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Brief paper

# Actuator fault tolerant control of systems with polytopic uncertainties using set-based diagnosis and virtual-actuator-based reconfiguration ☆

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ABSTRACT

In this paper, a robust actuator fault tolerant control (FTC) strategy for systems with polytopic uncertainty is proposed. Two types of model descriptions are investigated in this work: convex polytopic model uncertainty and linear-parameter-varying (LPV) convex polytopic model uncertainty; where, in the latter, the varying parameter is assumed to be measured. The proposed FTC strategy combines a robust fault detection and isolation (FDI) approach based on set separation with controller reconfiguration based on the use of a bank of virtual actuators (VA). Both, FDI and controller reconfiguration modules, use the same bank of VA. The robust FDI method is based on the separation of relevant sets defined for measurable residual signals, which are computed using the VA signals and taking into account system disturbances and model uncertainty. The closed-loop system is reconfigured by means of a VA which is adapted to the fault situation detected by the FDI unit. The performance of the resulting robust FTC scheme is analysed for the two types of model descriptions by means of a simulation example.



## 1. Introduction

Modern automatic control systems are subject to an increasing demand for highly reliable and safe operation. Fault tolerant control (FTC) systems combine fault detection and isolation (FDI) with controller reconfiguration (CR) principles in an overall scheme capable to maintain stability and minimise performance degradation under a range of fault situations—typically, those that are more frequent and/or have adverse impact on the process performance. Virtual sensors (VS) and virtual actuators (VA) have been proposed as an interesting approach for CR after faults (Blanke, Kinnaert, Lunze, & Staroswiecki, 2006; Steffen, 2005).

The LPV modelling approach has received major attention in recent years since it affords tractable mathematical descriptions for nonlinear systems. In LPV systems, the system matrices are considered to be functions of a time-varying 'parameter' that is measurable at the current time but whose future evolution is not known. Benefiting from the measurements, LPV systems typically employ scheduling control, where the coefficients of the control system are scheduled in real time according to the current value of the parameter. The fact that the future evolution of the varying parameter is not known a priori can be considered as a source of modelling uncertainty in LPV systems. This means that, although the parameter measurement availability allows for tighter scheduled controller implementation, further assumptions are needed to devise strategies with guaranteed properties for all future parameter evolutions. In the current work we achieve this by considering a convex polytopic modelling framework where the system matrices are assumed to vary in a matrix polytope with known vertices. We will also consider the convex polytopic modelling framework for the case where the system matrices belong to a matrix polytope (and are possibly time-varying), but cannot be described as a function of a measurable parameter; with some abuse of terminology we refer to this latter case as 'non-LPV'.

Research on FTC of LPV systems has become increasingly active in the last few years, with most works dealing separately with the FDI or the CR problems. Amongst papers that consider the CR problem in isolation, Rodrigues, Theilliol, Aberkane, and Sauter (2007) present a static output feedback control that can be reconfigured when multiple actuator faults occur assuming the availability of exact fault information. In Tabatabaeipour, Stoustrup, and Bak (2015), VS and VA are designed for the

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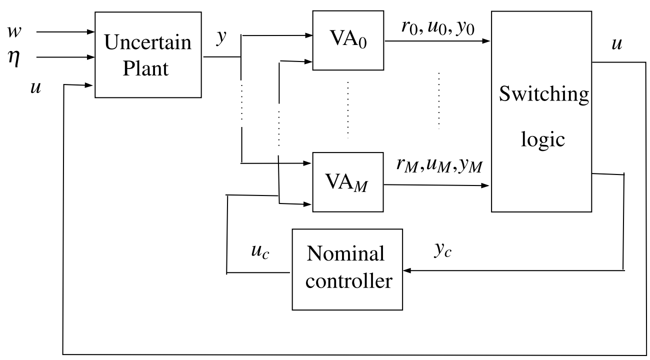


**Fig. 1.** Proposed FTC scheme with uncertain plant, bank of virtual actuators $\mathrm{VA}_0$ to $\mathrm{VA}_M$, switching logic and nominal controller.

reconfiguration of LPV systems after faults, and stability of the reconfigured system is established using input to state stability theory. The faulty system is assumed to be known, that is, no FDI or fault estimation is analysed. Other approaches, such as Rotondo, Nejjari, and Puig (2014), include the use of VS and VA for LPV systems with fault estimation formulated as a parameter estimation problem. These works do not discuss the stability properties of the overall scheme that integrates the fault estimator with the VS and/or VA in the closed-loop system. For the case of 'non-LPV' polytopic systems, where the uncertainty cannot be described as a function of a measurable parameter, we are not aware of any FDI or FTC approaches beyond the work of Shen, Yang, and Sun (2014), who also point out the lack of results treating this case.

In the present paper we focus on the integration of FDI and CR in an overall actuator FTC scheme for systems with convex polytopic model description, with guarantees of fault tolerance and closed-loop stability. In our initial work (Nazari, Seron, & De Doná, 2014) dealing with LPV systems under actuator faults, the scheme of Seron, De Doná, and Richter (2011) based on a bank of VA performing both FDI and CR tasks was extended to discrete-time systems with convex LPV model uncertainty. In the current paper, the idea of using a bank of VA (as in Seron et al., 2011 and Nazari et al., 2014) in an integrated FTC approach is employed for discrete-time systems with convex polytopic model uncertainties both for the LPV and non-LPV cases. In the latter case, since the use of scheduled control is no longer possible, the controller and the bank of VA are designed for the centre of the uncertainty polytope, which inevitably introduces an error in the controller reconfiguration leading to possible performance degradation. The performance of the integrated FTC strategy is simulated using the numerical example of Nazari et al. (2014) and the results are compared for the LPV and non-LPV cases.

## 2. Proposed scheme

The proposed FTC scheme is shown in Fig. 1. In this scheme, a bank of VA operates in closed-loop with an observer-based tracking controller designed for the nominal (fault free) plant. A suitable residual signal is associated to each VA. Correct FDI is guaranteed if appropriately defined residual sets have no intersection between each other. A switching logic monitors the residual signals to determine which set they belong to, and engages in the loop the VA that matches the currently diagnosed fault situation. In the following subsections the different elements of the scheme are described and their design and properties are explained in Section 4, after introducing the LPV and non-LPV modelling frameworks in Section 3.

### 2.1. Uncertain plant and actuator fault models

We consider an uncertain discrete-time system given by

$$x^+ = Ax + BFu + Ew, \quad (1a)$$
$$y = Cx + \eta, \quad (1b)$$
$$v = C_v x, \quad (1c)$$

where $x$ and $x^+ \in \mathbb{R}^n$ are the current and successor system states, $u \in \mathbb{R}^m$ is the control input, $w \in \mathbb{R}^r$ is a bounded process disturbance, $y \in \mathbb{R}^p$ is the plant measured output, $v \in \mathbb{R}^q$ is a performance output and $\eta \in \mathbb{R}^p$ is a bounded measurement noise. The system matrices $A$, $B$ and $C$ lie in the convex hull $\pi = Co\{(A_1, B_1, C_1), (A_2, B_2, C_2), \ldots, (A_N, B_N, C_N)\}$, that is, $A = \sum_{i=1}^{N} \alpha_i A_i$, $B = \sum_{i=1}^{N} \alpha_i B_i$ and $C = \sum_{i=1}^{N} \alpha_i C_i$ for certain known constant matrices $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $C_i \in \mathbb{R}^{p\times n}$, and uncertain (e.g., functions of an uncertain parameter) coefficients satisfying $\alpha_i \ge 0$ and $\sum_{i=1}^{N} \alpha_i = 1$. The matrix $C_v$ is a known constant matrix. The "fault matrix" $F \in \mathbb{R}^{m\times m}$ in (1a) models actuator faults. We consider a finite range of fault situations represented by the matrix $F$ taking $M+1$ different values $F \in \{F_0, F_1, \ldots, F_M\}$. In particular, $F_0 = I$ (the identity matrix) represents the "healthy" situation, that is, no actuator fault. We will say that an abrupt change in the actuator fault situation occurs if $F$ changes from $F = F_i$ to $F = F_j$, $i, j \in \{0, \ldots, M\}$, $j \ne i$, at some time step. It is assumed that the pairs $(A_i, C_i)$ are detectable and the pairs $(A_i, B_i F_j)$, for $i = 1, \ldots, N$ and $j = 0, 1, \ldots, M$ are stabilisable. In addition, the pairs $\left(\begin{bmatrix} A_i & 0 \\ C_v & I \end{bmatrix}, \begin{bmatrix} B_i F_j \\ 0 \end{bmatrix}\right)$ are stabilisable, for $i = 1, \ldots, N$ and $j = 0, 1, \ldots, M$ (this is required for the VA design). We will further assume that the disturbances satisfy $w(k) \in \mathscr{W}$ and $\eta(k) \in \mathscr{N}$ for all $k \ge 0$, where the bounding sets are defined as $\mathscr{W} \triangleq \{w \in \mathbb{R}^r : |w| \le \overline{w}\}$ and $\mathscr{N} \triangleq \{\eta \in \mathbb{R}^p : |\eta| \le \overline{\eta}\}$ for some nonnegative vectors $\overline{w} \in \mathbb{R}^r$ and $\overline{\eta} \in \mathbb{R}^p$ (inequalities and absolute values are taken elementwise).

### 2.2. Nominal controller

We employ the observer-based, reference tracking controller

$$u_c = -K(\hat{x} - x_{\mathrm{ref}}) + u_{\mathrm{ref}}, \quad (2)$$
$$\hat{x}^+ = \bar{A}\hat{x} + \bar{B}u_c + L(y_c - \bar{C}\hat{x}), \quad (3)$$
$$x_{\mathrm{ref}}^+ = \bar{A}x_{\mathrm{ref}} + \bar{B}u_{\mathrm{ref}}, \quad (4)$$

where $\hat{x}$ is the observer state and, under healthy conditions ($F = F_0 = I$), $u_c = u$, $y_c = y$ ($u$, $y$ are the signals in the plant (1), see Fig. 1). More generally, as we will see below, $u$, $u_c$, $y$ and $y_c$ are related through the VA selected by the switching logic (cf. (5)–(7)) according to the diagnosed fault situation. The design of the different matrices in (2)–(4) will be explained in Section 4.

**Remark 1** (*Reference System*)**.** The reference system (4) generates a bounded trajectory $(u_{\mathrm{ref}}, x_{\mathrm{ref}})$, designed such that the output $C_v x_{\mathrm{ref}}$, where $C_v$ is the plant performance output matrix in (1c), follows as closely as possible a desired bounded external signal $v^*$ in the absence of disturbances and under all possible fault situations. Since the reference system (4) is a design choice, it is immediate to obtain constant vectors $u_{\mathrm{ref}}^0 \in \mathbb{R}^m$, $\overline{u_{\mathrm{ref}}} \in \mathbb{R}^m$, $x_{\mathrm{ref}}^0 \in \mathbb{R}^n$ and $\overline{x_{\mathrm{ref}}} \in \mathbb{R}^n$ such that $u_{\mathrm{ref}}(k) \in \mathscr{U}_{\mathrm{ref}} = \{u \in \mathbb{R}^m : |u - u_{\mathrm{ref}}^0| \le \overline{u_{\mathrm{ref}}}\}$ and $x_{\mathrm{ref}}(k) \in \mathscr{X}_{\mathrm{ref}} = \{x \in \mathbb{R}^n : |x - x_{\mathrm{ref}}^0| \le \overline{x_{\mathrm{ref}}}\}$ for all $k \ge 0$. ○

### 2.3. Bank of virtual actuators

We consider a bank of VA with integral action (Steffen, 2005), each described by the following equations associated with the