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Design constraints and limits of networked feedback in disturbance attenuation: An information-theoretic analysis[☆]

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ABSTRACT

In this paper we investigate the intrinsic design constraints and performance limits of networked control systems. We propose new information measures and correspondingly, develop an information-theoretic paradigm for analyzing the performance trade-offs and limits in disturbance attenuation over information-constrained networked feedback, which is enabled by a cohesive development of new information measures, Bode-type integral inequalities, and performance bounds. The integrals and bounds incorporate the information measures and serve to quantify the trade-offs and limits in disturbance attenuation for broad classes of networked feedback systems consisting of linear time-invariant plants and causal, possibly nonlinear, time-varying stabilizing controllers communicating over general noisy channels with causal encoders and decoders. The notion of negentropy rate is introduced to address general, non-Gaussian disturbances. The channel blurredness, a newly proposed information measure for the quality of communication channels, is used to characterize the effect of communication channel noises on the integrals and henceforth the trade-offs in disturbance attenuation. Bounds on the power gain, a novel disturbance attenuation measure tailored for performance analysis of networked control systems, provide the fundamental limits of disturbance attenuation achievable by networked feedback.

1. Introduction

Control system design is intended to mitigate physical constraints and meet a variety of performance objectives. Bode sensitivity integral relation (Bode, 1945), arguably the most important result in performance limitation studies, plays a central role in quantifying the intrinsic design constraints and performance limits in feedback design, of which a canonical objective is disturbance attenuation. For a linear time-invariant (LTI) system, the disturbance attenuation property is characterized by the system's sensitivity function. Accordingly, fundamental trade-off of disturbance attenuation can be fully captured frequency-wise using the Bode sensitivity integral. Bode integral relations have had a profound impact and have inspired renewed research effort dated most recently, leading to various extensions and new results which seek to quantify design constraints and performance limitations by logarithmic integrals of Bode and Poisson type (Boyd & Desoer, 1985; Chen, 1995, 2000, 2014; Freudenberg & Looze, 1985; Middleton, 1991).

Closely related to the Bode integral relations are fundamental limits of performance achievable under system constraints. With performance indices addressing different design goals, performance limits and bounds have been obtained under the $\mathcal{H}_\infty$ criterion (Boyd & Desoer, 1985; Chen, 2000; Khargonekar & Tannenbaum, 1985), and for LQR and $\mathcal{H}_2$ performance indices, concerning, e.g., cheap regulator problem (Kwakernaak & Sivan, 1972), servomechanism problem (Qiu & Davison, 1993), and optimal tracking (Chen, Hara, & Chen, 2003; Chen, Qiu, & Toker, 2000; Morari & Zafiriou, 1989). Like Bode integral relations, the search for performance limits has also gone unabated. Most

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notably, recent developments in the networked control have led to fundamental limits of feedback stabilization over communication channels; more specifically, it has been found that for a system to be stabilized over a communication channel, the channel specification, such as data rate, capacity, quantization density, and signal-to-noise ratio (SNR), must in one way or another satisfy an intrinsic bound (Braslavsky, Middleton, & Freudenberg, 2007; Nair & Evans, 2000; Tatikonda & Mitter, 2004).

While the aforementioned developments have been substantial and are continuing to branch to different problem areas and for different system categories, unfortunately, neither the Bode integral relations nor the performance limits can be readily extended to networked feedback systems, due to the inherent communication constraints present in the feedback loop (Bemporad, Heemels, & Johansson, 2010; Hespanha, Naghshtabrizi, & Xu, 2007; Nair, Fagnani, Zampieri, & Evans, 2007). The impact of communication constraints on control performance calls for the incorporation of information and communication constraints into performance limitation studies, and in turn the integration of information and control theories for analysis and design of networked feedback systems. This constitutes the primary goal of our present paper.

Earlier information-theoretic developments of Bode integrals include Jonckheere, Hammad, and Wu (1993), Jonckheere and Wu (1992), Iglesias (2001) and Zang and Iglesias (2003). In Jonckheere et al. (1993) and Jonckheere and Wu (1992), relations were established between the Bode sensitivity integral and the Kolmogorov–Sinai entropy and the Shannon entropy rate. In Iglesias (2001) and Zang and Iglesias (2003), an information-theoretic approach was employed to characterize input–output invariance properties similar to that described by the Bode sensitivity integral. More recently, Martins and co-workers (Martins & Dahleh, 2008; Martins, Dahleh, & Doyle, 2007) studied the disturbance attenuation problem for single-input and single-output (SISO) networked feedback systems. Using also an information-theoretic approach, they obtained Bode-type integral inequalities incorporating channel capacity. Further along this line, Ishii, Okano, and Hara (2011) and Okano, Hara, and Ishii (2009) obtained similar results for multiple-input and multiple-output (MIMO) systems. Ensuing works sought after extensions to nonlinear systems (Yu & Mehta, 2010) and stochastic switched systems (Li & Hovakimyan, 2013). For conventional systems, applications of the Bode integral relation were found in, e.g., aircraft control (Stein, 2003), gas turbine combustion systems (Banaszuk, Mehta, Jacobson, & Khibnik, 2006), and steel milling processes (Goodwin, Graebe, & Salgado, 2001). More recently, for systems subject to communication constraints, Bode-type integral inequalities were employed to analyze molecular fluctuations (Lestas, Vinnicombe, & Paulsson, 2010) and vehicle platoon control systems (Zhao, Minero, & Gupta, 2014). Conversely, performance of communication systems has also to be characterized using the Bode integral relation; see, e.g., Elia (2004).

In this paper we develop Bode-type integral relations and performance bounds concerning the disturbance attenuation of networked feedback systems subject to information constraints. While following in part the spirit of the earlier works in, e.g., Martins and Dahleh (2008), Martins et al. (2007), Ishii et al. (2011) and Okano et al. (2009), the Bode-type integrals developed presently differ in several important aspects. First, we seek to develop new information-theoretic measures to quantify the effect of disturbances and communication channel constraints on the integrals. These measures include *negentropy rate* and *channel blurredness*. The introduction of the negentropy rate enables us to address more general, non-Gaussian disturbances than Gaussian disturbances; the latter are typically assumed in the aforementioned works. The channel blurredness, on the other hand, provides a measure of channel quality seen rather susceptible to characterizing the trade-off in disturbance attenuation, which appears more amenable than the traditional channel capacity and is directly pertinent to characterizing channel noise effect. Second, our integral inequalities based on the new measures are potentially tighter. Furthermore, since they concern a system's disturbance attenuation over the entire frequency range, our integral inequalities appear better suited to account for performance trade-offs in different frequency ranges. In this spirit, we derive two sets of Bode-type integral inequalities, with one quantifying the effect of unstable poles on the disturbance attenuation at the plant input and the other quantifying the effect of nonminimum phase zeros on the disturbance attenuation at the system's output.

The performance bounds derived in this paper have no analog in the previous networked control studies. These bounds incorporate the new information measures and provide fundamental limits of disturbance attenuation imposed by information constraints, which can neither be overcome nor circumvented by any controller design or channel coding. The bounds are developed for a rather general notion termed *power gain*, which measures the worst-case power reduction of the disturbance signal. They are also obtained under very general conditions: the controller and the encoder/decoder are all allowed to be nonlinear and time varying. It is seen that in general the disturbance response may grow exponentially in power with channel blurredness. This fact thus further exhibits the distinct impact of communication channel on control performance, and solidifies the role of channel blurredness in capturing it. Furthermore, when specialized to more specific channels such as an additive white Gaussian noise (AWGN) channel, for which an explicit expression can be found for the channel blurredness, the lower bound is seen to depend on the channel's noise-to-signal ratio explicitly. Interestingly, these bounds can also be applied to systems without communication channels, whereas both the controller and plant may be nonlinear and time varying.

In summary, the information-theoretic analysis conducted in this paper is rather general and allows for performance analysis of broad classes of networked feedback systems consisting of nonlinear, time-varying controllers and nonlinear, time-varying communication channels, together with possibly nonlinear, time-varying plants. This generality ensures that our results are applicable to networked systems with a wide variety of channels, in which both the physical-layer models and the higher-layer input–output properties of the communication channels can be accommodated; in the former case, according to standardized interfaces and layering principles (Gallager, 2008) in digital communication, the encoder may include quantizer, source encoder, and channel encoder, while the decoder may include channel decoder and source decoder. Particularly relevant are potential applications with filterbank-based encoder/decoder channels (Vaidyanathan, 1993), which constitute a core transmission prototype for, e.g., subband coding and transmultiplexers in digital communications (cf. Section 2). Note also that at a more specific level, AWGN channels can be used to approximate highly nonlinear quantization processes via the so-called dithered quantization, a widely used quantized model (Zamir & Feder, 1992). Thus, the primary benefits offered by this paper lie in that the Bode-type integrals and the power gain bounds can be used to analyze the performance trade-offs and achievable limits under networked feedback with a wide class of channel models; in contrast, the previous Bode-type integrals are largely pertinent to stabilization issues (cf. Section 3).

The remainder of this paper is organized as follows. Section 2 introduces the necessary notations and preliminaries, together with the newly proposed information-theoretic measures, including negentropy rate, power gain, and channel blurredness. In Section 3, the Bode-type integrals of networked feedback systems are derived. Section 4 presents the power gain bounds for networked feedback systems and feedback systems without channels. Section 5 provides an illustrative numerical example. Concluding remarks are given in Section 6.