Automatica 79 (2017) 11–16

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Prediction-based control for systems with state and several input delays☆

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ARTICLE INFO

Article history:
Received 14 December 2015
Received in revised form
19 September 2016
Accepted 9 January 2017

Keywords:
Systems with input and state delays
Delay compensation
Predictor-based control

ABSTRACT

In this contribution an extension of the prediction-based design scheme proposed in Tsubakino et al. (2015) for compensation of input delays to the case of systems with input and state delays is presented. To derive safely implementable control laws we propose to apply additional input filters and obtain a closed-loop system in the form of a set of retarded type differential equations. An example illustrating the design scheme is given.

1. Introduction

The predictive scheme for the compensation of input delay known now as Smith predictor has been proposed in Smith (1959). Starting from this publication a lot of new techniques to control systems with input delay have been developed, see Artstein (1982), Krstic (2009), Kwon and Pearson (1980), Manitius and Olbrot (1979) and references therein. In comparison to the case of systems with input delay, very few results are available for systems with both input and state delays, see Jankovic (2010), Kharitonov (2014) and Zhou (2014).

Recently, an interesting predictor-based scheme for the compensation of input delays has been proposed in Tsubakino, Roux Oliveira, and Krtic (2015). The scheme is destined for the computation of stabilizing control laws to systems where each control channel has an individual delay. In this contribution we present an extension of the prediction-based scheme to the case of systems with state and input delays. For simplicity of the presentation we treat the case of systems with one state delay and two input delays, but the exploited technique can be extended to the case of systems with multiple state and input delays.

Similar to the case of systems with input delays the design scheme is based on the variation-of-constants formula and involves several stages. At the first stage a control component with the minimal input delay is defined. Then, the component is used to design the control component with the next input delay, and so on. The obtained stabilizing controllers are described by a set of integral equations, similar to that in Tsubakino et al. (2015), with some additional terms due to the presence of the state delay in the system.

Exact implementation of these controllers is not possible due to the integral terms. In applications integrals are approximated by finite Riemann type sums according to quadrature rules. As a result the integral equations are replaced by the corresponding difference equations. Such modification of the control laws leads to a serious modification of their nature. While the original integral equations are of the retarded type, the modified difference equations are of the neutral type.

A detailed analysis of possible consequences of the approximation in the case of systems with only input delay undertaken in Engelborghs, Dambrine, and Roos (2001), Gu (2012) and Van Assche, Dambrine, Lafey, and Richard (1999) reveals that the new control laws lead to unstable closed-loop system when the integral equations are not internally stable. This means that the stability of the closed-loop system with the original integral control laws may not imply that with the approximated ones, even when quadrature rules of high accuracy are used. To overcome this technical limitation the idea to apply for the control input a low-pass filter has been proposed in Mondie and Michiels (2003). On the one hand, such a filter allows to maintain the retarded type of the control laws when the involved integrals are substituted by corresponding Riemann type sums. On the other hand, the application of the filters can be treated as augmentation of the system state by treating control variables as a part of the system state. Stability of the system with

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Fouad Giri under the direction of Editor Miroslav Krstic.

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http://dx.doi.org/10.1016/j.automatica.2017.01.028
0005-1098/

the extended state implies that of the original closed-loop system. It has been demonstrated in Mondie and Michiels (2003) that approximation of integrals in the extended system by Riemann type sums does not destroy stability.

In Zhou (2014) the idea to apply an auxiliary filter for the control input has been extended to the case of systems with both input and state delays. In this contribution we present an extension of the design procedure, proposed previously in Kharitonov (2015) for systems with one input delay, to the case of systems with several input delays. Since this procedure is based on the original control laws there is no need to design new control laws for the extended system as it has been proposed in Zhou (2014).

Section 2 starts with a description of time delay systems studied in the contribution. Some basic notations and an explicit expression for future values of the system state are given here. In Section 3 a two step design scheme is presented. In Section 4 a problem of safe implementation of the stabilizing controls is discussed. Here we provide a design procedure for the computation of safely implementable stabilizing controls. Section 4 is devoted to the computation of the characteristic function of the closed-loop system. In Section 6 an example, illustrating the design scheme, is presented.

2. Preliminaries

2.1. System description

Given a time-delay system

$$\frac{dx(t)}{dt} = A_0x(t) + A_1x(t-h) + B_1u^{(1)}(t-\tau_1) + B_2u^{(2)}(t-\tau_2), \quad (1)$$

where A_0, A_1 are real $n \times n$ matrices, B_1 is a real $n \times m_1$ matrix and B_2 is a real $n \times m_2$ matrix. The state delay h is positive and the control delays are ordered $0 \le \tau_1 \le \tau_2$.

2.2. Basic assumption

In the case $\tau_1 = \tau_2 = 0$ the system is of the form

$$\frac{dx(t)}{dt} = A_0x(t) + A_1x(t-h) + Bu(t). \quad (2)$$

Here $n \times (m_1 + m_2)$ matrix $B = (B_1, B_2)$ and

$$u(t) = \begin{pmatrix} u^{(1)}(t) \\ u^{(2)}(t) \end{pmatrix}.$$

Assume that for system (2) there exists a control law

$$u(t) = F_0x(t) + F_1x(t-h)$$

such that the closed-loop system

$$\frac{dx(t)}{dt} = (A_0 + BF_0)x(t) + (A_1 + BF_1)x(t-h) \quad (3)$$

is exponentially stable.

Partitioning the gain matrices F_0, F_1 as follows

$$F_j = \begin{pmatrix} F_{j1} \\ F_{j2} \end{pmatrix}, \quad j = 0, 1,$$

where F_{j1} is $m_1 \times n$ matrix, and F_{j2} is $m_2 \times n$ matrix we re-write the control law in the component-wise form

$$u^{(1)}(t) = F_{01}x(t) + F_{11}x(t-h), \quad (4)$$

$$u^{(2)}(t) = F_{02}x(t) + F_{12}x(t-h). \quad (5)$$

2.3. Problem statement

There are three possible distributions of the system delays. The first one is when

$$0 < \tau_1 < \tau_2 \le h,$$

in the second one

$$0 < \tau_1 \le h < \tau_2.$$

In this paper we study the case when the system delays are ordered as follows

$$0 < h \le \tau_1 < \tau_2.$$

The reason to address this particular case is the following: In the case of the first distribution we need prediction only for $x(t+\tau_1)$ and $x(t+\tau_2)$. In the case of the second distribution we need additionally prediction for $x(t+\tau_2-h)$. In the last case prediction for $x(t+\tau_1)$, $x(t+\tau_2)$, $x(t+\tau_1-h)$ and $x(t+\tau_2-h)$ will be needed. This means that this case is more involved compared to the previous ones.

Problem 1. Based on (4)–(5) find a scheme to compensate time delays in the control components.

2.4. State prediction

Let $K^{(1)}(t)$ be a fundamental matrix of system (1), see Bellman and Cooke (1963). For $t < 0$ this matrix is equal to the trivial one, $K^{(1)}(t) = 0_{n\times n}$, $K^{(1)}(0) = I_n$, and for $t \ge 0$ the matrix satisfies the equation

$$\frac{dK^{(1)}(t)}{dt} = A_0K^{(1)}(t) + A_1K^{(1)}(t-h).$$

Given $\eta > 0$ then along a solution of system (1) the following variation-of-constants formula holds (Bellman & Cooke, 1963),

$$\begin{aligned} x(t+\eta) = {} & K^{(1)}(\eta)x(t) \\ & + \int_{-h}^{0} K^{(1)}(\eta - h - \theta)A_1x(t+\theta)d\theta \\ & + \int_{-\tau_1}^{\eta-\tau_1} K^{(1)}(\eta - \tau_1 - \xi)B_1u^{(1)}(t+\xi)d\xi \\ & + \int_{-\tau_2}^{\eta-\tau_2} K^{(1)}(\eta - \tau_2 - \xi)B_2u^{(2)}(t+\xi)d\xi. \end{aligned} \quad (6)$$

3. Design scheme

Here a designed scheme for stabilization of system (1) is presented. The scheme is based on procedure previously proposed in Kharitonov (2015) for systems with delay in the state variable and one input delay, and consists of two steps. At the first one a control law corresponding component $u^{(1)}$ is derived. At the second one component $u^{(2)}$ is defined.

3.1. First step

Based on (4) the first control component is defined as follows

$$u^{(1)}(t) = F_{01}x(t+\tau_1) + F_{11}x(t+\tau_1-h).$$

To compensate the control delay τ_1 we apply (6) and replace the advanced values of the state variable on the right hand side of the