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Adaptive leader-following consensus for a class of higher-order nonlinear multi-agent systems with directed switching networks☆

Wei Liu, Jie Huang [1]

Department of Mechanical and Automation Engineering, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong

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ABSTRACT

In this paper, we study the leader-following consensus problem for a class of uncertain nonlinear multi-agent systems under jointly connected directed switching networks. The uncertainty includes constant unbounded parameters and external disturbances. We first extend the recent result on the adaptive distributed observer from global asymptotical convergence to global exponential convergence. Then, by integrating the conventional adaptive control technique with the adaptive distributed observer, we present our solution by a distributed adaptive state feedback control law. Our result is illustrated by the leader-following consensus problem for a group of van der Pol oscillators.

1. Introduction

In the past few years, the cooperative control problems for multi-agent systems have attracted extensive attention due to their wide applications in engineering systems such as sensor networks, robotic teams, satellite clusters, unmanned air vehicle formations and so on. The consensus problem is one of the basic cooperative control problems, whose objective is to design a distributed control law for each agent such that the states (or outputs) of all agents approach the same value. Depending on whether or not a multi-agent system has a leader, the consensus problem can be divided into two classes: leaderless and leader-following. The leaderless consensus problem aims to make the states (or outputs) of all agents asymptotically synchronize to a same trajectory, while the leader-following consensus problem requires the states (or outputs) of all agents to asymptotically track a desired trajectory which is generated by the leader system.

The consensus problem of linear multi-agent systems has been extensively studied. For example, the leaderless case was studied in Olfati-Saber and Murray (2004), Ren (2008), Seo, Shim, and Back (2009) and Tuna (2008), the leader-following case was studied in Hong, Chen, and Bushnell (2008); Hong, Hu, and Gao (2006), Hu and Hong (2007), Hu and Zheng (2014) and Ni and Cheng (2010), and both two cases were studied in Jadbabaie, Lin, and Morse (2003) and Su and Huang (2012a). In particular, the linear multi-agent system considered in Hu and Zheng (2014) contains some time-varying disturbances, and the adaptive control technique has been used to deal with these disturbances. Recently, more attention has been paid to the consensus problem of nonlinear multi-agent systems. For example, in Liu, Xie, Ren, and Wang (2013), Mei, Ren, and Ma (2013), Song, Cao, and Yu (2010); Song, Liu, Cao, and Yu (2013) and Yu, Chen, Cao, and Kurths (2010), the consensus problem was studied for several classes of nonlinear systems satisfying the global Lipschitz condition or the global Lipschitz-like condition. In Liu and Huang (2015, 2017), Su and Huang (2013) and Wang, Xu, and Hong (2014), the leader-following consensus problem was studied via the output regulation theory and the nonlinear systems considered in Liu and Huang (2015, 2017), Su and Huang (2013) and Wang et al. (2014) contain both disturbance and uncertainty, but the boundary of the uncertainty is known. In Hu and Hu (2010), the authors designed a nonlinear observer-based filter to track a single second-order linear Gaussian target and analyzed the stability of the proposed filter in the sense of mean square. Based on the adaptive control technique, the leader-following consensus problem was studied for first-order nonlinear multi-agent systems in Das and Lewis (2010) and Yu and Xia (2012), for second-order nonlinear multi-agent systems in Liu and Huang (2016), and for multiple uncertain

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E-mail addresses: wliu@mae.cuhk.edu.hk (W. Liu), jhuang@mae.cuhk.edu.hk (J. Huang).

[1] Fax: +852 39436002.

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rigid spacecraft systems in Cai and Huang (2016b). In Das and Lewis (2010) and Zhang and Lewis (2012), the neutral networks method was used to study uncertain nonlinear multi-agent systems subject to static networks and the designed control laws can make the tracking errors uniformly ultimately bounded for initial conditions in some prescribed compact subset.

In this paper, we will further consider the leader-following consensus problem for a class of uncertain nonlinear multi-agent systems. Our paper has the following features. First, the order of our system is generic and the nonlinearity does not have to satisfy the global Lipschitz-like condition which excludes some benchmark nonlinear systems such as van der Pol systems, Duffing systems and so on. Thus, the linear control techniques as used in Liu et al. (2013), Mei et al. (2013), Song et al. (2010, 2013) and Yu et al. (2010) do not apply to our system. Second, our system contains both constant uncertain parameters and external disturbances and the uncertain parameters can take any constant value. Thus, the robust control approaches in Liu and Huang (2015, 2017), Su and Huang (2013) and Wang et al. (2014) do not apply to our system either. Third, our networks satisfy the jointly connected condition, which is the mildest condition on the communication network since it allows the network to be disconnected at any time, and contains the static network case (Das & Lewis, 2010; Zhang & Lewis, 2012) and the every time connected switching network case (Liu & Huang, 2015) as special cases. Finally, compared with Das and Lewis (2010) and Zhang and Lewis (2012), our result is global and the consensus can be achieved exactly. As a result of these features, the problem is much more general than the existing results and cannot be handled by the techniques in the literatures. To solve our problem, we have integrated the classical adaptive control technique and the recently developed adaptive distributed observer to obtain a distributed adaptive control law. We have also furnished a detailed stability analysis for the closed-loop system.

It should be noted that the leader-following consensus problem for a class of multiple uncertain Euler–Lagrange systems has been studied in Cai and Huang (2016a), where the adaptive distributed observer method has been first proposed. However, the system considered in Cai and Huang (2016a) contains only parameter uncertainty but no disturbance, and the communication network is assumed to be undirected jointly connected. In this paper, we extend the network from the undirected case to the directed case.

The rest of this paper is organized as follows. In Section 2, we present our problem formulation and two assumptions. In Section 3, we introduce some concepts for the adaptive distributed observer and establish a technical lemma. In Section 4, we present our main result. In Section 5, we provide an example to illustrate our design. Finally, in Section 6, we close the paper with some concluding remarks.

Notation. For any column vectors a_i, $i = 1, \ldots, s$, denote $\mathrm{col}(a_1, \ldots, a_s) = [a_1^T, \ldots, a_s^T]^T$. $\otimes$ denotes the Kronecker product of matrices. Vector $\mathbf{1}_N$ denotes an N-dimensional column vector with all elements being 1. $\|x\|$ denotes the Euclidean norm of vector x. $\|A\|$ denotes the induced norm of matrix A by the Euclidean norm. $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the maximum eigenvalue and the minimum eigenvalue of a symmetric real matrix A, respectively. We use $\sigma(t)$ to denote a piecewise constant switching signal $\sigma : [0, +\infty) \to \mathcal{P} = \{1, 2, \ldots, n_0\}$, where n_0 is a positive integer, and $\mathcal{P}$ is called a switching index set. We assume that all switching instants $t_0 = 0 < t_1 < t_2, \ldots$ satisfy $t_{i+1} - t_i \geq \tau_0 > 0$ for some constant τ_0 and all $i \geq 0$, where τ_0 is called the dwell time.

2. Problem formulation

Consider a class of nonlinear multi-agent systems as follows:

$$\begin{aligned} \dot{x}_{si} &= x_{(s+1)i}, \quad s = 1, 2, \ldots, r-1 \\ \dot{x}_{ri} &= f_i^T(x_i, t)\theta_i + d_i(w) + u_i, \quad i = 1, \ldots, N \end{aligned} \tag{1}$$

where $x_i = \mathrm{col}(x_{1i}, \ldots, x_{ri}) \in \mathbb{R}^r$ is the state, $u_i \in \mathbb{R}$ is the input, $f_i : \mathbb{R}^r \times [0, +\infty) \to \mathbb{R}^m$ is a known function satisfying locally Lipschitz condition with respect to x_i uniformly in t, $\theta_i \in \mathbb{R}^m$ is an unknown constant parameter vector, $d_i(w)$ denotes the disturbance with $d_i : \mathbb{R}^{n_w} \to \mathbb{R}$ being a known $\mathcal{C}^1$ function, and w is generated by the following linear exosystem system

$$\dot{w} = S_b w \tag{2}$$

with $w \in \mathbb{R}^{n_w}$ and $S_b \in \mathbb{R}^{n_w \times n_w}$. It is assumed that the reference signal is also generated by a linear exosystem as follows:

$$\dot{x}_0 = S_a x_0 \tag{3}$$

where $x_0 \in \mathbb{R}^r$ and $S_a \in \mathbb{R}^{r \times r}$. Let $v = \mathrm{col}(x_0, w)$ and $S = \mathrm{diag}(S_a, S_b)$. Then we can put (2) and (3) together as follows:

$$\dot{v} = Sv. \tag{4}$$

The system (1) and the exosystem (4) together can be viewed as a multi-agent system of $(N+1)$ agents with (4) as the leader and the N subsystems of (1) as N followers. With respect to the plant (1), the exosystem (4), and a given switching signal $\sigma(t)$, we can define a time-varying digraph $\bar{\mathcal{G}}_{\sigma(t)} = (\bar{\mathcal{V}}, \bar{\mathcal{E}}_{\sigma(t)})$[2] with $\bar{\mathcal{V}} = \{0, 1, \ldots, N\}$ and $\bar{\mathcal{E}}_{\sigma(t)} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$ for all $t \geq 0$, where the node 0 is associated with the leader system (4) and the node i, $i = 1, \ldots, N$, is associated with the ith subsystem of system (1). For $i = 1, \ldots, N$, $j = 0, 1, \ldots, N$, and $i \neq j$, $(j, i) \in \bar{\mathcal{E}}_{\sigma(t)}$ if and only if u_i can use the information of the jth subsystem for control at time instant t. Let $\bar{\mathcal{A}}_{\sigma(t)} = [\bar{a}_{ij}(t)] \in \mathbb{R}^{(N+1)\times(N+1)}$ be the weighted adjacency matrix of $\bar{\mathcal{G}}_{\sigma(t)}$. Let $\bar{\mathcal{N}}_i(t) = \{j, (j, i) \in \bar{\mathcal{E}}_{\sigma(t)}\}$ denote the neighbor set of agent i at time t. Let $\mathcal{G}_{\sigma(t)} = (\mathcal{V}, \mathcal{E}_\sigma(t))$ be the subgraph of $\bar{\mathcal{G}}_{\sigma(t)}$, where $\mathcal{V} = \{1, \ldots, N\}$ and $\mathcal{E}_{\sigma(t)} \subseteq \mathcal{V} \times \mathcal{V}$ is obtained from $\bar{\mathcal{E}}_{\sigma(t)}$ by removing all edges between the node 0 and the nodes in $\mathcal{V}$. Clearly, the case where the network topology is fixed can be viewed as a special case of switching network topology when the switching index set contains only one element.

Let us describe our control law as follows.

$$\begin{aligned} u_i &= h_i(x_i, \zeta_i, x_j, \zeta_j) \\ \dot{\zeta}_i &= l_i(x_i, \zeta_i, x_j, \zeta_j, j \in \bar{\mathcal{N}}_i(t)), \quad i = 1, \ldots, N \end{aligned} \tag{5}$$

where h_i and l_i are some nonlinear functions.

A control law of the form (5) is called a distributed control law since u_i only depends on the information of its neighbors and itself. Our problem is described as follows.

Problem 1. Given the multi-agent system composed of (1) and (4), and a switching graph $\bar{\mathcal{G}}_{\sigma(t)}$, design a control law of the form (5), such that, for any initial states $x_i(0)$, $\zeta_i(0)$ and $v(0)$, the solution of the closed-loop system exists for all $t \geq 0$, and satisfies $\lim_{t\to+\infty}(x_i(t) - x_0(t)) = 0$.

To solve our problem, we introduce two assumptions as follows.

Assumption 1. All the eigenvalues of S are distinct with zero real parts.

Remark 2.1. Under Assumption 1, the exosystem (4) can generate arbitrarily large constant signals and multi-tone sinusoidal signals with arbitrarily unknown initial phases and amplitudes and arbitrarily known frequencies. Since, under Assumption 1, all the eigenvalues of S_a are distinct, the minimal polynomial of S_a is

[2] See Appendix for a summary of graph.