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Multi-sinusoidal disturbance rejection for discrete-time uncertain stable systems☆

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ABSTRACT

Linear single-input single-output discrete-time systems $P(z)$ with unknown parameters, order and relative degree are considered, which are perturbed by biased sinusoidal disturbances. Under the assumption that the disturbance frequencies, the sign of the static gain $P(1)$, the sign of either $\mathrm{Re}[P(e^{j\omega_i})]$ or $\mathrm{Im}[P(e^{j\omega_i})]$, for any disturbance frequency ω_i, are known, linear disturbance compensators are proposed which achieve exponential disturbance suppression. Such results are then locally extended to the case of disturbances with unknown frequencies.

1. Introduction and problem statement

The problem of disturbance rejection or disturbance attenuation on the controlled output of dynamical systems is one of the main problems which arise in control applications. The disturbance signal may be of stochastic type or of deterministic type and in the latter case very often it can be modeled as the output of an exogenous system (exosystem). For example, sinusoidal disturbances and periodic disturbances (with a limited number of harmonics) belong to such a class and are largely found in practical control problems (Ben Amara, Kabamba, & Ulsoy, 1999; Hong, Du, Tee, & Ge, 2010; Landau, Alma, Constantinescu, Martinez, & Noe, 2011). The disturbances may be either of known frequencies or of unknown (or partially known) frequencies so that the uncertainty on the exosystem may refer only to the initial conditions or also to the exosystem parameters.

Even though a large amount of papers is available for continuous-time systems (see Brown & Zhang, 2004; Esbrook, Tan, & Khalil, 2013; Fedele & Ferrise, 2013; Pigg & Bodson, 2010 and references contained therein), only a few are available for the discrete-time counterpart. Since the control algorithms are generally discretized to obtain digital controllers, the study of discrete-time control laws is of great interest. In fact, if we consider the linear scalar system

$$\dot{x} = -x + u + d(t) \tag{1}$$

and the disturbance compensator

$$\begin{aligned} \dot{\eta}_1 &= \eta_2 + 2x \\ \dot{\eta}_2 &= -\omega^2 \eta_1 \\ u &= -\eta_1 \end{aligned} \tag{2}$$

in which ω is the frequency of the sinusoidal disturbance $d(t) = d_0 \sin(\omega t + \phi)$, it is easy to see that the closed loop system is asymptotically stable. However, if an Euler first-order discretized version of (2) is applied to system (1), an easy computation shows that when $d = 0$ the zero-order hold discrete-time system obtained from (1), in closed-loop with the Euler discretized version of (2) is unstable, for some sampling time T and for some frequency ω. For example, for $\omega = 3$, $T = 0.1$ we obtain the following eigenvalues: $(-0.8289, -0.0856 \pm j3.2941)$ in continuous-time, $(0.9205, 0.9921 \pm j0.3282)$ in discrete-time.

The most interesting contributions to the solution of the disturbance rejection problem for discrete-time linear systems are discussed in the sequel. Under the assumption that the plant model is known while the disturbance model is of known order but unknown, two adaptive regulators are proposed in Landau et al. (2011). In Hoagg, Santillo, and Bernstein (2008), the parameters of the process and those of the exosystem are supposed to be unknown and a fully adaptive control law is proposed for minimum-phase multi-input multi-output linear systems

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with known relative degree. The proposed controller achieves global asymptotic output error vanishing but the convergence is not exponential, so that robustness with respect to unmodeled disturbances is not guaranteed. The approach followed in Guo and Bodson (2009) consists in estimating amplitudes, phases and frequencies of the unknown periodic disturbance so that it can be compensated by the control input. Even in this case, the plant model is required to be known. In Marino and Santosuosso (2011), an exponentially converging global regulator is designed for known systems affected by an unknown number (with known upper bound) of sinusoidal disturbances. Several contributions referring to a benchmark example are given in Airimitoaie, Castellanos Silva, and Landau (2013), Aranovskiy and Freidovich (2013), Castellanos Silva, Landau, and Airimitoaie (2013), Chen and Tomizuka (2013), de Callafon and Fang (2013), Karimi and Emedi (2013) and Wu and Ben Amara (2013). All of them assume that the benchmark is accurately identified and robustness to model uncertainties are analyzed only for the case of known frequencies in Aranovskiy and Freidovich (2013) and de Callafon and Fang (2013), where it is shown that stability is preserved for a little amount of parameter uncertainties. In Jafari, Ioannou, Fitzpatrick, and Wang (2015), small model uncertainties are allowed when the disturbance frequencies are known, while ultimately bounded error is guaranteed in the case of unknown frequencies.

The assumption of known plant model is very restrictive since the plant parameters may be different from the nominal ones for several reasons: different operating conditions, abruptly changes as a consequence of faults, tolerance range allowed by manufacturing industries. For instance, the system

$$y(z) = \frac{(1+z)[u(z)+d(z)]}{(z-0.1)(z-0.5)} \tag{3}$$

may reduce to

$$y(z) = \frac{u(z)+(1+z)d(z)}{(z-0.1)(z-0.5)} \tag{4}$$

as a consequence of sensor/actuator fault, so that even its relative degree may be different (see further details in Section 5). In this paper, motivated by the recent results obtained for linear continuous-time systems (Marino & Tomei, 2015, 2016), we consider single-input single-output linear stable discrete-time systems whose order, relative degree and parameters are completely unknown, perturbed by additive biased multi-sinusoidal disturbances. However, the sign of the static gain $P(1)$ of the transfer function $P(z)$ along with either the sign of $\mathrm{Re}[P(e^{j\omega_i})]$ or the sign of $\mathrm{Im}[P(e^{j\omega_i})]$, for any disturbance frequency ω_i, must be known. The problem we are considering is precisely stated in the following definition.

Definition 1.1 (*Disturbance Rejection Problem*). Consider the linear system

$$x(k+1) = Ax(k) + B[u(k)+d(k)],\ x(0) = x_0$$
$$y(k) = Cx(k) + D[u(k)+d(k)] \tag{5}$$

in which $x \in \mathbb{R}^n$, $y \in \mathbb{R}$, $u \in \mathbb{R}$, $d(k) = d_0 + \sum_{i=1}^{q} d_i \cos(\Omega_i kT + \phi_i)$ is a matching biased multi-sinusoidal disturbance, with $d_i \geq 0$, $0 \leq i \leq q$, $\Omega_i > 0$, $1 \leq i \leq q$, $0 \leq \phi_i < 2\pi$, $1 \leq i \leq q$ and T being the sampling time. Assume that A is a Schur matrix (i.e. all its eigenvalues are strictly inside the unit disk) and denote by

$$P(z) = C(zI-A)^{-1}B + D \tag{6}$$

the transfer function between $u(z)$ and $y(z)$. The disturbance rejection problem is solvable for the linear system (5), if there exists a linear output feedback compensator (see Fig. 1)

$$\xi(k+1) = A_u\xi(k) + B_u y(k), \qquad \xi(0) = \xi_0$$

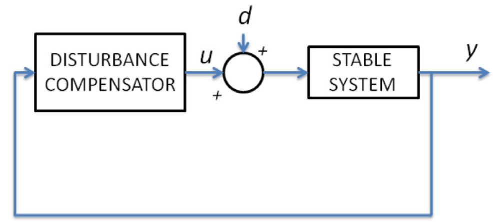

Fig. 1. Block diagram of the disturbance compensator.

$$u(k) = C_u\xi(k) + D_u y(k) \tag{7}$$

such that for the closed-loop system (5), (7), the state vector $x(k)$ and the disturbance compensator error $u(k)+d(k)$ converge exponentially to zero as k tends to infinity, for any initial condition (x_0, ξ_0). □

Remark 1.1. Note that also systems with not-matching disturbances can be reduced to the form (5). In fact, consider the system

$$x(k+1) = Ax(k) + Bu(k) + M\bar{d}(k)$$
$$y(k) = Cx(k) + Du(k) + N\bar{d}(k) \tag{8}$$

in which $\bar{d}(k)$ is a biased multi-sinusoidal disturbance. Assume that an input $u_r(k)$ exists such that (for suitable initial conditions) the disturbance has no effect on the output, i.e.

$$x_r(k+1) = Ax_r(k) + Bu_r(k) + M\bar{d}(k)$$
$$0 = Cx_r(k) + Du_r(k) + N\bar{d}(k). \tag{9}$$

Defining $\tilde{x} = x - x_r$, from (8) and (9), we obtain

$$\tilde{x}(k+1) = A\tilde{x}(k) + B[u(k) - u_r(k)]$$
$$y(k) = C\tilde{x}(k) + D[u(k) - u_r(k)]. \tag{10}$$

so that by setting $d(k) = -u_r(k)$, we re-obtain (5). □

We show that a copy of the disturbance exosystem (internal model, Francis & Wonham, 1976) driven by the output error fed back by a sufficiently small gain is sufficient to achieve global exponential disturbance rejection. We begin in Section 2 by considering constant and pure sinusoidal disturbances while the general case of biased multi-sinusoidal disturbances is addressed in Section 3. In Section 4, we consider disturbances with unknown frequencies. A local solution for the disturbance rejection problem is proposed in which the unknown frequencies are estimated by a first order updating difference equation for each frequency to be estimated. By means of the averaging method, local converging properties are demonstrated. Finally, two numerical examples are simulated in Section 5 to illustrate the performance of the proposed compensator: the first one is referred to a system with variable relative degree as a consequence of fault; the second one deals with a learning problem in the case of unknown period.

2. Constant and single-frequency disturbances

In this section, some preliminary results are stated and demonstrated. In particular, the problem of disturbance rejection is examined when disturbances are either constant or purely sinusoidal. The constructive proofs of the theorems proposed in this section will be useful to gain a better understanding of the more complex results given in the next section.

Theorem 2.1 (*Constant Disturbance*). *Consider the linear system (5) with $d(k)$ a constant disturbance. Assume that $P(1) \neq 0$ with known sign. Then, there exists a $g^* > 0$ such that for any $0 < g \leq g^*$, the dynamic output feedback compensator*

$$\hat{\eta}(k+1) = \hat{\eta}(k) + g\ \mathrm{sign}[P(1)]y(k)$$