Automatica 79 (2017) 27–34

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Sliding mode control for singular stochastic Markovian jump systems with uncertainties[☆]

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ARTICLE INFO

Article history:
Received 26 June 2014
Received in revised form
13 November 2016
Accepted 18 December 2016

Keywords:
Singular systems
Stochastic systems
Markovian jump systems
Sliding mode control

ABSTRACT

This paper considers sliding mode control design for singular stochastic Markovian jump systems with uncertainties. A suitable integral sliding function is proposed and the resulting sliding mode dynamics is an uncertain singular stochastic Markovian jump system. A set of new sufficient conditions is developed which not only guarantees the stochastic admissibility of the sliding mode dynamics, but also determines all the parameter matrices in the integral sliding function. Then, a sliding mode control law is synthesized such that reachability of the specified sliding surface can be ensured. Finally, three examples are given to demonstrate the effectiveness of the results.

1. Introduction

Markovian jump systems (MJSs) have the advantage of better representing physical systems with random changes in both structure and parameters. Much recent attention has been paid to the investigation of these systems (Fang & Loparo, 2002; Xiong & Lam, 2006; Yue & Han, 2005). Singular systems have extensive applications in fields related to electrical circuits and power systems (Lewis, 1986; Yang, Zhang, & Zhou, 2006). When singular systems experience abrupt changes in their structure, it is natural to model them as singular Markovian jump systems (SMJSs) (Boukas, 2008; Huang & Mao, 2010). In practice, these systems are often corrupted by noise, for example Brownian motion. Therefore it is of significance to study singular stochastic Markovian jump systems (SSMJSs).

Sliding mode control (SMC) has been recognized as an effective strategy for control of systems with uncertainties and nonlinearity (Hung, Gao, & Hung, 1993; Ma & Boukas, 2009). The sliding mode dynamics is a reduced-order system and completely insensitive to matched uncertainties (Edwards & Spurgeon, 1998; Utkin, Guldner, & Shi, 1999). Sliding mode methods can also be applied to systems in the presence of mismatched uncertainties (Yan, Spurgeon, & Edwards, 2005). To obtain similar levels of robustness from a classical linear state feedback controller, high gain is required (Young, Utkin, & Özgüner, 1999) which can be limiting in terms of controller saturation and practical application. A novel augmented sliding mode observer is presented for the augmented system of MJSs and is utilized to eliminate the effects of sensor faults and disturbances (Li, Gao, Shi, & Zhao, 2014). Sliding mode methods are successfully applied to uncertain time-delay systems (Alwi & Edwards, 2008; Fridman, Gouaisbaut, Dambrine, & Richard, 2003; Yan, Spurgeon, & Edwards, 2013), interconnected systems (Yan, Spurgeon, & Edwards, 2010), stochastic systems (Niu, Ho, & Wang, 2007; Shi, Xia, Liu, & Rees, 2006), SMJSs (Wu & Daniel, 2010; Wu, Su, & Shi, 2012; Wu & Zheng, 2009). When a linear sliding function is used, the dimension of the resulting sliding motion

☆ This work was supported by the National Natural Science Foundation of P. R. China under grant No. 61673099,61603055, 61304054 and 61273008, respectively and the Royal Academy of Engineering of United Kingdom under grant No. 12/13RECI027. The initial version of this paper was completed during the period from Oct. 19, 2013 to Jan. 18, 2014 when the first author, Professor Qingling Zhang, visited the University of Kent, funded by the Royal Academy of Engineering of the United Kingdom under grant No. 12/13RECI027. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Fabrizio Dabbene under the direction of Editor Richard Middleton.

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http://dx.doi.org/10.1016/j.automatica.2017.01.002
0005-1098/

will be reduced and the regular form typically used for sliding mode control design (Edwards & Spurgeon, 1998) is necessary in order to solve the corresponding existence problem. When considering singular systems, this regular form is available only if the column vector of the input matrix B is a linear representation of that of the derivative matrix E. In comparison, the integral-type sliding function introduces a compensator whose dimension is equal to the dimension of the input vector and the resulting sliding motion is of full order. In this case the regular form typically adopted for sliding mode controller design is not required and the integral-type sliding function (Wu & Daniel, 2010; Wu et al., 2012; Wu & Zheng, 2009) is suitable for any singular system. In Wu et al. (2012) and Wu and Zheng (2009), parameter matrices G_i (G) in the sliding function need to be designed in advance. If the selection of these parameter matrices is not appropriate, additional conservatism will be introduced into the stability analysis of the resulting sliding mode dynamics. In order to decrease the conservatism, these parameter matrices need be redesigned but no constructive design approach is given. In Wu and Daniel (2010), although a method of how to design all the parameter matrices in the sliding function is given, a particular constraint must be satisfied so that EB_i for system matrices E and B_i must have full column rank.

This paper considers the design of a SMC for a class of uncertain SSMJSs. Key questions to be addressed are stated as follows:

Q1. How to design a suitable sliding function such that conditions developed for the stochastic admissibility of the resulting sliding mode dynamics can determine all the parameter matrices in the sliding function complementing existing design methods?

Q2. How to analyze and synthesize a SMC law so that the proposed approach can effectively reject the effect of Markovian switching on the desired dynamic performance of uncertain SSMJSs?

2. System representation and preliminaries

Consider a nonlinear SSMJS described as follows:

$$\begin{aligned} E\mathrm{d}x(t) &= [(A(r_t)+\Delta A(r_t,t))\,x(t) \\ &\quad + B(r_t)(u(t)+f(x(t),r_t))]\,\mathrm{d}t \\ &\quad + D(r_t)\,x(t)\,\mathrm{d}\varpi(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input and $\varpi(t)$ is a one-dimensional Brownian motion satisfying $\mathcal{E}\{\mathrm{d}\varpi(t)\} = 0$ and $\mathcal{E}\{\mathrm{d}\varpi^2(t)\} = \mathrm{d}t$, $\mathcal{E}\{\cdot\}$ denotes the mathematical expectation of the stochastic process or vector. The matrix $E \in \mathbb{R}^{n\times n}$ may be singular. It is assumed that $\mathrm{rank}(E) = r \le n$. Matrices $A(r_t), B(r_t)$ and $D(r_t)$ are known and real with appropriate dimensions where $B(r_t)$ has full column rank, $\Delta A(r_t,t)$ is uncertain and satisfies

$$\Delta A(r_t,t) = M(r_t)\,F(r_t,t)\,N(r_t) \tag{2}$$

where matrices $M(r_t)$ and $N(r_t)$ are known, and the function matrix $F(r_t,t)$ is unknown and Lebesgue-measurable with

$$F^{\mathrm{T}}(r_t,t)\,F(r_t,t) \le I$$

for all $t \ge 0$; $\{r_t, t\ge 0\}$ is a continuous-time Markov process with right continuous trajectories taking values in a finite set $\mathcal{S} = \{1,2,\ldots,N\}$ with the transition rate matrix (TRM) $\Pi \triangleq \{\pi_{ij}\}$ given by

$$\mathcal{P}\{r_{t+h}=j\,|r_t=i\} = \begin{cases} \pi_{ij}h + o(h) & i\ne j \\ 1+\pi_{ii}h+o(h) & i=j \end{cases} \tag{3}$$

where $h > 0$, $\lim_{h\to 0}^{o(h)/h} = 0$; $\pi_{ij} \ge 0$ for $j \ne i$ is the transition rate from mode i at time t to j at time $t+h$, which satisfies $\pi_{ii} = -\sum_{j=1,j\ne i}^{N}\pi_{ij}$; the nonlinear term $f(x(t),r_t) \in \mathbb{R}^m$ represents the system nonlinearity satisfying

$$\|f(x(t),r_t)\| \le \vartheta_{r_t}\|x(t)\| \le \vartheta\,\|x(t)\|, \quad r_t\in\mathcal{S} \tag{4}$$

where $\vartheta_{r_t} > 0$ is a constant and $\vartheta \triangleq \max_{i\in\mathcal{S}}(\vartheta_i)$.

For each $r_t = i \in \mathcal{S}$, corresponding matrices or vectors relating to r_t in the system (1) are denoted with the index i, for example, $A(r_t) = A_i$, $\Delta A(r_t,t) = \Delta A_i(t)$, and $f(x(t),r_t) = f_i(x)$ etc.

The unforced nominal system of the system (1) can be described as

$$E\mathrm{d}x(t) = A_i x(t)\,\mathrm{d}t + D_i x(t)\,\mathrm{d}\varpi(t)\,. \tag{5}$$

A basic assumption and a definition are first introduced.

Assumption 1. For $i\in\mathcal{S}$, $\mathrm{rank}(E) = \mathrm{rank}([E\ D_i])$.

Definition 1 (*Xu & Lam, 2006*).

(i) The continuous SSMJS (5) is said to be regular if $\det(sE - A_i)$ is not identically zero for every $i\in\mathcal{S}$.
(ii) The continuous SSMJS (5) is said to be impulse-free if $\deg(\det(sE-A_i)) = \mathrm{rank}(E)$ for every $i\in\mathcal{S}$.
(iii) The continuous SSMJS (5) is said to be stochastically stable if for any $x_0\in\mathbb{R}^n$ and $r_0\in\mathcal{S}$, there exists a scalar $\tilde{M}(x_0,r_0) > 0$ such that

$$\lim_{t\to\infty}\mathcal{E}\left\{\int_0^t x^{\mathrm{T}}(s,x_0,r_0)\,x(s,x_0,r_0)\,\mathrm{d}s\,|x_0,r_0\right\} \le \tilde{M}(x_0,r_0)$$

where $x(t,x_0,r_0)$ denotes the solution under the initial condition x_0 and r_0.
(iv) The continuous SSMJS (5) is said to be stochastically admissible if it is regular, impulse-free and stochastically stable.

Lemma 1 (*Xu, Zhang, & Hu, 2007*). *For given matrices E, $X>0$, Y, if $E^{\mathrm{T}}X + Y\Lambda^{\mathrm{T}}$ is nonsingular, then there exist matrices $S>0$, L such that $ES + L\Theta^{\mathrm{T}} = \left(E^{\mathrm{T}}X+Y\Lambda^{\mathrm{T}}\right)^{-1}$, where $X,S\in\mathbb{R}^{n\times n}$, $Y,L\in\mathbb{R}^{n\times(n-r)}$, and $\Lambda,\Theta\in\mathbb{R}^{n\times(n-r)}$ are any matrices with full column rank satisfying $E^{\mathrm{T}}\Lambda = 0$, $E\Theta = 0$.*

Lemma 2. *Let M,F,N and P be real matrices of appropriate dimensions with $P>0$, $F^{\mathrm{T}}F\le I$ and a scalar $\varepsilon>0$. Then*

$$MFN + N^{\mathrm{T}}F^{\mathrm{T}}M^{\mathrm{T}} \le \varepsilon MP^{-1}M^{\mathrm{T}} + \frac{1}{\varepsilon}N^{\mathrm{T}}F^{\mathrm{T}}PFN.$$

The proof is trivial, so it is omitted. □

Remark 1. From Lemma 2, when $F = I$, it follows that $MN + N^{\mathrm{T}}M^{\mathrm{T}} \le \varepsilon MP^{-1}M^{\mathrm{T}} + \frac{1}{\varepsilon}N^{\mathrm{T}}PN$, and when $P = I$, it follows that $MFN + N^{\mathrm{T}}F^{\mathrm{T}}M^{\mathrm{T}} \le \varepsilon MM^{\mathrm{T}} + \frac{1}{\varepsilon}N^{\mathrm{T}}N$.

3. SMC synthesis

In this section, a sliding surface is designed and the corresponding sliding motion is analyzed. Then sliding mode controllers are synthesized such that the closed-loop system has the desired performance.

For the system (1), consider the following integral sliding function:

$$s(t) = B_i^{\mathrm{T}}\bar{P}_iEx(t) - \int_0^t B_i^{\mathrm{T}}\bar{P}_i(A_i+B_iK_i)\,x(\theta)\,\mathrm{d}\theta \tag{6}$$

where $\bar{P}_i\in\mathbb{R}^{n\times n}$ and $K_i\in\mathbb{R}^{m\times n}$ are real matrices to be designed with $B_i^{\mathrm{T}}\bar{P}_iB_i$ being nonsingular. It should be noted that due to the