Automatica 75 (2017) 127–132

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

# Continuous observer design for a class of multi-output nonlinear systems with multi-rate sampled and delayed output measurements[☆]

Yanjun Shen[a], Daoyuan Zhang[a], Xiaohua Xia[b]

[a] Hubei Provincial Collaborative Innovation Center for New Energy Microgrid, China Three Gorges University, Yichang, Hubei, 443002, China

[b] Department of Electrical, Electronic and Computer Engineering, University of Pretoria, Pretoria 0002, South Africa

ARTICLE INFO

*Article history:*
Received 25 October 2014
Received in revised form
18 July 2016
Accepted 20 August 2016

*Keywords:*
Hybrid systems
Continuous observer
Multi-rate
Delayed measurements

ABSTRACT

In this paper, continuous observer is designed for a class of multi-output nonlinear systems with multi-rate sampled and delayed output measurements. The time delay may be larger or less than the sampling intervals. The sampled and delayed measurements are used to update the observer whenever they are available. Sufficient conditions are presented to ensure global exponential stability of the observation errors by constructing a Lyapunov–Krasovskii function. A numerical example is given to illustrate the effectiveness of the proposed methods.



## 1. Introduction

Recently, the problem of design global convergent observers for nonlinear systems has made great progress. For the observation of nonlinear systems, one can use extended Luenberger observers (Zeitz, 1987), normal form observers (Bestle & Zeitz, 1983; Krener & Isidori, 1983; Xia & Gao, 1988, 1989), Lyapunov based observers (Raghavan & Hedrick, 1994; Thau, 1973), high-gain observers (Gauthier, Hammouri, & Othman, 1992; Gauthier & Kupka, 1994), sliding mode observers (Haskara, Özgüner, & Utkin, 1998) and moving horizon/optimization based observers (Michalska & Mayne, 1995). Among these methods, high-gain observers play an important role and can be used to a large class of nonlinear systems with a triangular structure after a coordinate change. New developments of high gain observers have been carried out in various directions (Andrieu, Praly, & Astolfi, 2009; Deza, 1991; Deza, Bossanne, Busvelle, Gauthier, & Rakotopara, 1993; Gauthier et al., 1992; Praly, 2003). For example, the result of Gauthier et al. (1992) is extended to a class of nonlinear systems where the nonlinear terms admit an incremental rate depending on the measured output (Praly, 2003). In Deza (1991), the authors considered observer design for multi-input and multi-output (MIMO) nonlinear systems. The result has been extended to a class of MIMO nonlinear systems, in which interconnection between the blocks are not allowed (Deza et al., 1993). Based on the observer normal form, another extension for the multi-output systems has been studied in Rudolph and Zeitz (1994). However, the nonlinearity of each block does not allow the unmeasurable states of its own block. Under the conditions of observability and triangular structure, a nonlinear system can be transformed into the block low triangular form considered in Shim, Son, and Seo (2001) by a coordinate transformation. Then, semi-global observer has been designed for nonlinear systems with interconnections between the subsystems (Shim et al., 2001). The nonlinear system with block lower triangular form is rather general when nonlinear changes of coordinates are allowed. It includes the control-affine multi-input and single-output (MISO) nonlinear systems which are strongly observable for any input (Gauthier et al., 1992) and the control-affine MIMO nonlinear systems which are strongly observable for any input for each output taken separately (Deza, 1991). Moreover, it can be used to express some physical systems. For example, the dynamical equations of a permanent magnet stepper motor can be transformed into the block lower triangular form Mahmoud and Khalil (2002). The estimation errors can converge to the origin in finite-time by using high gain observers

☆ This work was supported by the National Science Foundation of China (61374028, 61273183), the Scientific Innovation Team Project of Hubei Provincial Department of Education (T201504). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Dragan Nesic under the direction of Editor Andrew R. Teel.

*E-mail addresses:* shenyj@ctgu.edu.cn (Y. Shen), 944732896@qq.com (D. Zhang), xxia@up.ac.za (X. Xia).

http://dx.doi.org/10.1016/j.automatica.2016.09.028
0005-1098/

in conjunction with applications of geometric homogeneity and Lyapunov theories (Li, Xia, & Shen, 2013; Shen & Huang, 2009; Shen & Xia, 2008).

It should be noted that the above results on observer design are based on continuous-time analysis. However, for a networked control system, the output is only available at discrete-time instants since it is usually transmitted through a shared band-limited digital communication network. Therefore, observer design for continuous systems with sampled and delayed output measurements has attracted the control community wide attention. There exist three main approaches to design observer for continuous systems with sampled and delayed measurements, for example, discrete time analysis based on a discretized model (Arcak & Nešić, 2004; Barbot, Monaco, & Normand-Cyrot, 1999; Nešić, Teel, & Kokotović, 1999), continuous time analysis followed by discretization (Khalil, 2004; Nešić & Teel, 2004; Postoyan & Nešić, 2012; Wang, Nešić, & Postoyan, 2015), and a mixed continuous and discrete time analysis without discretization (Ahmed-Ali & Lamnabhi-Lagarrigue, 2012; Ahmed-Ali, Van Assche, Massieu, & Dorléans, 2013; Deza, Busvelle, Gauthier, & Rakotopora, 1992; Karafyllis & Kravaris, 2009; Nadri, Hammouri, & Grajales, 2013; Raff et al. Raff, Kögel, & Allgöwer, 2008; Van Assche, Ahmed-Ali, Ham, & Lamnabhi-Lagarrigue, 2011; Zhang, Shen, & Xia, 2014). More specifically, two classes of global exponential observers have been presented for a class of continuous systems with sampled and delayed measurements in Ahmed-Ali, Van Assche et al. (2013). By using the same methods, exponential convergent observers were proposed for nonlinear systems with sampled and delayed measurements in Ahmed-Ali, Karafyllis, and Lamnabhi-Lagarrigue (2013). The observers designed in Ahmed-Ali, Karafyllis et al. (2013) and Ahmed-Ali, Van Assche et al. (2013) are in essence discontinuous. The authors in Zhang et al. (2014) proposed a continuous observer for a class of nonlinear systems with sampled and delayed measurements based on an auxiliary integral technique. But there is a constraint condition on time delay, that is the maximum delay must be less than the minimum sampling interval as in Ahmed-Ali, Karafyllis et al. (2013) and Ahmed-Ali, Van Assche et al. (2013).

In this paper, we address continuous observer design for a class of multi-output nonlinear systems with multi-rate sampled and delayed output measurements. The considered nonlinear systems are in continuous time while the outputs are in discrete time. In order to overcome the difficulties in analysis, we represent the sampled-data system as a continuous time system with successive delay components by some transformations. The time delays are more general than those in Ahmed-Ali, Karafyllis et al. (2013), Ahmed-Ali, Van Assche et al. (2013) and Zhang et al. (2014) since they may be larger or smaller than the sampling periods. Our main contributions include the following: (a) Continuous observer is designed for a class of multi-output nonlinear systems whenever the sampled and delayed measurements are available. (b) The observer is transformed into a continuous nonlinear system with time-varying delay by time delay method. Then, by constructing a Lyapunov–Krasovskii function, sufficient conditions are presented to ensure that the observation errors are globally exponentially stable. (c) Different high gains are used to dominate the nonlinear terms in each block. Then, upper bounds on each sampling period and time delay are also achieved.

This paper is organized as follows. In Section 2, continuous observers are presented for a class of multi-output nonlinear systems with multi-rate sampled and time delayed measurements. In Section 3, an example is used to illustrate the validity of the proposed design methods. Finally, Section 4 concludes the paper.

Throughout this paper, let $\mathbb{R}^n$ denote $n$-dimension real space, $I$ denote an identity matrix, diag{} denote a diagonal matrix, and the superscript "$\top$" stand for matrix transposition. For any $x \in \mathbb{R}^n$, let $\|x\| = (x^\top x)^{1/2}$. For a continuous function $f : \mathbb{R} \to \mathbb{R}$ and $t \in \mathbb{R}$, let $\lim_{s\to t^-} f(s) = \lim_{s\to t, s<t} f(s)$. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the largest and the smallest eigenvalues of $P \in \mathbb{R}^{n\times n}$, respectively.

## 2. Main results

In this section, we consider the following multi-output nonlinear systems:

$$\begin{cases}\dot{x}(t) = Ax(t) + B(x(t), u(t)),\\ y(t) = Cx(t) = [C_1x^1(t), \ \dots \ C_mx^m(t)]^\top,\end{cases} \tag{1}$$

where the state $x(t) \in \mathbb{R}^n$, the input $u(t) \in \mathbb{R}^p$, the output $y(t) \in \mathbb{R}^m$, $x(t) = [x^1(t)^\top, \dots, x^m(t)^\top]^\top$, $x^i(t) \in \mathbb{R}^{\lambda_i}$ $(1 \le i \le m)$ is the $i$th partition of the state $x(t)$; $A = \mathrm{diag}\{A_1, \dots, A_m\}$, $A_i$ is $\lambda_i \times \lambda_i$ matrix of Brunovsky form, that is $A_i = \begin{bmatrix}0 & 1 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & 1\\ 0 & 0 & \cdots & 0\end{bmatrix}$, $C = \mathrm{diag}\{C_1, \dots, C_m\}$, $C_i = [1, 0, \dots, 0]_{1\times\lambda_i}$, and $B(x(t), u(t)) = [b^1(x(t), u(t))^\top, \dots, b^m(x(t), u(t))^\top]^\top$ in which the $j$th element of $b^i(\cdot)$, $b^i_j(\cdot)$ has the following structural dependence on the states:

$$b^i_j(t) = b^i_j(x^1(t), \dots, x^{i-1}(t); x^i_1(t), \dots, x^i_j(t); u(t)),$$

for all $1 \le i \le m$ and $1 \le j \le \lambda_i$. Thus, $b^i_j$ is independent of the lower states $(x^i_{j+1}, \dots, x^i_{\lambda_i})$ of the $i$th block and the states of the lower blocks $(x^{i+1}, \dots, x^m)$. The $i$th block of the above system can be expressed as follows:

$$\begin{cases}\dot{x}^i_1(t) = x^i_2(t) + b^i_1(x(t)^{[1,i-1]}; x^i_1(t); u(t)),\\ \quad\vdots\\ \dot{x}^i_{\lambda_i-1}(t) = x^i_{\lambda_i}(t) + b^i_{\lambda_i-1}(x(t)^{[1,i-1]}; x(t)^i_{[1,\lambda_i-1]}; u(t)),\\ \dot{x}^i_{\lambda_i}(t) = b^i_{\lambda_i}(x(t)^{[1,i-1]}; x(t)^i_{[1,\lambda_i]}; u(t)),\end{cases} \tag{2}$$

where $x^i_j(t)$ is the $j$th element of the $i$th block $x^i(t)$. The abbreviation $x(t)^{[1,k]} := [x^1(t)^\top, \dots, x^k(t)^\top]^\top$ and $x(t)^i_{[1,j]} := [x^i_1(t), \dots, x^i_j(t)]^\top$ can be used to simplify the notation. We assume that there are $m$ sensors in $m$ channels to sample the output $y$ at sampling instants $t^i_k$, and $t^i_k < t^i_{k+1}$ $(i = 1, \dots, m$ and $k = 0, 1, 2, \dots, \infty)$, where $\{t^i_k\}$ $(i = 1, \dots, m)$ are strictly increasing sequences and satisfy that $\lim_{k\to\infty} t^i_k = \infty$. The sampled measures are available at instants $t^i_k + \tau^i_k$ $(i = 1, \dots, m)$, where $\tau^i_k > 0$ $(i = 1, \dots, m)$ denote the transmission delay, which are unknown but have an upper bound $\bar{\tau}_i$. The nonlinear terms $b^i_j(\cdot)$ are assumed to satisfy the following global Lipschitz conditions with Lipschitz constant $l_1 > 0$,

$$\begin{aligned}&\big|b^i_j(x^1, \dots, x^{i-1}; x^i_1, \dots, x^i_j; u) - b^i_j(\hat{x}^1, \dots, \hat{x}^{i-1}; \hat{x}^i_1,\\ &\quad \dots, \hat{x}^i_j; u)\big| \le l_1\big(|x^1_1 - \hat{x}^1_1| + |x^1_2 - \hat{x}^1_2| + \cdots |x^i_j - \hat{x}^i_j|\big),\\ &\qquad 1 \le i \le m, \ 1 \le j \le \lambda_i.\end{aligned} \tag{3}$$

Now, the explicit form of the $i$th block of the observer is given as follows:

$$\begin{cases}\dot{\hat{x}}^i_1(t) = \hat{x}^i_2(t) + L_i a^i_1 e^i_1(t^i_k) + b^i_1(\hat{x}(t)^{[1,i-1]}; \hat{x}^i_1(t); u(t)),\\ \quad\vdots\\ \dot{\hat{x}}^i_{\lambda_i-1}(t) = \hat{x}^i_{\lambda_i}(t) + L_i^{\lambda_i-1} a^i_{\lambda_i-1} e^i_1(t^i_k)\\ \quad + b^i_{\lambda_i-1}(\hat{x}(t)^{[1,i-1]}; \hat{x}(t)^i_{[1,\lambda_i-1]}; u(t)),\\ \dot{\hat{x}}^i_{\lambda_i}(t) = L_i^{\lambda_i} a^i_{\lambda_i} e^i_1(t^i_k) + b^i_{\lambda_i}(\hat{x}(t)^{[1,i-1]}; \hat{x}(t)^i_{[1,\lambda_i]}; u(t)),\\ \hat{x}^i_j(t^i_{k+1} + \tau^i_{k+1}) = \lim\limits_{t\to t^i_{k+1}+\tau^i_{k+1}\,^-} \hat{x}^i_j(t),\\ \quad j = 1, 2, \dots, \lambda_i, \ t \in [t^i_k + \tau^i_k, t^i_{k+1} + \tau^i_{k+1}), \ k \ge 0,\end{cases} \tag{4}$$