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A variational approach for coupling kinematically incompatible structural models

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Abstract

In this work an extended variational framework aimed at properly addressing the coupling of kinematically incompatible structural models is presented. The main goal is to variationally state the theoretical bases to deal with the coupling of structural models with different dimensionality. In this approach, the coupling conditions are naturally derived from the governing variational principle formulated at the continuous level. Furthermore, by means of a real parameter γ we manage to build different continuous mechanical models that have different mechanical and mathematical features. In particular, the coupling of 3D solid models and 2D shell models, under Naghdi hypothesis, is treated by introducing the corresponding kinematical assumptions into the proposed extended variational principle. Also, the coupling between 3D solid and 1D beam models, under Bernoulli hypothesis, is presented. Moreover, for the continuous 3D–2D coupled problem a numerical approximation is addressed via the finite element method and some numerical results are given, comparing the responses of the system when the discrete model varies by changing the value of the parameter γ. Finally, a discussion comprising the main conclusions of the work is given.

Keywords: Incompatible kinematics; Multidimensional models; Variational formulation; Coupling conditions; Structural models

1. Introduction

In structural analysis it is common to make use of reduced models in order to represent the main phenomena involved in the problem. Those models are built by taking advantage of the particular form of the geometry of the structural component and of the loading acting on it. In this way, full 3D models can be condensed into shells, plates or even beams. This kind of reduction in the dimension of the problem, within the context of primal variational formulations, is given by deeming suitable kinematical assumptions, that is, assuming a particular form for the displacement vector field.

The situation we are interested in involves a structural component whose geometry and loadings take a very general configuration on a given portion of the domain of analysis, for which it is necessary to work with the full 3D model, whereas they take a particular configuration over the rest of the domain, where it is possible to introduce some kinematical restrictions. Research oriented to this field was formerly conducted in the 1980s in works that dealt with the problem of junctions between plates, and between plates and 3D elastic bodies [1,4,7]. Lately, the problem was also extended to junctions between shells, while the numerical study of junctions between elastic bodies and plates continued [2,3,9]. In the 1990s, some works covered the asymptotic analysis for the coupling between a 3D elastic body and a dimensionally reduced structure [10,13,14]. In all the aforementioned works the problem of performing a junction was analyzed from a very different standpoint than

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the one presented in this work. Indeed, in the present work the problem of coupling structural components is generalized. It will be seen that it is possible to handle any kind of junction in spite of the possibility of non-linear constitutive behaviors, as well as non-linearities arisen from large displacements. Thus, all those problems treated in the aforesaid literature can be embraced in the ideas developed here. It is worth noting that, up to the authors' knowledge, there is no previous works that deal into a unified continuous variational framework with the coupling between different structural solid models. Previous ideas regarding the coupling of models of different dimensionality from a purely kinematical point of view were explored in [8,17], but they have recently been variationally stated for the fluid flow problem in [5].

The concepts introduced here can also be useful in the context of domain decomposition techniques as a clever alternative approach to formulate a problem with non-matching meshes. Although in this situation there exists kinematical compatibility at the continuous level, this compatibility is lost when passing to the discrete level by introducing different approximations for the different partitions of the domain of analysis. Thus, the theory is also applicable for handling partitioned systems, and from this perspective it involves several well-known methods as those proposed in [15,16].

According to what was said in a previous paragraph, we have a structural component in which, for simplicity but without loss of generality, two possible incompatible kinematics coexist (this number can be arbitrary). Since the involved fields may suffer jumps over the locations where the kinematics changes, the original governing variational principle is not correctly stated. Then, the need for an extended variational formulation can be interpreted as a consequence of working with non-matching underlaying kinematics defined over complementary portions of the domain of analysis. This formulation is mathematically founded on the Lagrange multiplier theory, and can be understood from a mechanical viewpoint by means of introducing, into the original variational principle, some terms related to the jump in the fields, allowing discontinuities to occur over an artificial internal boundary where the kinematics changes. In this manner, the extended governing variational principle yields, as the natural Euler–Lagrange equations, besides the equilibrium equations, the coupling conditions between both models. These natural coupling conditions depend exclusively upon the kinematics adopted for each domain. It will be seen that, according to the way in which the jumps over the internal boundary are introduced, different possibilities regarding the final Euler–Lagrange equations emerge. This is done by means of a real parameter γ. By changing the value of this real number, which was chosen to be in the closed interval [0,1], it is possible to arrive at two different mechanical models within the continuous setting. Each one of these models comprise important mechanical and mathematical characteristics. This issue is addressed in each particular case when dealing with each particular coupling problem. Moreover, this extended variational principle holds the property of consistency in the following sense: when no difference between the kinematics is considered, the Euler–Lagrange equations ensure exactly the same solution of the original problem without any discontinuities.

Having taken into account the possible occurrence of discontinuities we are in position to perform any kind of kinematical restriction over just a portion of the domain of analysis, reducing the full 3D model to, for example, a beam or a shell model. It is worthwhile to mention here that when taking such a restriction over the displacement field we are also altering the way in which the involved duality product is defined. This aspect entails interesting outcomes for the setting of the whole problem, such as changes in the regularity conditions in the part of the domain complementary to that in which the kinematics changed. Then, once the foundations of the coupled problem are well established, a numerical approximation constitutes the next step in the modeling process. In this regard any suitable numerical scheme may eventually be chosen. In the present work a particular computational implementation is developed and an analysis concerning the features of the coupled discrete system is conducted through some numerical examples. Also, the dependence of the attributes of the discrete mechanical models with respect to the real parameter γ is evaluated.

The organization of this paper is as follows. In Section 2 the original and extended variational principles are presented. The coupling between 3D full models and 2D shell models, under Naghdi hypothesis [11,12], is derived in Section 3, while in Section 4 the coupling between 3D full models and 1D beam models, under Bernoulli hypothesis, is obtained. In all cases we formulate the equilibrium problem for a general constitutive behavior, although a simple example involving linear elastic materials in small displacements is presented to prove existence and uniqueness of the solution. The theory is introduced for any material, but the constitutive modelling problem is left aside in this work since it can be managed according to the classical constitutive theory well established for each model. Numerical results for the problem involving the coupling between a 3D solid and a 2D plate (infinite shell curvature) under Naghdi hypothesis are presented in Section 5 together with the corresponding finite element approximation. In Section 6 final remarks and general conclusions are given.

2. The variational framework

In this section the usual variational principle is recalled, then the extended governing variational formulation is devised in order to allow a discontinuity in the displacement field to occur at a given location of the domain of analysis. As a consequence of that, some relation between the original problem and the extended one has to be assured. Therefore, the added terms play the important role of making compatible both problems in terms of the corresponding Euler–Lagrange equations.