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Brief paper

Frequency domain weighted nonlinear least squares estimation of parameter-varying differential equations ☆

Jan Goos, John Lataire, Ebrahim Louarroudi, Rik Pintelon

ELEC, Vrije Universiteit Brussel, Pleinlaan 2, 1050 Brussels, Belgium

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ABSTRACT

This paper presents a frequency domain identification technique for estimation of Linear Parameter-Varying (LPV) differential equations. In a band-limited setting, it is shown that the time derivatives of the input and output signals can be computed exactly in the frequency domain, even for non-periodic inputs and parameter variations. The method operates in an errors-in-variables framework (noisy input and output), but the scheduling signal is assumed to be known. Under these conditions, the proposed estimator is proven to be consistent.

1. Introduction

A good example of an LPV system is a construction crane, which is basically a pendulum of varying length $l_{(t)}$. The cable length directly influences the poles of the system, thereby determining its eigenfrequency. In the linear parameter varying (LPV) framework (Rugh & Shamma, 2000; Tóth, 2010), we call variables like $l_{(t)}$ scheduling parameters, and denote them as $p_{(t)}$. The dynamic relation between input $u_{(t)}$ and output $y_{(t)}$ is still linear, but it depends on the (continuously varying) scheduling parameter.

There are two main classes of LPV identification techniques: local and global approaches. In the local LPV framework (Bruzelius & Breitholtz, 2001; De Caigny, Camino, & Swevers, 2011), a nonlinear or parameter-varying model is linearized at different operating points. The result is a set of LTI models, which are then interpolated over the operating range. Here, we opt to directly estimate an LPV model, from a single, global experiment, where the scheduling parameter $p_{(t)}$ varies during the measurement, covering its entire operating range. Amongst other advantages, a global modeling approach captures transient dynamics, when the plant shifts from one operating point to another. Additionally, the rate of change of the scheduling parameter can be directly accounted for.

During the past decade, a lot of research has been dedicated to the identification of LPV input–output equations, but mostly in discrete-time (DT) and mostly for non-periodic scheduling signals (Laurain, Gilson, Tóth, & Garnier, 2010; Tóth, Laurain, Gilson, & Garnier, 2012). For an overview, see Laurain, Gilson, Garnier, and Tóth (2011). Only in Felici, van Wingerden, and Verhaegen (2007) time domain subspace methods have been developed for DT systems with dedicated, periodically varying scheduling sequences. However, it was shown in Goos and Pintelon (2014) that these results can be improved upon, using a frequency domain approach. In this paper we study the frequency domain identification of parameter-varying differential equations. The advantages of continuous-time (CT) LPV modeling are:

- Most physical phenomena are continuous-time, hence CT-LPV models are closer to the physics than DT-LPV models. For example, the discrete-time approximation of a differential equation with static dependency on the scheduling parameter can have a dynamic dependency on the scheduling [10].
- Although the final implementation is in discrete-time, advanced LPV control design methods are mostly based on continuous-time models (Lovera, Navara, Lopes dos Santos, & Rivera, 2011).
- Computer aided design/engineering software tools mostly use CT models.

In Laurain, Tóth, Gilson, and Garnier (2010), the first steps are taken towards direct identification of an LPV differential equation, using an instrumental variables approach. However, the input signal is assumed to be known, and the time derivatives are approximated using filtering operations. The frequency domain approach proposed in this paper has the following advantages:

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E-mail addresses: jan.goos@vub.ac.be (J. Goos), john.lataire@vub.ac.be (J. Lataire), ebrahim.louarroudi@uantwerpen.be (E. Louarroudi), rik.pintelon@vub.ac.be (R. Pintelon).

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- The time derivatives can be computed exactly.
- It is easy to select the frequency band(s) of interest, meaning no additional filters should be designed.
- Both input and output signals can be corrupted by colored noise.
- Easy use of nonparametric noise models in the weighted nonlinear least squares cost function.

The proposed identification algorithm is based on the Linear Time-Varying (LTV) identification algorithm described in Lataire and Pintelon (2011). A first key difference is that the time-varying coefficients are now replaced by functions of the scheduling parameter $p_{(t)}$.

$$\sum_{n=0}^{N_a} a_n(p_{(t)})\frac{\mathrm{d}^n y_{0(t)}}{\mathrm{d}t^n} = \sum_{n=0}^{N_b} b_n(p_{(t)})\frac{\mathrm{d}^n u_{0(t)}}{\mathrm{d}t^n} \tag{1}$$

where the subscript of x_0 denotes a noiseless quantity. The coefficients of the LPV Input–Output (IO) model (1) are approximated by linear combinations of known/chosen basis functions in $p(t)$, *viz.*

$$\begin{bmatrix} a_n(p_{(t)}) \\ b_n(p_{(t)}) \end{bmatrix} = \sum_{i=0}^{N_p} \begin{bmatrix} a_{[n,i]} \\ b_{[n,i]} \end{bmatrix} \phi_i(p_{(t)}). \tag{2}$$

The results in this paper also hold if the scheduling parameter is multivariable. As an alternative to (2), Support Vector Machines (SVMs) (Laurain, Tóth, Zheng, & Gilson, 2012) or Gaussian Processes (GPs) can be used to model the coefficient functions.

A second key difference with (Lataire & Pintelon, 2011), is that the full covariance matrix is used to weigh the residual errors, to ensure consistency. The consistency and correctness of the proposed Linear Parameter-Varying Input–Output (IO) estimator is proven, and illustrated on a simulation example. The results hold for arbitrary non-steady-state, non-periodic data. Even though the identification problem is considered in the frequency domain, the input $u_{(t)}$ and scheduling $p_{(t)}$ do not have to be periodic.

2. The sampled LPV differential equation in the frequency domain

The Fourier transform of (1) will be computed from the measured time domain signals, which are sampled uniformly at a sample frequency $f_s = 1/T_s$ (T_s is the sample time). A total of N samples of each signal is acquired.

Definition 1. The Discrete Fourier Transform (DFT), at the angular frequencies $\omega_k = 2\pi k f_0\ \forall k \in [0, N-1]$, with $f_0 = f_s/N$, is defined as

$$\mathrm{DFT}\{x_{(nT_s)}\} = X_{(k)} = \sum_{n=0}^{N-1} x_{(nT_s)} e^{-j\omega_k n T_s}. \tag{3}$$

Definition 2. Similarly, the inverse Discrete Fourier Transform (iDFT) is defined as

$$\mathrm{iDFT}\{X_{(k)}\} = \frac{1}{N}\sum_{k=0}^{N-1} X_{(k)} e^{j\omega_k n T_s}. \tag{4}$$

We denote the DFTs of the input $u_{(t)}$ and the output $y_{(t)}$ with $U_{(k)}$ and $Y_{(k)}$ respectively.

Assumption 3. Band-limited excitation: the Fourier transforms of the true input and scheduling signals are zero beyond the Nyquist frequency: $|U_{0(j\omega_k)}| = 0$ and $|P_{0(j\omega_k)}| = 0$ for $kf_0 \geq f_{\mathrm{nyq}} = f_s/2$. Furthermore, the basis functions are band-limited w.r.t. f_{nyq}.

The continuous time signals are windowed, because only the time frame $[0, T]$ is considered. If a rectangular window $w_{(t)}$ is used, the Fourier transform becomes

$$\mathcal{F}\left\{w_{(t)}\frac{\mathrm{d}^n x_{(t)}}{\mathrm{d}t^n}\right\}\Big|_{j\omega_k} = (j\omega_k)^n X_{(j\omega_k)} + \underbrace{\sum_{r=0}^{n-1}(j\omega_k)^{n-1-r}\left(x^{(r)}{}_{(T_-)} - x^{(r)}{}_{(0_+)}\right)}_{=T_x^n(j\omega_k)} \tag{5}$$

where $x^{(r)}$ is the rth time derivative of x, $X_{(j\omega_k)}$ is the Fourier transform of $x_{(t)}$ and $\omega_k = 2\pi k f_0$ is the angular frequency $\forall k \in [0, N-1]$. Eq. (5) is proven in Appendix 5.B of Pintelon and Schoukens (2012) using integration by parts. Note that the difference between the initial and end conditions of the signal determine the polynomial $T_x^n{}_{(j\omega_k)}$. By including the transient term $T_x^n{}_{(j\omega_k)}$ in (5), the derivatives of arbitrary signals can be represented exactly in the frequency domain.

2.1. Frequency domain model

Taking the affine approximation (2) into account, the DFT of the sampled and windowed (1) equals

$$\sum_{n=0}^{N_a}\sum_{i=0}^{N_p} a_{[n,i]}\Phi_i\{p\} * \left[{}_{(j\omega_k)}{}^n Y_{(k)} + T_y^n{}_{(j\omega_k)}\right]$$
$$= \sum_{n=0}^{N_b}\sum_{i=0}^{N_p} b_{[n,i]}\Phi_i\{p\} * \left[{}_{(j\omega_k)}{}^n U_{(k)} + T_u^n{}_{(j\omega_k)}\right] \tag{6}$$

where $\Phi_i\{p\} = \mathrm{DFT}\{\phi_i(p_{(t)})\}$ are the DFT basis functions of the scheduling parameter, and $*$ represents the circular convolution product.

Assumption 4. Weierstrass approximation theorem: the basis functions $\phi(p_{(t)})$ can be approximated by a polynomial in t of degree m in the finite interval $t \in [0, T]$.

Assumption 5. The basis functions $\phi(p_{(t)})$ are periodic in the time window T, and can therefore be represented exactly by their Fourier series over the interval $t \in [0, T]$.

Theorem 6. *Assumptions 4 and 5 are alternative. If either one of them holds, the convolution of the basis functions and the polynomials $\Phi_i\{p\} * T_y^n{}_{(j\omega_k)}$ and $\Phi_i\{p\} * T_u^n{}_{(j\omega_k)}$ are also polynomials in $j\omega_k$. For the proof, we refer to Appendix A.*

Corollary 7. *The polynomials $T_y^n{}_{(j\omega_k)}$ and $T_u^n{}_{(j\omega_k)}$ can be extracted from the summation in (6), and grouped into one transient polynomial $T_{uy}^{N_T}{}_{(j\omega_k)} = \sum_{i=0}^{N_T}\gamma_i\,{}_{(j\omega_k)}{}^i$, of order $N_T = \max\{N_a, N_b\} - 1$.*

$$T_{uy}^{N_T}{}_{(j\omega_k)} = -\sum_{n=0}^{N_a}\sum_{i=0}^{N_p} a_{[n,i]}\Phi_i\{p\} * T_y^n{}_{(j\omega_k)} + \sum_{n=0}^{N_b}\sum_{i=0}^{N_p} b_{[n,i]}\Phi_i\{p\} * T_u^n{}_{(j\omega_k)}. \tag{7}$$

Computing derivatives of the signals is exact in the frequency domain. On the other hand, the computationally complex convolution can be avoided by multiplying the signals in the time domain.