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Brief paper

Distributed average tracking for multiple signals generated by linear dynamical systems: An edge-based framework[☆]

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ABSTRACT

This paper studies the distributed average tracking problem for multiple time-varying signals generated by linear dynamics, whose reference inputs are nonzero and not available to any agent in the network. In the edge-based framework, a pair of continuous algorithms with, respectively, static and adaptive coupling strengths is designed. Based on the boundary layer concept, the proposed continuous algorithm with static coupling strengths can asymptotically track the average of multiple reference signals without the chattering phenomenon. Furthermore, for the case of algorithms with adaptive coupling strengths, average tracking errors are uniformly ultimately bounded and exponentially converge to a small adjustable bounded set. Finally, a simulation example is presented to show the validity of theoretical results.

1. Introduction

In the past two decades, there have been lots of interests in the distributed cooperative control (Cao & Ren, 2012; Hong, Chen, & Bushnell, 2008; Li, Liu, Ren, & Xie, 2013; Liu & Geng, 2015; Liu, Zhao, & Geng, 2016; Olfati-Saber, Fax, & Murray, 2007; Ren, Beard, & Atkins, 2007; Tuna, 2008; Wen, Zhao, Duan, Yu, & Chen, 2016; Zhang, Lewis, & Das, 2011; Zhao & Duan, 2015; Zhao, Duan, Wen, & Chen, 2016; Zhao, Liu, Duan, & Wen, 2016), and (Ji, Ferrari-Trecate, Egerstedt, & Buffa, 2008), for multi-agent systems due to its potential applications in formation flying, path planning and so forth. Besides, the clock synchronization problems were also discussed in Bolognani, Carli, Lovisari, and Zampieri (2016), Carli and Zampieri (2014), Franceschelli, Pisano, Giua, and Usai (2012), Mills (1991), and Sundararaman, Buy, and Kshemkalyani (2005), which are very important to design distributed algorithms. Distributed average tracking, as a generalization of consensus and cooperative tracking problems, has received increasing attentions and been applied in many different perspectives, such as distributed sensor networks (Bai, Freeman, & Lynch, 2011; Spanos & Murray, 2005) and distributed coordination (Sun & Lemmon, 2007; Yang, Freeman, & Lynch, 2008). For practical applications, distributed average tracking should be investigated for signals modeled by more and more complex dynamical systems.

The objective of distributed average tracking problems is to design a distributed algorithm for multi-agent systems to track the average of multiple reference signals. The motivation of this problem comes from the coordinated tracking for multiple camera systems. Spurred by the pioneering works in Freeman, Yang, and Lynch (2006), and Spanos, Olfati-Saber, and Murray (2005) on the distributed average tracking via linear algorithms, real applications of related results can be found in distributed sensor fusion (Bai et al., 2011; Spanos & Murray, 2005), and formation control (Yang et al., 2008). In Bai and Lynch (2010), distributed average tracking problems were investigated by considering the robustness to initial errors in algorithms. The above-mentioned results are important for scientific researchers to build up a general framework to investigate this topic. However, a common assumption in the above works is that the multiple reference signals are constants (Freeman et al., 2006) or achieving to values (Spanos et al., 2005). In practical applications, reference signals may be produced by more general dynamics. For this reason, a class of nonlinear algorithms was designed in Nosrati, Shafiee, and Menhaj (2012) to track multiple reference signals with bounded deviations. Then, based on non-smooth control approaches, a couple of distributed algorithms

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were proposed in Chen, Cao, and Ren (2012); Chen, Ren, Lan, and Chen (2015) for agents to track arbitrary time-varying reference signals with bounded deviations and bounded second deviations, respectively. Using discontinuous algorithms, further, Zhao, Duan, and Li (2014) studied the distributed average tracking problems for multiple signals generated by linear dynamics.

Motivated by the above mentioned observations, this paper is devoted to solving the distributed average tracking problem with continuous algorithms, for multiple time-varying signals generated by general linear dynamical systems, whose reference inputs are assumed to be nonzero and not available to any agent in networks. First of all, based on relative states of neighboring agents, a class of distributed continuous control algorithms is proposed and analyzed. Then, a novel class of distributed algorithms with adaptive coupling strengths is designed by utilizing an adaptive control technique. Different from Cao and Ren (2012) and Li et al. (2013), where the nonlinear signum function was applied to the whole neighborhood (node-based algorithm), the proposed algorithms in this paper are designed along the edge-based framework as in Chen et al. (2012, 2015) and Zhao et al. (2014). Compared with the above existing results, the contributions of this paper are three-fold. First, main results of this paper extend the reference signals which were generated by first and second-order integrators in Chen et al. (2012, 2015), respectively, to signals generated by linear dynamical systems, which can describe more complex signals. An advantage of edge-based algorithms designed here is that they have a certain symmetry, which is very important to get the average value of multiple signals under an undirected topology. By utilizing this property, the edge-based algorithms obtained in this paper successfully solve distributed average tracking problems for multiple signals generated by general linear systems with bounded inputs. Second, by using adaptive control approaches, the requirements of all global information are removed, which greatly reduce the computational complexity for large-scale networks. Third, compared with existing results in Zhao et al. (2014), new continuous algorithms are redesigned via the boundary layer concept with clock synchronization devices. Since there exist differences between the local times of the agents, which may effect the distributed average tracking result, the clock synchronization is introduced in this paper. The clock synchronization problem has been solved in many existing papers such as Bolognani et al. (2016), Carli and Zampieri (2014), Franceschelli et al. (2012), Mills (1991) and Sundararaman et al. (2005). With the help of the existing results on clock synchronization in Bolognani et al. (2016), Carli and Zampieri (2014), Franceschelli et al. (2012), Mills (1991) and Sundararaman et al. (2005), the first step before beginning computation is to set the local clock to synchronize the local times. Thus, the boundary layer concept with clock synchronization devices plays a vital role to reduce the chattering phenomenon. Continuous algorithms in this paper is more appropriate for real engineering applications.

Notations. Let R^n and $R^{n\times n}$ be sets of real numbers and real matrices, respectively. I_n represents the identity matrix of dimension n. Denote by $\mathbf{1}$ a column vector with all entries equal to one. The matrix inequality $A > (\geq)B$ means that $A - B$ is positive (semi-) definite. Denote by $A\otimes B$ the Kronecker product of matrices A and B. For a vector $x = (x_1, x_2, \ldots, x_n)^T \in R^n$, let $\|x\|$ denote the 2-norm of x, $\mathrm{sig}^{\frac{1}{2}}(x) = (\mathrm{sig}^{\frac{1}{2}}(x_1), \mathrm{sig}^{\frac{1}{2}}(x_2), \ldots, \mathrm{sig}^{\frac{1}{2}}(x_n))^T$. For a set V, $|V|$ represents the number of elements in V.

2. Preliminaries

2.1. Graph theory

An undirected (simple) graph $\mathcal{G}$ is specified by a vertex set $\mathcal{V}$ and an edge set $\mathcal{E}$ whose elements characterize the incidence relations between distinct pairs of $\mathcal{V}$. The notation $i \sim j$ is used to denote that node i is connected to node j, or equivalently, $(i,j) \in \mathcal{E}$. We make use of the $|\mathcal{V}| \times |\mathcal{E}|$ incidence matrix, $D(\mathcal{G})$, for a graph with an arbitrary orientation, i.e., a graph whose edges have a head (a terminal node) and a tail (an initial node). The columns of $D(\mathcal{G})$ are then indexed by the edge set, and the ith row entry takes the value 1 if it is the initial node of the corresponding edge, -1 if it is the terminal node, and zero otherwise. The diagonal matrix $\Delta(\mathcal{G})$ of the graph contains the degree of each vertex on its diagonal. The adjacency matrix, $A(\mathcal{G})$, is the $|\mathcal{V}| \times |\mathcal{V}|$ symmetric matrix with zero in the diagonal and one in the (i,j)th position if node i is adjacent to node j. The graph Laplacian (Godsil & Royle, 2001) of $\mathcal{G}$, $L := \frac{1}{2}D(\mathcal{G})D(\mathcal{G})^T = \Delta(\mathcal{G}) - A(\mathcal{G})$, is a rank deficient positive semi-definite matrix.

An undirected path between node i_1 and node i_s on undirected graph means a sequence of ordered undirected edges with the form $(i_k; i_{k+1})$, $k = 1, \ldots, s-1$. A graph $\mathcal{G}$ is said to be connected if there exists a path between each pair of distinct nodes.

Assumption 1. Graph $\mathcal{G}$ is undirected and connected.

Lemma 1 (*Godsil & Royle, 2001*)**.** *Under Assumption 1, zero is a simple eigenvalue of L with $\mathbf{1}$ as an eigenvector and all the other eigenvalues are positive. Moreover, the smallest nonzero eigenvalue λ_2 of L satisfies $\lambda_2 = \min_{x\neq 0, \mathbf{1}^T x = 0} \frac{x^T L x}{x^T x}$.*

Define $M = I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T$. Then M satisfies following properties: Firstly, it is easy to see that 0 is a simple eigenvalue of M with $\mathbf{1}$ as the corresponding right eigenvector and 1 is the other eigenvalue with multiplicity $N-1$, i.e., $M\mathbf{1} = \mathbf{1}^T M = 0$. Secondly, since $L^T = L$, one has $LM = L(I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T) = L - \frac{1}{N}L\mathbf{1}\mathbf{1}^T = L = L - \frac{1}{N}\mathbf{1}\mathbf{1}^T L = (I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T)L = ML$. Finally, $M^2 = M(I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T) = M - \frac{1}{N}M\mathbf{1}\mathbf{1}^T = M$.

3. Distributed average tracking for multiple reference signals with general linear dynamics

Suppose that there are N time-varying reference signals, $r_i(t) \in R^n$, $i = 1, 2, \ldots, N$, which generated by the following linear dynamical systems:

$$\dot{r}_i(t) = Ar_i(t) + Bf_i(t), \tag{1}$$

where $A \in R^{n\times n}$ and $B \in R^{n\times p}$ both are constant matrices with compatible dimensions, $r_i(t) \in R^n$ is the state of the ith signal, and $f_i(t) \in R^p$ represents the reference input of the ith signal. Here, we assume that $f_i(t)$ is continuous and bounded, i.e., $\|f_i(t)\| \leq f_0$, for $i = 1, 2, \ldots, N$, where f_0 is a positive constant. Suppose that there are N agents with $x_i \in R^n$ being the state of the ith agent in distributed algorithms. It is assumed that agent i has access to $r_i(t)$, and agent i can obtain the relative information from its neighbors denoted by $\mathcal{N}_i$. Besides, let $|\mathcal{N}_i|$ represent the number of elements in the set $\mathcal{N}_i$, $i = 1, 2, \ldots, N$.

Assumption 2. (A, B) is stabilizable.

The main objective of this paper is to design a class of distributed algorithms for agents to track the average of multiple signals $r_i(t)$ generated by the general linear dynamics (1) with bounded reference inputs $f_i(t)$, $i = 1, 2, \ldots, N$.

Therefore, a distributed algorithm is proposed as follows:

$$\begin{aligned}
\dot{s}_i(t) &= As_i(t) + Bu_i(t),\\
u_i(t) &= c_1 \sum_{j\in\mathcal{N}_i}[K(x_i(t) - x_j(t))] + c_2 \sum_{j\in\mathcal{N}_i} h_i[K(x_i(t) - x_j(t)), t_i],\\
x_i(t) &= s_i(t) + r_i(t),
\end{aligned} \tag{2}$$