estimation with power constraint. To satisfy the requirement of power consumption, we propose a stochastic link activation scheme, where each sensor equipped with a distributed estimator sends data to its neighboring sensors according to different probabilities. First, we design the optimal estimator gain of each sensor to minimize the state estimation error covariance. Then, we find an upper bound of the expected state estimation error covariance and provide a sufficient condition to guarantee the stability of the proposed estimator. Finally, we formulate the link activation problem as an optimization problem, and convert it to a convex optimization.

1. Introduction

Spurred by applications in various fields including battlefield surveillance, intelligent transportation, environment monitoring, and health care, recently there has been a surge of interest in distributed state estimation using a wireless sensor network (WSN). A wireless sensor network consists of a large number of geographically distributed sensor nodes, which are capable of measuring certain parameters of interest, such as temperature, humidity, position and velocity of a vehicle. In the last decade, many works on consensus-based distributed estimation were reported, typically under the assumption that each sensor can observe the target state and exchange the estimates with its neighbors (Cattivelli & Sayed, 2010; Cui, Zhang, Lam, & Ma, 2013; Demetriou, 2010; Dong, Wang, Lam, & Gao, 2014; Li & Ghassan, 2007; Li & Guo, 2014; Olfati-Saber, 2009; Ren, Beard, & Kingston, 2005; Schizas, Ribeiro, & Giannakis, 2008; Shen, Wang, Shu, & Wei, 2011; Spanos, Saber, & Murray, 2005; Stanković, Stanković, & Stipanović, 2009; Xi, He, & Liu, 2010; Yan, Qian, Zhang, Yang, & Guo, 2016; Zhang, He, & Chen, 2016). Distributed estimation strategy does not rely on the network topology, has lower energy cost, and is more flexible for ad-hoc deployment when compared with centralized and decentralized estimations (Anderson & Moore, 1979; Iftar, 1993; Rao, Durrant-Whyte, & Sheen, 1993; Sanders, Tacker, & Linton, 1974). Such distributed state estimation, however, faces a common issue that the energy of sensors distributed in a complex environment is usually limited and the onboard batteries are difficult to recharge. In most cases, a sensor consumes a considerable amount of energy when sending data to its neighboring sensors, which is often unnecessary. For a sensor, effectively selecting some of its neighboring sensors to send data to can efficiently extend the lifetime of its power source while guaranteeing a desired level of estimation quality.

In recent years, communication power consumption minimization and battery lifetime maximization for sensor networks have been extensively investigated, where sensor scheduling as an effective methodology has been carefully studied (Gupta, Chung, Hassibi, & Murray, 2006; Joshi & Boyd, 2009; Liu, Wang, He, & Zhou, 2015; Mo, Ambrosino, & Sinopoli, 2011a; Mo, Garone, Casavola, & Sinopoli, 2011b; Ren, Cheng, Chen, Shi, & Zhang, 2014; Shi, Chen, & Shi, 2014; Shi, Cheng, & Chen, 2011; Yang et al., 2015; Zhang, Cheng, Shi, & Chen, 2016). The existing works can be classified into three categories: (1) Deterministic scheduling

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Martin Enqvist under the direction of Editor Torsten Söderström.

E-mail addresses: weny@ecust.edu.cn (W. Yang), yangchao@ecust.edu.cn (C. Yang), hbshi@ecust.edu.cn (H. Shi), eesling@ust.hk (L. Shi), eegchen@cityu.edu.hk (G. Chen).

http://dx.doi.org/10.1016/j.automatica.2016.09.009

(Joshi & Boyd, 2009; Mo et al., 2011a; Shi et al., 2011; Yang & Shi, 2015; Zhang et al., 2016): the sequence indicates when to use sensors to send data, where timing is fixed at each step. In Joshi and Boyd (2009), Mo et al. (2011a) and Yang and Shi (2015), a convex optimization problem is formulated to select an optimal subset of the sensors subject to limited power or channel bandwidth. In Shi et al. (2011), a sensor scheduling scheme based on minimizing the estimation error is discussed for two scenarios: the sensor has sufficient computational capability and the sensor has limited computational capability. (2) Stochastic scheduling (Gupta et al., 2006; Mo et al., 2011b): each sensor sends data to a remote estimator according to a given probability at each time step, and an optimal probability distribution is obtained by solving an optimization problem. In Gupta et al. (2006), a stochastic sensor selection strategy is proposed to schedule multiple sensors which cannot send data simultaneously. (3) Event-triggered scheduling (Liu et al., 2015; Ren et al., 2014; Shi et al., 2014): these schedules utilize real-time measurements from the selected sensors. In Ren et al. (2014), the power scheduling problem is formulated as a Markov decision process, and a simple optimal dynamic schedule is developed, which minimizes the average estimation error under the energy constraint. In Shi et al. (2014), the state estimation is considered based on the information from multiple sensors that provide their measurement updates according to separate event-triggering conditions. In Liu et al. (2015), an event-based recursive distributed filter is examined under a pre-determined Send-on-Delta data transmission condition.

In this work, we consider optimal stochastic sensor scheduling for distributed estimation subject to limited power constraint. For deterministic sensor scheduling, finding an optimal sequence of binary values, which indicates when a sensor sends data, is in general difficult. Compared with a deterministic schedule, a stochastic one is more flexible, which provides more feasible schedules and distributes the energy of the sensors more uniformly. On the other hand, although event-triggered scheduling based on real-time measurements leads to better estimation accuracy, it often has high computational complexity. Moreover, the analysis of event-triggered schedules for distributed estimate is much more difficult in general.

Most of the aforementioned works focused on scheduling sensors efficiently with a single estimator, i.e., on a network composed of multiple sensing devices and one fusion center. To the best of our knowledge, very few studies are devoted to scheme design of sensor scheduling for distributed state estimation because the tight coupling among the sensors bring additional challenges to analyzing the estimator stability and performance, not to say designing optimal scheduling scheme. In Yang, Chen, Wang, and Shi (2014), we investigate a particular scenario of this problem, where each sensor is activated with an identical probability to transmit its estimates. When a sensor is activated, it can transmit data to all of its neighboring sensors. We refer to this method of obtaining estimates as a stochastic *sensor activation scheme*. Such a stochastic scheme is very easy to implement in practice, under which the communication loads of the sensors are equally distributed. The design with an identical activating probability of the sensors, however, is unnecessary in most cases, which will limit the applications of the scheme. For example, in a heterogeneous sensor network, it is reasonable to assign the activating probability to a sensor by jointly considering its power and estimation accuracy.

In this paper, we extend the study of Yang et al. (2014) to a more challenging and practical case, where each sensor decides to send its data to any one of its neighboring sensors *with different probabilities* subject to *power constraint*. As such, the scheme will activate the communication link between each pair of sensors, referred to as the *link activation scheme*. Thus, sensor activation scheme with an identical probability becomes a special case of the current one. Since more feasible activation solutions become available, the new link activation scheme leads to better estimation accuracy with more adaptive and efficient sensor allocation. In addition, it is more robust in a time-varying circumstance. For example, when some sensors are corrupted or attacked, it is easier to adjust the activation probability of the communication link than the deterministic transmitting sequence.

The main contributions of this paper are summarized as follows.

(1) Different from the existing works (Cattivelli & Sayed, 2010; Olfati-Saber, 2009; Schizas et al., 2008; Stanković et al., 2009), we investigate distributed filtering under stochastic link activation of a sensor network with limited energy;
(2) We obtain an optimal estimator for each sensor by minimizing its estimation error covariance for known link probabilities;
(3) We derive an upper bound of the expected estimation error covariance and provide a sufficient condition for its stability, which relates to the network topology and scheduling sequence;
(4) We obtain a set of optimal probabilities by relaxing the optimal stochastic link activation to a convex problem that can be easily solved.

The remainder of the paper is organized as follows. In Section 2, we introduce the system model and derive the optimal gain for the distributed estimators. In Section 3, we study the stability of the proposed estimator using stochastic link activation, and derive an upper bound of the expected estimation error covariance. In Section 4, we formulate the stochastic link activation under power constraint as an optimization problem, which involves a series of linear matrix inequalities (LMIs). We present a numerical example in Section 5 to illustrate the performance of the optimal activation scheme. We provide some concluding remarks in the end.

Notation. $\mathrm{tr}(\cdot)$ denotes the trace of a matrix. $\mathrm{vec}(A)$ is the vector formed by "stacking" the columns of A in the natural order. $\mathrm{diag}(A_i)$ denotes a block diagonal matrix with main diagonal block equal to A_i. $|A|$ denotes the cardinality of a set A. For matrices A and B, $A \otimes B$ is their Kronecker product. $A \geq 0$ if A is positive semi-definite, and $A \geq B$ if $A - B \geq 0$. Moreover, $A > 0$ if A is positive-definite, and $A > B$ if $A - B > 0$. $\mathbf{1}$ denotes a vector of arbitrary dimension with each component equal to one. I denotes the identity matrix. $\circ$ denotes the Hadamard product. To simplify the symmetrical notation, $(X)Y(\cdot)^T$ means $(X)Y(X)^T$, and $(X)(\cdot)^T$ means $(X)(X)^T$.

2. Problem statement

Consider the following discrete linear time-invariant system:

$$x(k+1) = Ax(k) + w(k), \tag{1}$$

where $x(k) \in \mathbb{R}^m$ is the system state vector, $w(k) \in \mathbb{R}^m$ is the process noise. Assume that $x(0)$ and $w(k)$ are independent zero-mean Gaussian random vectors with covariances Π_0 and Q, respectively.

A sensor network composed of n sensors is used to measure $x(k)$. The measurement equation of the ith sensor is given by

$$y_i(k) = H_i x(k) + v_i(k), \tag{2}$$

where $v_i(k) \in \mathbb{R}^{m_i}$ is zero-mean white Gaussian with covariance matrix $R_i > 0$ which is independent of $x(0)$, $w(k)$ $\forall k,\ i$, and is independent of $v_j(s)$ when $i \neq j$ or $k \neq s$.

We model the sensor network as a directed graph $G = (V, E)$ with the nodes $V = \{1, 2, \ldots, n\}$ being the sensors and the edges $E \subset V \times V$ representing the communication links. An edge (i, j) means the existence of a link from sensor j to sensor i. The in-neighboring sensors of sensor i is denoted by $N_i = \{j : (i, j) \in E\}$, and its dimension is denoted by $d_i = |N_i|$. Also, denote the set of