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Brief paper

Robust cooperative learning control for directed networks with nonlinear dynamics☆

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ABSTRACT

This paper studies a class of robust cooperative learning control problems for directed networks of agents (a) with nonidentical nonlinear dynamics that do not satisfy a global Lipschitz condition and (b) in the presence of switching topologies, initial state shifts and external disturbances. All uncertainties are not only time-varying but also iteration-varying. It is shown that the relative formation of nonlinear agents achieved via cooperative learning can be guaranteed to converge to the desired formation exponentially fast as the number of iterations increases. A necessary and sufficient condition for exponential convergence of the cooperative learning process is that at each time step, the network topology graph of nonlinear agents can be rendered quasi-strongly connected through switching along the iteration axis. Simulation tests illustrate the effectiveness of our proposed cooperative learning results in refining arbitrary high precision relative formation of nonlinear agents.

1. Introduction

Networks of multiple interacting agents can be used to represent groups of individuals, such as flocking birds, schooling fishes, swarming bees, and people in social networks. In such complex systems an important class of problems is what kind of interaction rules will lead to cooperation among the agents. Of particular interest are rules that involve only nearest neighbors or local information, where consensus/synchronization and formation are two widely-considered problems (see, e.g., Abdessameud & Tayebi, 2013; Cao, Morse, & Anderson, 2008; Jadbabaie, Lin, & Morse, 2003; Lin, Francis, & Maggiore, 2005; Moreau, 2005; Olshevsky & Tsitsiklis, 2009; Ren & Beard, 2005; Schenato & Fiorentin, 2011). These types of cooperative control problems have important practical applications in many areas, such as spacecrafts, mobile robots and unmanned aerial vehicles (see, e.g., Bullo, Cortés, & Martínez, 2009; Cao, Yu, Ren, & Chen, 2013; Olfati-Saber, Fax, & Murray, 2007 and references therein).

A recent focus in cooperative control concerns situations where agents in a network learn to cooperate, see, e.g., Ahn and Chen (2009), Li and Li (2014), Liu and Jia (2012a,b), Meng, Jia, Du, and Zhang (2014), Meng, Jia, Du, and Zhang (2015b) and Xu, Zhang, and Yang (2011) for formation keeping, Li and Li (2013), Shi, He, Wang, and Zhou (2014), Xiong, Yu, Chen, and Gao (in press), Yang, Xu, and Yu (2013) and Yang, Xu, Huang, and Tan (2014) for leader or reference trajectory following and Meng, Jia, and Du (2015a) for consensus seeking. It is common to observe groups of individual agents that learn to cooperate in practice. Take for example a marching band, which motivated the authors' recent paper (Meng & Moore, 2016). Through repetitively practicing, a group of performers in the marching band march to seek desired formations as well as playing instruments with a specified music (http://www.wikihow.com/Practice-Marching-Band-Formations). In such a picture of cooperative learning for a group of performers, the key ideas are repetition by individuals and communication between neighbors. Other similar examples include synchronized swimming, formation soldiers marching and aerobatic flights displaying (see also Meng & Moore, 2016). In addition, there are a number of applications where cooperative learning is

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desirable, such as when high precision control performance is required, see, e.g., Ahn, Moore, and Chen (2010) for satellites formation flying, Chen and Jia (2010) for robot formation running and (Sun, Hou, & Li, 2013) for coordinated trains' trajectory tracking.

Studies to develop analysis tools for cooperative learning are far from complete. The related results all benefit from the use of iterative learning control (ILC) approaches (see the ILC surveys (Ahn, Chen, & Moore, 2007; Bristow, Tharayil, & Alleyne, 2006; Xu, 2011) for more details on this approach), for which there are two main problems. One is because of the switching topology of interactions among agents, especially with respect to iteration, which renders the cooperative learning processes to be dependent on iteration-varying parameters. This may violate one of the fundamental requirements on ILC—repetitiveness (Meng & Moore, 2016). As a result many of the existing cooperative learning results are applicable for only iteration-invariant topology (see, e.g., Ahn & Chen, 2009; Li & Li, 2013, 2014; Shi et al., 2014; Xiong et al., in press; Xu et al., 2011; Yang et al., 2014). When considering iteration-varying topologies, most existing results require the agents to be strongly or quasi-strongly connected at all iterations in order for the learning processes to meet the contraction mapping principle and thus achieve convergence as the number of iterations increases (see, e.g., Liu & Jia, 2012a,b; Meng et al., 2015a,b; Yang et al., 2013). The other main problem is related to the dynamics of agents when those dynamics are nonlinear. In ILC, the global Lipschitz assumption is usually imposed as an assumption on nonlinear systems (Xu, 2011). In the cooperative learning problem most existing results for nonlinear agents require their dynamics to satisfy the global Lipschitz assumption (see, e.g., Ahn & Chen, 2009; Liu & Jia, 2012a,b; Meng et al., 2014; Xu et al., 2011; Yang et al., 2014) or to be locally Lipschitz but exactly known (see, e.g., Li & Li, 2013, 2014).

In this paper, we further investigate the cooperative learning problem of Meng et al. (2014) and Meng and Moore (2016) for formation control of nonlinear agents. We should point out that we extend the results of Meng et al. (2014) and Meng and Moore (2016) by relaxing the constraints on the agents' dynamics so that a global Lipschitz condition is not required and iteration-varying uncertainties are admitted. The contributions of the developed results are stated especially by comparison with the existing results as follows.

(1) We consider networks of agents with nonidentical time-varying nonlinear dynamics, which are not required to satisfy the global Lipschitz condition and which simultaneously deal with iteration-varying initial state shifts and external disturbances. This extends the results of Meng et al. (2014) and Meng and Moore (2016) for networks of agents with time-invariant nonlinear dynamics to more general networks with iteration-varying uncertainties.
(2) Although ILC-motivated formation of agents with nonlinear dynamics not fulfilling the global Lipschitz condition is handled in Meng et al. (2015b), the switching topologies are required to satisfy the quasi-strong connectivity (or spanning tree) condition for each time step and each iteration. The same problem exists in Meng et al. (2015a) although it can address the robust issues on initial state shifts and external disturbances. In contrast to Meng et al. (2015a,b), we only need the joint quasi-strong connectivity condition of switching topologies along the iteration axis. This retains the idea of basic convergence results for cooperative learning of agents in Meng and Moore (2016).
(3) Our cooperative learning results contain a necessary and sufficient robust consensus result for networks with both multiplicative and additive disturbances, which improve the traditional consensus results of, e.g., Schenato and Fiorentin (2011).

In addition, we show that cooperative learning among agents in networks with nearest neighbor communications can be robust against exponentially convergent iteration-varying uncertainties, but in general cannot deal with those bounded and/or non-exponentially convergent uncertainties from the standard stability point of view (Rugh, 1996). We also give simulation examples for nonlinear agents to demonstrate the validity of our results.

We organize the remainder of this paper as follows. We give the problem statements in Section 2 for cooperative learning of nonlinear agents subject to switching topologies that change in two directions (time and iteration). In Section 3, we present the two-dimensional (2-D) dynamics analysis for nonlinear agents under the action of an ILC-motivated distributed algorithm, based on which we obtain the necessary and sufficient cooperative learning results (with an exponential convergence speed) in Section 4. We provide simulation examples, and then conclusions, in Sections 5 and 6, respectively. In the Appendix, we present the proofs of lemmas and theorems.

Notations: We use I_m, 0 and diag$\{\cdot\}$ to represent the mth-order identity matrix, the null matrix with required dimensions and the (block) diagonal matrix, respectively. We adopt $\mathscr{I}_n = \{1, 2, \ldots, n\}$, $\mathbb{Z}_N = \{0, 1, \ldots, N\}$, $\mathbb{Z}_N \setminus \{0\} = \{1, 2, \ldots, N\}$, $1_n = [1, 1, \ldots, 1]^{\mathrm{T}} \in \mathbb{R}^n$ and $\prod_{i=l}^{j} A_i = A_j \cdots A_{l+1}A_l$ if $j \geq l$, $\prod_{i=l}^{j} A_i = I_n$ if $j < l$ and $\sum_{i=0}^{-1} A_i f_i = 0$ for some appropriately dimensioned sequences $\{A_i\}$ and $\{f_i\}$. In addition, $\|A\|_\infty$ is the maximum row sum norm (respectively, l_∞ norm) of a matrix (respectively, vector) A and $A \circ B$ (respectively, $A \otimes B$) is the Hadamard (respectively, Kronecker) product of matrices A and B. We say that a matrix $A \geq 0$ is nonnegative if all its entries are nonnegative and a nonnegative matrix $A \in \mathbb{R}^{n\times n}$ is stochastic if $A1_n = 1_n$.

2. Problem statement

2.1. High-precision formation via ILC

Consider networks with n nonlinear agents. We are interested in high-precision formation tasks that are achieved via ILC (see also Liu & Jia, 2012b; Meng et al., 2014; Meng et al., 2015b; Meng & Moore, 2016). Thus, the agents' dynamics evolve along two directions: a finite time axis for $t \in \mathbb{Z}_N$ and an infinite iteration axis over $k \in \mathbb{Z}_+$. Assume that each agent v_i has the following nonidentical nonlinear dynamics:

$$\begin{cases} x_{i,k}(t+1) = f_i\left(x_{i,k}(t), t\right) + u_{i,k}(t) + w_{i,k}(t) \\ x_{i,k}(0) : \left\|x_{i,k}(0) - x_{i0}\right\|_\infty \leq \boldsymbol{\psi}_i \boldsymbol{\sigma}_i^k \\ w_{i,k}(t) : \left\|w_{i,k}(t) - w_i(t)\right\|_\infty \leq \boldsymbol{\vartheta}_i(t)\boldsymbol{\upsilon}_i^k(t) \end{cases}, \quad i \in \mathscr{I}_n \tag{1}$$

where $x_{i,k}(t) \in \mathbb{R}^m$ is the state; $u_{i,k}(t) \in \mathbb{R}^m$ is the protocol or control input; $w_{i,k}(t) \in \mathbb{R}^m$ and $w_i(t) \in \mathbb{R}^m$ are (unknown) disturbances; $x_{i0} \in \mathbb{R}^m$ is a constant vector; $\boldsymbol{\psi}_i > 0, 0 \leq \boldsymbol{\sigma}_i < 1$, $\boldsymbol{\vartheta}_i(t) > 0$ and $0 \leq \boldsymbol{\upsilon}_i(t) < 1$ are (unknown) scalars; and $f_i(y,t) \triangleq [f_{i1}(y,t), f_{i2}(y,t), \ldots, f_{im}(y,t)]^{\mathrm{T}} \in \mathbb{R}^m$ is a vector-valued nonlinear function for any $y \in \mathbb{R}^m$ and $t \in \mathbb{Z}_N$. Note that the system (1) is described in a 2-D ILC framework, with time step t and iteration number k as two independent variables (Ahn et al., 2007). For each iteration k, the system (1) operates over $t \in \mathbb{Z}_N$, which is repeated for the next iteration $k+1$ after the control input $u_{i,k}(t)$ is updated by $u_{i,k+1}(t)$ with the application of ILC algorithms (see, e.g., the following ILC algorithm of (5)).

In this paper, we aim at addressing ILC-motivated formation tasks formally given in the following definition (see, e.g., Liu & Jia, 2012a; Meng et al., 2014; Meng & Moore, 2016).

Definition 1. We say that the agents of (1) under a designed protocol achieve formation exponentially fast if there exists some