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Brief paper

Robust sliding mode control for uncertain discrete singular systems with time-varying delays and external disturbances☆

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ABSTRACT

In this paper, a sliding mode control (SMC) of uncertain discrete singular systems with external disturbances and time-varying delays is under consideration. By use of the free weighting matrices and the Lyapunov–Krasovskii functional, a delay-dependent sufficient condition is given in strict linear matrix inequality (LMI) format to guarantee the sliding mode dynamics to be admissible (regular, causal and stable). Furthermore, a proposed SMC law and an adaptive SMC law are synthesized to make sure that the trajectories of system can be driven to a region near equilibrium point in finite time. Finally, a numerical example is designed to display the effectiveness of the control scheme. All these results are expected to propose a new approach for the research on SMC of discrete time-delay singular systems.

1. Introduction

Singular systems have the characteristics of preserving the structure of practical systems and applications in robotic systems, networks and power systems, therefore study on singular systems is an important part in control theory and applications (Ding, Zhu, & Zhong, 2011; Wu & Zheng, 2009). In particular, discrete singular systems have more complicated form than regular ones in asymptotic stability, regularity and causality, and have more research value (Jiao, 2012; Li, Z X, H Y, Y, & Wu, 2013; Lin, Fei, & Gao, 2012; Wu, Park, Su, & Chu, 2012; Xin, Zhang, Chun-Yu, Yong-Yun, & Zhan, 2010). Meanwhile, time-delay which often takes place in practical systems and affects the stability and performance of systems, should be taken into account in the stability analysis of systems (Ramakrishnan & Ray, 2013; Wu, He, She, & Liu, 2004). By now, control of singular delay systems has attracted lots of researchers' attention.

Sliding mode control is widely adopted in lots of complex and engineering systems, including time-delay systems (Chen, Hwang, & Tomizuka, 2002; Feng & Lam, 2012; Ginoya, Shendge, & Phadke, 2014; Goyal, Deolia, & Sharma, 2015; Xia, Zhu, Li, Yang, & Zhu, 2010), stochastic systems (Niu, Ho, & Lam, 2005; Wu & Ho, 2010), and Markovian jumping systems (Ding, Zhu, Zhong, & Zeng, 2012; Li, Wu, Shi, & Lim, 2015). As we know, system performance may be degraded by the affection of presence of nonlinearities and external disturbances. When nonlinearities and external disturbances are considered, the results in Ramakrishnan and Ray (2013) cannot be used to ascertain delay-dependent stability. So it is necessary to aim to deal with the systems with nonlinearities and external disturbances by SMC approach. On the other hand, uncertainties make the system modeling more complicated and are inevitable in practical systems (Feng & Lam, 2012; Lin et al., 2012). To solve this problem, it is essential to use SMC which is a useful design control method to keep the insensitivity of systems to the uncertainties on the sliding surface.

However, SMC law in Wu and Zheng (2009) was synthesized for continuous singular systems with time-varying delays. It is common knowledge that the designed control law would be incapable of dealing with the problems in discrete systems, because discrete SMC cannot be synthesized lightly by counterpart equivalent of the continuous systems. Xin et al. (2010) focused on SMC problem for discrete time-delay systems and sufficient conditions were derived in the LMI framework, but these results should not be used in singular systems for the existence of the singular matrix E. Guo and Gao (2009) and Liu, Fu, Cai, and Song

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(2013) employed SMC method in discrete singular systems, while the former did not consider time-varying delay and the latter was complex in decomposing the system into two low dimensional subsystems firstly. As far as we know, the problem of SMC for uncertain discrete singular systems with time-varying delays and external disturbances has not been fully studied yet and still keep challenging.

This paper will investigate robust SMC for uncertain discrete singular systems with time-varying delays and external disturbances. The main results of this paper can be summarized as follows: (i) proving that the sliding mode dynamics are regular, causal and stable by designing a linear sliding surface for the existence of the singular matrix E, (ii) proving that the states of system can be driven to a region near equilibrium point in a finite time by designing a SMC law and an adaptive SMC law, (iii) showing the effectiveness of the proposed approach by conducting a numerical example.

Notations: In the whole paper, the superscript "T" represents matrix transposition, $\mathbf{R}^n$ denotes the n-dimensional Euclidean space, $A > 0$ (<0) stands for A is a symmetric positive (negative) definite matrix, $A \geq 0$ (≤ 0) means that matrix A is real positive (negative) semi-definite, A^{-1} indicates the inverse of matrix A, I and 0 are used to represent an identity matrix and zero matrix of appropriate dimensions respectively. The notation $*$ is represented the symmetric terms in a symmetric matrix.

2. Problem formulation

In this paper, the following uncertain discrete singular system with time-varying delays and external disturbances is considered:

$$\begin{cases} Ex(k+1) = (A+\Delta A)x(k) + (A_d+\Delta A_d) \\ \qquad\qquad \times\ x(k-d(k)) + B(u(k)+\omega(x(k),k)) \\ x(k) = \varphi(k), \quad k = -d_M, -d_M+1, \ldots, 0, \end{cases} \tag{1}$$

where $x(k) \in \mathbf{R}^n$ is the state vector, $d(k)$ is the time-varying delay with known lower and upper bounds satisfying $0 \leq d_m \leq d(k) \leq d_M$, $u(k) \in \mathbf{R}^m$ is the control input vector, $\omega(x(k),k) \in \mathbf{R}^m$ is the external disturbances vector satisfying $\|\omega(x(k),k)\| \leq \gamma\|x(k)\|$, where $\gamma > 0$ is a known constant. $\varphi(k)$ is the initial vector function of the system. $E \in \mathbf{R}^{n\times n}$ may be singular, and is assumed rank(E)$= r \leq n$. A, A_d and B are known real constant matrices with appropriate dimensions and rank(B)$= m$, The system parameter uncertainties ΔA and ΔA_d are constrained by $[\Delta A\ \Delta A_d] = CF(k)[D\ D_d]$, where C, D and D_d are known real constant matrices with appropriate dimensions, and $F(k)$ satisfies $F^T(k)F(k) \leq I$.

The unforced discrete singular system with time-varying delays of the system (1) can be written as

$$Ex(k+1) = Ax(k) + A_d x(k-d(k)). \tag{2}$$

Definition 1 (*Jiao, 2012*).

(1) The pair (E, A) is said to be regular, if $\det(zE - A)$ is not identically zero.

(2) The pair (E, A) is said to be causal if $\deg(\det(zE - A)) = \text{rank}(E)$.

(3) For given integers $d_m > 0, d_M > 0$, the discrete singular system (2) is said to be regular and causal for any time delay $d_i(k)$ satisfying $d_m \leq d_i(k) \leq d_M$, $(i = 1, 2, \ldots, n)$, if the pair (E, A) is regular and causal.

(4) System (2) is said to be admissible if it is regular, causal, and stable.

Lemma 1 (*Wu & Zheng, 2009*). *Given a scalar $\epsilon > 0$, Σ_1 and Σ_2 are assumed to be real matrices with appropriate dimensions. Then for any matrix Δ meeting the requirement of $\Delta^T\Delta \leq I$, the following inequality holds:*

$$\Sigma_1\Delta\Sigma_2 + (\Sigma_1\Delta\Sigma_2)^T \leq \epsilon^{-1}\Sigma_1\Sigma_1^T + \epsilon\Sigma_2^T\Sigma_2. \tag{3}$$

Lemma 2 (*Xin et al., 2010*). *Given positive integers β_1 and β_2 meeting the requirements of $1 \leq \beta_1 \leq \beta_2$, then for any constant matrix $\Gamma \geq 0$, $\Gamma \in \mathbf{R}^{n\times n}$, $\Psi(j) \in \mathbf{R}^n$, the following inequality holds:*

$$\begin{aligned} &-(\beta_2-\beta_1+1)\sum_{j=\beta_1}^{\beta_2}\Psi^T(j)\Gamma\Psi(j) \\ &\quad \leq -\Big(\sum_{j=\beta_1}^{\beta_2}\Psi(j)\Big)^T\Gamma\Big(\sum_{j=\beta_1}^{\beta_2}\Psi(j)\Big). \end{aligned} \tag{4}$$

Lemma 3 (*Ding et al., 2011*). *Let $X \in \mathbf{R}^{n\times n}$ be symmetric such that $E_L^TXE_L > 0$ and $T \in \mathbf{R}^{(n-r)\times(n-r)}$ be nonsingular. Therefore, $XE + W^TTH^T$ is nonsingular, and its inverse can be showed by the following equation:*

$$(XE + W^TTH^T)^{-1} = \mathbb{X}E^T + W\mathbb{T}H, \tag{5}$$

where $\mathbb{X}$ is symmetric and $\mathbb{T}$ is a nonsingular matrix with $E_R^T\mathbb{X}E_R = (E_L^TXE_L)^{-1}$, $\mathbb{T} = (H^TH)^{-1}T^{-1}(WW^T)^{-1}$, where W and H are any matrices with full row rank and satisfy $WE = 0$ and $EH = 0$, respectively. E is decomposed as $E = E_LE_R^T$ with $E_L \in \mathbf{R}^{n\times r}$ and $E_R \in \mathbf{R}^{n\times r}$ are of full column rank.

3. Main results

In this section, a linear sliding surface will be built by using the LMI. Moreover, a reaching motion control law will be designed to ensure the reachability of the quasi-sliding mode, and the states of the closed-loop system can be driven to a region near equilibrium point in finite time.

3.1. Sliding surface design

Usually, SMC design contains two steps, which includes sliding surface design and control signal design. In this work, we design the following switching surface function:

$$s(k) = GEx(k) - G(A+BK)x(k-1), \tag{6}$$

where $K \in \mathbf{R}^{m\times n}$ is a real matrix to be designed and $G \in \mathbf{R}^{m\times n}$ is to be chosen such that GB is nonsingular.

It is important to note that the ideal quasi-sliding mode clearly satisfies the following formula:

$$s(k+1) = s(k) = 0. \tag{7}$$

When the state trajectories of the system enter into the ideal quasi-sliding mode, we obtain the equivalent control law of the sliding motion from (1), (6) and (7) as:

$$\begin{aligned} u_{eq}(k) = &-(GB)^{-1}G\big[(\Delta A - BK)x(k) \\ &+ (A_d+\Delta A_d)x(k-d(k))\big] - \omega(x(k),k). \end{aligned} \tag{8}$$

By substituting (8) into (1) and defining $\widetilde{G} \triangleq I - B(GB)^{-1}G$ for simplicity, the sliding mode dynamics can be formulated as:

$$\begin{cases} Ex(k+1) = \overline{A}x(k) + \overline{A}_d x(k-d(k)) \\ x(k) = \varphi(k), \quad k = -d_M, -d_M+1, \ldots, 0, \end{cases} \tag{9}$$

where $\overline{A} = \widetilde{A} + \Delta\widetilde{A}$, $\widetilde{A} = A + BK$, $\Delta\widetilde{A} = \widetilde{G}\Delta A$, $\overline{A}_d = \widetilde{A}_d + \Delta\widetilde{A}_d$, $\widetilde{A}_d = \widetilde{G}A_d$, $\Delta\widetilde{A}_d = \widetilde{G}\Delta A_d$ and the uncertainties $\Delta\widetilde{A}$ and $\Delta\widetilde{A}_d$ can be written as $[\Delta\widetilde{A}\quad \Delta\widetilde{A}_d] = \widetilde{C}F(k)[D\quad D_d]$, where $\widetilde{C} = \widetilde{G}C$.