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Relaxed conditions for stability of time-varying delay systems☆

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ABSTRACT

In this paper, the problem of delay-dependent stability analysis of time-varying delay systems is investigated. Firstly, a new inequality which is the modified version of free-matrix-based integral inequality is derived, and then by aid of this new inequality, two novel lemmas which are relaxed conditions for some matrices in a Lyapunov function are proposed. Based on the lemmas, improved delay-dependent stability criteria which guarantee the asymptotic stability of the system are presented in the form of linear matrix inequality (LMI). Two numerical examples are given to describe the less conservatism of the proposed methods.

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1. Introduction

Time delay is a natural phenomenon in real world. It is well known that the existence of time delay often causes the oscillation, deterioration of system performance, and even instability, so the stability analysis of time-delay systems strongly requires before experimental stage. As these reason, the stability analysis of time-delay system has formed a sturdy research field during the past years (Gu, Kharitonov, & Chen, 2003).

Let us consider the following time-varying delay systems:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t - h(t)), \\ x(t) = \phi(t), \quad t \in [-h, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $h(t)$ is the time-varying delay satisfying $0 \leq h(t) \leq h$ and $-\mu \leq \dot{h}(t) \leq \mu < 1$, $\phi(t)$ is initial function, and A, B are known real constant matrices with appropriate dimensions.

In stability problems of time-delay systems, to derive less conservative criteria guaranteeing the stability of the system (1) is a key purpose. The maximal allowable upper bound (MAUB) of time-delay is one of the important indexes to check conservatism of stability criteria in the system. Therefore, many researchers have tried to develop such conditions which ensure the stability for MAUB of time-delay as large as possible. In line with this, several remarkable approaches have been reported such as free-weighting matrix approach, delay partitioning approach, reciprocally convex approach, augmented Lyapunov method, and reduction approach for Jensen's inequality (Kim, 2016; Seuret & Gouaisbaut, 2013; Zeng, He, Wu, & She, 2015a,b).

Recently, authors in Xu, Lam, Zhang, and Zou (2015) gave a new insight for reducing the conservatism of stability criteria. Most existing works on the stability of time-delay systems require the positive definiteness of all matrices in Lyapunov functions to meet their positive definiteness. In Xu et al. (2015), a relaxed condition for a matrix in the Lyapunov function instead of its positive definiteness was proposed, i.e. the matrix does not need to be positive definite. After this work, several works about relaxed conditions were reported (Zhang, Lam, & Xu, 2015a,b).

Motivated by above discussion, this paper focuses on to develop relaxed conditions for time-varying delay systems because above commented works on relaxed conditions can be applied only integral terms with constant time-delay interval, i.e. $\int_{t-h}^{t} f(s)ds$. To this end, a new inequality is derived based on free-matrix-based integral inequality, and then by utilizing this new inequality two new relaxed conditions are presented.

Notation I denotes the identity matrix with appropriate dimensions. $\star$ in a matrix represents the elements below the main diagonal of a symmetric matrix. $Sym\{X\}$ indicates $X + X^T$. $X_{[f(t)]} \in \mathbb{R}^{m\times n}$ means that the elements of the matrix X include the values of $f(t)$. For $X \in \mathbb{R}^{m\times n}$, $X^{\perp}$ denotes a basis for the null-space of X.

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2. Preliminaries

The following lemmas will play a key role to derive main results.

Lemma 1 (Zeng et al., 2015a). *Let x be a differentiable function: $[\alpha, \ \beta] \rightarrow \mathbb{R}^n$. For symmetric matrices $R \in \mathbb{R}^{n\times n}$ and $Z_1, Z_3 \in \mathbb{R}^{3n\times 3n}$, and any matrices $Z_2 \in \mathbb{R}^{3n\times 3n}$ and $N_1, N_2 \in \mathbb{R}^{3n\times n}$ satisfying*

$$\begin{bmatrix} Z_1 & Z_2 & N_1 \\ \star & Z_3 & N_2 \\ \star & \star & R \end{bmatrix} \geq 0,$$

the following inequality holds:

$$-\int_\alpha^\beta \dot{x}^T(s)R\dot{x}(s)ds \leq \varpi_1^T(\alpha, \beta)\Psi_1\varpi_1(\alpha, \beta),$$

where

$$\varpi_1(\alpha, \beta) = \left[x^T(\beta), \quad x^T(\alpha), \quad \frac{1}{\beta-\alpha}\int_\alpha^\beta x^T(s)ds\right]^T,$$

$$\Psi_1 = (\beta-\alpha)\left(Z_1 + \frac{1}{3}Z_3\right) + Sym\Big\{N_1[I, \ \ -I, \ \ 0] + N_2[-I, \ \ -I, \ \ 2I]\Big\}.$$

Lemma 2. *Let $x \in \mathbb{R}^n$ be a continuous function and admits a continuous derivative differentiable function in $[\alpha, \ \beta]$. For symmetric matrices $R \in \mathbb{R}^{n\times n}$ and $Z_1 \in \mathbb{R}^{2n\times 2n}$, and any matrix $Z_2 \in \mathbb{R}^{2n\times n}$ satisfying*

$$\begin{bmatrix} Z_1 & Z_2 \\ \star & R \end{bmatrix} \geq 0,$$

the following inequality holds:

$$-\int_\alpha^\beta x^T(s)Rx(s)ds \leq \varpi_2^T(x, \alpha, \beta)\Psi_2\varpi_2(x, \alpha, \beta),$$

where

$$\varpi_2(x, \alpha, \beta) = \left[\int_\alpha^\beta x^T(s)ds, \quad \frac{1}{\beta-\alpha}\int_\alpha^\beta\int_v^\beta x^T(s)dsdv\right]^T,$$

$$\Psi_2 = \frac{\beta-\alpha}{3}Z_1 + Sym\Big\{Z_2[-I, \ \ 2I]\Big\}.$$

Proof. Lemma 2 can be easily obtained as the same procedure to Lemma 4 of Zeng et al. (2015a) with $\zeta(s) = \left[f(s)\varpi_2^T(x, \alpha, \beta), \ x^T(s)\right]^T$. So, it is omitted here. ■

Remark 1. In Zeng et al. (2015a), authors have stated that Lemma 1 is more generalized inequality because depending on matrices, Z_1, Z_2, Z_3, N_1, and N_2, it could be Lemma 2 of Zhang, Wu, She, and He (2005) or Corollary 5 of Seuret and Gouaisbaut (2013). But Lemma 1 can only provide the relation between $J_R(\dot{x}, \alpha, \beta)$ and $\varpi_1(\alpha, \beta)$ where $J_R(\dot{x}, \alpha, \beta) \triangleq -\int_\alpha^\beta \dot{x}^T(s)R\dot{x}(s)ds$. In other words, Lemma 1 cannot suggest the upper bound of $J_R(x, \alpha, \beta)$ or $J_R(\int_s^\beta x(v)dv, \alpha, \beta)$, and so on. On the other hand, Lemma 2 can present their relation. In addition, Lemma 2 would also be reduced to Corollary 5 of Seuret and Gouaisbaut (2013) when we put $\dot{x}$ into it with $Z_1 = \frac{1}{(\beta-\alpha)^2}\begin{bmatrix} -6R & 9R \\ \star & -16R \end{bmatrix}$ and $Z_2 = \frac{1}{\beta-\alpha}\begin{bmatrix} -R & R \end{bmatrix}^T$. Also, for $J_R(\dot{x}, \alpha, \beta)$, Lemma 2 becomes the same to Lemma 2 of Zhang et al. (2005). These show the generality of Lemma 2.

Lemma 3. *For the system (1) with given a positive constant h, if there exist positive definite matrices $Q_i \in \mathbb{R}^{3n\times 3n}$ $(i = 1, 2)$, symmetric matrices $P \in \mathbb{R}^{5n\times 5n}$, $G_i \in \mathbb{R}^{6n\times 6n}$ $(i = 1, 2)$, and any matrices $H_i \in \mathbb{R}^{6n\times 3n}$ $(i = 1, 2)$ satisfying the following LMIs: $\forall h(t) \in \{0, h\}$, $\forall h^2(t) \in \{0, h^2\}$, $\forall h^3(t) \in \{0, h^3\}$*

$$\Sigma_{[\mathcal{H}]} > 0, \tag{2}$$

$$\begin{bmatrix} G_1 & H_1 \\ \star & Q_1 \end{bmatrix} \geq 0, \qquad \begin{bmatrix} G_2 & H_2 \\ \star & Q_2 \end{bmatrix} \geq 0, \tag{3}$$

then the following function is positive definite for $0 \leq h(t) \leq h$:

$$V_a(t) = \eta_1^T(t)P\eta_1(t) + \int_{t-h(t)}^t \eta_2^T(v)Q_1\eta_2(v)dv + \int_{t-h}^{t-h(t)} \eta_3^T(v)Q_2\eta_3(v)dv, \tag{4}$$

where $\mathcal{H} = [h(t), h^2(t), h^3(t)]$ and

$$\eta_1(t) = \left[x^T(t), \ x^T(t-h(t)), \ x^T(t-h), \int_{t-h(t)}^t x^T(s)ds, \ \int_{t-h}^{t-h(t)} x^T(s)ds\right]^T,$$

$$\eta_2(v) = \left[x^T(v), \quad \dot{x}^T(v), \quad \int_v^t \dot{x}^T(s)ds\right]^T,$$

$$\eta_3(v) = \left[x^T(v), \quad \dot{x}^T(v), \quad \int_v^{t-h(t)} \dot{x}^T(s)ds\right]^T,$$

$$\Pi_{1[h(t)]} = \left[r_1, \quad r_2, \quad r_3, \quad h(t)r_4, \quad (h-h(t))r_5\right],$$

$$\Pi_{2[h(t)]} = \left[h(t)r_4, \ r_1-r_2, \ h(t)(r_1-r_4), \ r_6, \ r_1-r_4, \ \frac{h(t)}{2}r_1 - r_6\right],$$

$$\Pi_{3[h(t)]} = \left[2r_6 - h(t)r_4, \quad r_1+r_2-2r_4, \quad h(t)r_4 - 2r_6\right],$$

$$\Pi_{4[h(t)]} = \left[(h-h(t))r_5, \ r_2-r_3, \ (h-h(t))(r_2-r_5), \ r_7, \ r_2-r_5, \ \frac{h-h(t)}{2}r_2 - r_7\right],$$

$$\Pi_{5[h(t)]} = \left[2r_7 - (h-h(t))r_5, \ r_2+r_3-2r_5, \ (h-h(t))r_5 - 2r_7\right],$$

$$\Upsilon_{1[\mathcal{H}]} = \frac{h(t)}{3}\Pi_{2[h(t)]}G_1\Pi_{2[h(t)]}^T + Sym\Big\{\Pi_{2[h(t)]}H_1\Pi_{3[h(t)]}^T\Big\},$$

$$\Upsilon_{2[\mathcal{H}]} = \frac{h-h(t)}{3}\Pi_{4[h(t)]}G_2\Pi_{4[h(t)]}^T + Sym\Big\{\Pi_{4[h(t)]}H_2\Pi_{5[h(t)]}^T\Big\},$$

$$\Sigma_{[\mathcal{H}]} = \Pi_{1[h(t)]}P\Pi_{1[h(t)]}^T - \Upsilon_{1[\mathcal{H}]} - \Upsilon_{2[\mathcal{H}]}$$

and r_i $(i = 1, \ldots, 7) \in \mathbb{R}^{7n\times n}$ (for example, $r_3 = [0, 0, I, 0, 0, 0, 0]$) are block entry matrices.

Proof. Let a vector be

$$\chi(t) = \left[x^T(t), \quad x^T(t-h(t)), \quad x^T(t-h), \ \frac{1}{h(t)}\int_{t-h(t)}^t x^T(s)ds, \quad \frac{1}{h-h(t)}\int_{t-h}^{t-h(t)} x^T(s)ds, \ \frac{1}{h(t)}\int_{t-h(t)}^t\int_v^t x^T(s)dsdv, \ \frac{1}{h-h(t)}\int_{t-h}^{t-h(t)}\int_v^{t-h(t)} x^T(s)dsdv\right]^T.$$

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