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Errors-in-variables identification using maximum likelihood estimation in the frequency domain ☆

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ABSTRACT

This paper deals with the identification of errors-in-variables (EIV) models corrupted by additive and uncorrelated white Gaussian noises when the noise-free input is an arbitrary signal, not required to be periodic. In particular, a frequency domain maximum likelihood (ML) estimator is proposed and analyzed in some detail. As some other EIV estimators, this method assumes that the ratio of the noise variances is known. The estimation problem is formulated in the frequency domain. It is shown that the parameter estimates are consistent. An explicit algorithm for computing the asymptotic covariance matrix of the parameter estimates is derived. The possibility to effectively use lowpass filtered data by using only part of the frequency domain is discussed, analyzed and illustrated.

1. Introduction

In this paper the problem of identifying a linear dynamic system from noisy input–output measurements is addressed. System representations where both inputs and outputs are affected by additive errors are called errors-in-variables (EIV) models and play an important role in several engineering and other applications, Van Huffel (1997), Van Huffel and Lemmerling (2002) and Zhang, Pintelon, and Schoukens (2013). Many system identification methods have been proposed for the EIV problems. For some surveys in the field, see Söderström (2007, 2012) and Guidorzi, Diversi, and Soverini (2008).

In many EIV contexts the additive noises are assumed as white. In these cases, if the assumptions of Gaussianity are fulfilled, it is feasible to use a maximum likelihood (ML) approach. In this work the EIV ML problem is addressed by using frequency domain techniques, when the noise-free input is an arbitrary sequence and the noise variance ratio is known. The frequency domain approach has some special features, not present in time domain methods. In particular, filtering can be reduced to the selection of appropriate frequencies in a limited band of the signal spectrum. Moreover, continuous-time and discrete-time models can be handled with equal difficulties, McKelvey (2002) and Ljung (1999). From a theoretic point of view, there is a full equivalence between time and frequency domain identification methods, also for finite data records, Agüero, Yuz, Goodwin, and Delgado (2010). Some ideas of the approach described here were first presented in Soverini and Söderström (2014). The derivation here is quite different and much more direct. Further, this paper also contains analysis of the consistency and accuracy properties of the parameter estimates. In its approach the paper differs from other previous work on an ML formulation of the EIV problem in the time and frequency domains, cf. Diversi, Guidorzi, and Soverini (2007), Pintelon and Schoukens (2007, 2012), see also Zhang et al. (2013). The relation of these papers to the present one is discussed in Section 2, see Remark 2.1 and the discussion thereafter.

The organization of the paper is as follows. Section 2 defines the EIV identification problem in frequency domain. Section 3 describes the EIV set up in the frequency domain, while Section 4 presents the ML solution. Section 5 analyses the properties of the parameter estimates. In Section 6 the properties of the proposed ML algorithm are verified by means of Monte Carlo simulations. Finally some concluding remarks are reported in Section 7.

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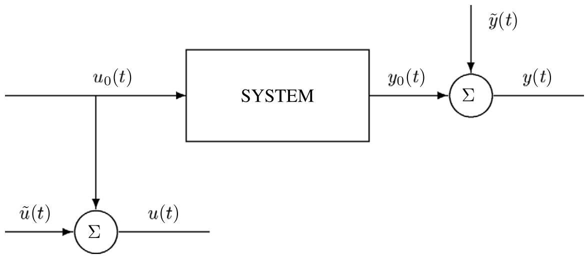

Fig. 1. The basic setup for a dynamic errors-in-variables problem.

2. Statement of the problem

Consider the linear time-invariant SISO system described in Fig. 1. The noise-free input and output $u_0(t)$, $y_0(t)$ are linked by the difference equation

$$A(q^{-1})\, y_0(t) = B(q^{-1})\, u_0(t), \tag{1}$$

where $A(q^{-1})$ and $B(q^{-1})$ are polynomials in the backward shift operator q^{-1}

$$\begin{aligned} A(q^{-1}) &= 1 + a_1 q^{-1} + \cdots + a_{n_a} q^{-n_a} \\ B(q^{-1}) &= b_0 + b_1 q^{-1} + \cdots + b_{n_b} q^{-n_b}. \end{aligned} \tag{2}$$

In the EIV environment the input and output measurements are assumed to be corrupted by additive noise so that the available observations are

$$u(t) = u_0(t) + \tilde{u}(t) \tag{3}$$

$$y(t) = y_0(t) + \tilde{y}(t). \tag{4}$$

In the sequel, the following assumptions will be considered as satisfied.

A1. The system (1) is asymptotically stable.
A2. $A(q^{-1})$ and $B(q^{-1})$ do not share any common factor.
A3. The polynomial degrees n_a and n_b are assumed to be *a priori* known.
A4. The noiseless input $u_0(t)$ is a zero-mean ergodic process and is persistently exciting of sufficiently high order.
A5. $\tilde{u}(t)$ and $\tilde{y}(t)$ are mutually uncorrelated zero-mean Gaussian white processes with variances λ_u and λ_y, respectively.
A6. $\tilde{u}(t)$ and $\tilde{y}(t)$ are uncorrelated with the noise-free input $u_0(t)$.

Let $\{u(t)\}_{t=0}^{N-1}$ and $\{y(t)\}_{t=0}^{N-1}$ be a set of input and output observations at N equidistant time instants. The corresponding Discrete Fourier Transforms (DFTs) are defined as

$$U(\omega_k) = \frac{1}{\sqrt{N}} \sum_{t=0}^{N-1} u(t)\, \mathrm{e}^{-\mathrm{i}\omega_k t}\,, \tag{5}$$

$$Y(\omega_k) = \frac{1}{\sqrt{N}} \sum_{t=0}^{N-1} y(t)\, \mathrm{e}^{-\mathrm{i}\omega_k t}\,, \tag{6}$$

where $\omega_k = 2\pi k/N$ and $k = 0, \ldots, N-1$. In frequency domain, the problem under investigation can be stated as follows.

Problem 1. Let $U(\omega_k)$, $Y(\omega_k)$ be a set of noisy measurements generated by an EIV system of type (1)–(4), under Assumptions A1–A6, where $\omega_k = 2\pi k/N$ and $k = 0, \ldots, N-1$. Estimate the system parameters a_i $(i = 1, \ldots, n_a)$, b_i $(i = 0, \ldots, n_b)$ and possibly the noise variances λ_u, λ_y.

Remark 2.1. Under the previous Assumptions A1–A6, the system is not identifiable in the very general case. To achieve identifiability, one has to add one of the following assumptions.

(1) The properties of the noise-free input are given by a model of finite order, for example an ARMA model.
(2) The noise-free input is known to be periodic. Two periods of the data are then sufficient.
(3) The ratio of the noise variances is known.

One can always debate which assumption is more realistic or more general than the others.

The first assumption is applied in the so called joint output method, Söderström (1981). A frequency domain variant is developed in Pintelon and Schoukens (2007).

A reason why the last two assumptions are quite similar in character goes as follows: if the second assumption applies, by subtracting data of one period from the other period, one gets an estimate of the pure noisy data, and then the noise variances can be estimated separately. This is essentially the SML approach proposed in Schoukens, Pintelon, Vandersteen, and Guillaume (1997).

There are also several methods in the literature where none of the conditions (1)–(3) is imposed, but then the information in the data is not fully exploited. Still, consistency is achieved. Examples of such methods are the instrumental variable method, the generalized instrumental variable method, Söderström (2011), including its special cases bias-eliminating least squares and the Frisch scheme, and the covariance matching approach, Söderström, Mossberg, and Hong (2009).

Methods for EIV identification do also differ in what quantities are estimated in addition to the polynomial coefficients in (2). Both in Zhang et al. (2013) and in this paper the realization $u_0(0), \ldots, u_0(N-1)$ of the noiseless input is regarded as a deterministic sequence to be estimated.

In this paper the following additional assumption is applied, with the aim of deriving, exploiting and analyzing the frequency domain ML estimator.

A7. The noise variances λ_u and λ_y are unknown but their ratio $\rho = \lambda_y/\lambda_u$ is assumed as known, with $0 < \rho < \infty$.

For situations where $\tilde{u}(t)$ and $\tilde{y}(t)$ are due to sensor noise, it can be possible to obtain separate measurement data of the noise only. From such records, estimates of λ_u and λ_y (and also ρ) can be obtained.

Remark 2.2. It is worth noting that Assumption A7 is necessary also for estimators based on total least squares (TLS), to be consistent, cf. Cadzow and Solomon (1986), Van Huffel (1997) and Van Huffel and Lemmerling (2002).

The paper (Zhang et al., 2013) uses frequency domain properties as in this paper, and both papers take effects of transients into account. While this paper deals fully with the ML estimation, the estimation method considered in Zhang et al. (2013) is different and only partly based on equations derived from the gradient of the likelihood function.

The following frequency domain property is worth recalling.

Property. *Let $x(t)$ be an arbitrary signal. Consider the frequency domain data $X(\omega_k) \triangleq X_k$, $k = 0, \ldots, N-1$ of $x(t)$, see (5). The number of data N is usually even, however the following consideration holds also when N is odd. It can be observed that*

$$\begin{aligned} X(\omega_{N-1-k}) &= \frac{1}{\sqrt{N}} \sum_{t=0}^{N-1} x(t)\, \mathrm{e}^{-\mathrm{i}\frac{N-1-k}{N}2\pi t} \\ &= \frac{1}{\sqrt{N}} \sum_{t=0}^{N-1} x(t)\, \mathrm{e}^{-\mathrm{i}\frac{-(1+k)}{N}2\pi t} = X^*(\omega_{1+k})\,, \end{aligned} \tag{7}$$

where $X^*(\cdot)$ is the conjugate of $X(\cdot)$. The conclusion is that it is enough to consider half of the sequence $X_0, \ldots, X_{N-1}$, since the