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Brief paper

Selective modal control for vibration reduction in flexible structures ☆

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ABSTRACT

The design of a controller for selective reduction of vibrations in flexible low-damped structures is presented. The objective of the active feedback control law is to increase damping of selected modes only, in frequency regions where a disturbance is likely to produce largest effect. Moreover, the stabilizing controller is required to be band-pass, in order to filter out high-frequency sensor noise and low-frequency accelerometer drift, and stable to increase robustness to uncertain parameters. The control design is based on the Inverse Optimal Design approach, through the solution of a matrix Stein equation, resulting in the solution of an optimal $\mathcal{H}_\infty$ control problem. A grey-box identification approach of the authors is employed for obtaining the model from experimental data or from detailed Finite Element Model (FEM) simulators. The problem of optimal actuator/sensor location is also addressed. Detailed simulation results are provided to show the effectiveness of the strategy.

1. Introduction

Active control of flexible structure to reduce vibrations caused by exogenous disturbances is a topic ubiquitous in Control Theory. Applications include structural vibration reductions in any transportation system (automotive, railways, aircraft are the areas more involved in the study) and noise reduction, since often reducing the vibration also reduces noise. Different control strategies have been proposed, since the classic velocity-feedback discussed in the seminal paper by Balas (Balas, 1982) to positive position feedback (Fanson & Caughey, 1990). In most cases, the selection of collocated sensor/actuator pairs is of paramount importance, as it renders the flexible structure *minimum phase* (Calafiore, Carabelli, & Bona, 1997). Adaptive control is proposed in different papers, e.g. in Khoshnood and Moradi (2014) adaptation is on the estimate of the frequencies of vibration. Another methodology proposed is Sliding Mode Control (Cavallo, De Maria, & Setola, 1999), where selective modal rejection is considered based on a state feedback and a robust observer. Finally, optimal $\mathcal{H}_2$ and $\mathcal{H}_\infty$ controllers have been considered (Cavallo, De Maria, Natale, & Pirozzi, 2006, 2008, 2010) for output feedback with strong stability and bandpass properties of the controller. The controller proposed in this paper follows the same philosophy as Cavallo et al. (2008), namely the objective is to produce a bandpass controller so that low frequencies can be cut because using accelerometers as sensors, low frequency drift has to be filtered out. Moreover, also high frequency has to be cut to avoid sensor noise and spill-over effect (Balas, 1982) caused by unavoidable model order truncation on a structure that is theoretically characterized by infinite modes. The closed-loop design turns out to be the solution of a suitable $\mathcal{H}_\infty$ control problem, where the performance index is deduced from specifications on damping increase in selected frequency bands (Canciello & Cavallo, 2015). The proposed approach requires that the flexible structure model is in a suitable space-state form, that is produced by a frequency-domain identification procedure proposed in Cavallo, De Maria, Natale, and Pirozzi (2007). The model is thus obtained directly from frequency data, either from experiments or generated by detailed FEM software. The proposed controller, differently from Cavallo et al. (2008), can select the modes to control, i.e., it can increase the damping of a prescribed set of modes only. This capability reveals very useful when broad-band disturbances act in a frequency region where undamped modes may magnify the effect of the disturbances. Moreover, to maximize the effect of the control, the selection of the optimal location of the actuator/sensor pairs is briefly discussed. Conditions are also provided for the stability of the stabilizing controller, to increase robustness and simplify practical implementation of the closed-loop control, e.g., avoiding actuators saturation. Detailed simulation results are presented to test the approach.

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2. The model

A flexible system has theoretically infinite modes, thus practical use always requires truncation. This issue is not trivial, as pointed out in Anderson and Liu (1989), where it was shown that only a frequency-weighted order reduction results into robustness against uncertainties due to unmodelled dynamics. A commonly used approach is to resort to FEM models, possibly of high order, and then using reduction of the system's order by retaining only the first, say n, modes. Thus, a linear $2n$-order dynamical system is obtained. Another possibility, addressed in this paper, is the use of a grey-box identification procedure.

Describing the model in modal coordinates (Cavallo et al., 2010), the state is $x^T = (x_1^T, x_2^T) = (\eta_1(t), \ldots, \eta_n(t), \dot{\eta}_1(t), \ldots, \dot{\eta}_n(t))$, where $\eta_i(t)$ is the generalized coordinate of the ith mode shape, and the model is described in the classical TITO (Two Input–Two Output) form

$$\dot{x} = Ax + B_w w + B_u u \quad (1)$$

$$z = C_z x + D_{zu} u \quad (2)$$

$$y = C_y x + D_{yw} w \quad (3)$$

$u, y \in \mathbb{R}^m$ are control input and output, respectively, i.e., force actuators (e.g., magnetostrictive DVA (Cavallo, Natale, Pirozzi, & Visone, 2005)) and velocity sensors, $w, z \in \mathbb{R}^{n+m}$ are disturbances and performance output, respectively.

$$A = \begin{pmatrix} 0 & \mathrm{I} \\ -\Omega & -\Lambda \end{pmatrix}, \qquad B_u = \begin{pmatrix} 0 \\ B_{u2} \end{pmatrix}, \qquad C_y = \begin{pmatrix} 0 & B_{u2}^T, \end{pmatrix} \quad (4)$$

$$\Omega = \mathrm{diag}(\omega_1^2, \ldots, \omega_n^2), \qquad \Lambda = 2\mathrm{diag}(\zeta_1\omega_1, \ldots, \zeta_n\omega_n) \quad (5)$$

$$B_w = \begin{pmatrix} 0 & 0 \\ B_{w2} & 0 \end{pmatrix}, \qquad C_z = B_w^T, D_{zu} = \begin{pmatrix} 0 \\ I \end{pmatrix}, \qquad D_{yw} = D_{zu}^T. \quad (6)$$

The structure of the matrices B_w and C_z has a physical justification (Cavallo et al., 2008). Identity matrices in D_{zu}, D_{yw} result from scaling (Skogestad & Postlethwaite, 2005). As it will be shown in Section 3, the design of the controller heavily depends on the structure of the model (1)–(6). The grey-box frequency identification procedure in Cavallo et al. (2007), produces a model with the desired structure, starting from frequency data obtained experimentally (Cavallo et al., 2008). However, in some cases experimental data may not be available, as in the preliminary design phase of the mechanical structure where the location of some of the actuators/sensors pairs has not been defined yet. In this case a detailed structural model can be produced by any FEM software, possibly employing multiphysics modules, to take into account also complex geometries and smart sensors/actuators. Then synthetic frequency data are generated and the best mathematical model of the form (1)–(6) approximating the data is obtained. This approach allows the designer to easily change configurations in earlier stages of the design of the mechanical structure, embedding control efficiency considerations into the design of mechanical parts.

3. Controller design

The proposed controller has to be a bandpass filter, in order to filter out low frequencies, rejecting possible drifts due to sensors, and to filter out unmodelled high frequency dynamics, as shown also in Cavallo et al. (2008), where the approach was validated on both simulation and experimental data. This is accomplished by designing a strictly proper controller with m transmission zeros at $s = 0$. Moreover, the stabilizing controller must dampen only some system modes that are located within prescribed bandwidth where also disturbances are present. The following theorem addresses the above issues.

Theorem 1. *Consider the system (1)–(6), choose n scalars $0 \le \delta_i < 1$, $i = 1, \ldots, n$ and let $\Delta = \mathrm{diag}(\delta_1, \ldots, \delta_n)$. Select a scalar $k > 0$ and let*

$$W = k^2 \Delta B_{u2} B_{u2}^T \Delta + 2k\Delta\Lambda. \quad (7)$$

Assume the solution S of the Stein equation

$$S + \Delta S \Delta = W \quad (8)$$

to be positive semidefinite and let $B_{w2} = S^{1/2}$. Then the controller

$$K_\infty(s) = C_\infty(sI - A_\infty)^{-1} B_\infty \quad (9)$$

where

$$A_\infty = \begin{pmatrix} 0 & \mathrm{I} \\ -\Omega & -\Lambda_\infty \end{pmatrix} \quad (10)$$

$$B_\infty = \begin{pmatrix} 0 \\ k\Delta\left(\mathrm{I} - \Delta^2\right)^{-1} B_{u2} \end{pmatrix}, \qquad C_\infty = -\begin{pmatrix} 0 & kB_{u2}^T \Delta \end{pmatrix}. \quad (11)$$

$\Lambda_\infty = \Lambda + kB_{u2}B_{u2}^T\Delta - k^{-1}S\Delta + k\Delta\left(I - \Delta^2\right)^{-1} B_{u2}B_{u2}^T$ and Ω as in (5), is band-pass, stabilizes the system (1)–(6) and is such that the closed-loop has norm $\|T_{zw}\|_\infty < k$, where $T_{zw} = \mathrm{LFT}(P, K_\infty)$, and LFT denotes the Linear Fractional Transformation (Zhou & Doyle, 1998).

Proof. Preliminarily, note that the Stein equation (8) has always a unique solution (Klein & Spreij, 2005), due to the structure of the matrix Δ and the selection of the δ_i's. As it is well-known (Zhou & Doyle, 1998), a controller H_∞ stabilizes the system in closed loop if: (i) $H_\infty \in dom(Ric)$ and $X_\infty = Ric(H_\infty) \ge 0$, (ii) $J_\infty \in dom(Ric)$ and $Y_\infty = Ric(J_\infty) \ge 0$, (iii) $\rho(X_\infty Y_\infty) < k^2$, where H_∞ and J_∞ are two Hamiltonian matrices, X_∞ and Y_∞ denote the solutions of Riccati equations associated to the Hamiltonian matrices, and $\rho(F)$ denotes the spectral radius of the matrix F. Note that the particular structure of the Riccati equations, due to $B_w = C_z^T$, implies block-diagonal solutions X_∞ and Y_∞. In particular, the solution $X_{\infty 2}$ of

$$-X_{\infty 2}\Lambda - \Lambda X_{\infty 2} + \tilde{B}_{w2} + X_{\infty 2}(k^{-2}\tilde{B}_{w2} - \tilde{B}_{u2})X_{\infty 2} = 0 \quad (12)$$

where $\tilde{F} = FF^T$, is $X_{\infty 2} = k\Delta$. Then the explicit solutions X_∞ and Y_∞ can be deduced as

$$X_\infty = k\begin{pmatrix} \Omega\Delta & 0 \\ 0 & \Delta \end{pmatrix}, \qquad Y_\infty = k\begin{pmatrix} \Omega^{-1}\Delta & 0 \\ 0 & \Delta \end{pmatrix}. \quad (13)$$

Hence also condition (iii) is satisfied. The band-pass property of the controller follows from Lemma 1 in Canciello and Cavallo (2015).

Remark 2. The estimate of the closed-loop norm has to be carefully understood. Usually, high values of the closed-loop $\mathcal{H}_\infty$ norm are associated to *poor* performances. However, in the proposed approach the performance matrices depend on k, so comparing the closed-loop norms for different values of k does not make sense. It is better to refer to the overall *gain* of the controller to have a sensible interpretation of the effect of k. Indeed, from (10), (11) it is clear that by increasing k both the bandwidth and the gain (the latter, roughly, by a factor k^2) are increased. Thus, rather counter-intuitively, closed-loop performances are enhanced by *increasing* k.

The solution of the Stein equation (8) is easily computed by using the Kronecker product and vectorization. Indeed (8) can be rewritten as $(I + \Delta \otimes \Delta)\mathrm{vec}(S) = \mathrm{vec}(W)$, whose explicit solution is simply

$$s_{ij} = \frac{1}{1 + \delta_i\delta_j} w_{ij}, \quad i = 1, \ldots, n, \; j = i, \ldots, n. \quad (14)$$