Automatica 79 (2017) 78–83

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Event-based consensus of second-order multi-agent systems with discrete time ☆

Wei Zhu [a], Huizhu Pu [a], Dandan Wang [a], Huaqing Li [b]

[a] Research Center of System Theory and Application, Chongqing University of Posts and Telecommunications, Chongqing 400065, China

[b] College of Electronic and Information Engineering, Southwest University, Chongqing, 400715, China

ARTICLE INFO

Article history:
Received 17 December 2015
Received in revised form
20 November 2016
Accepted 3 January 2017
Available online 2 March 2017

Keywords:
Event-based control
Consensus
Discrete time
Multi-agent systems

ABSTRACT

Event-based control has received considerable attention due to its irreplaceable advantage in resource-limited systems. In this paper, event-based consensus of second-order discrete-time multi-agent systems is investigated. The event-based controller is designed for each agent, which is based on the state measurement error among neighborhood agents at its own triggering time instants. Some sufficient conditions are obtained for achieving consensus. To avoid observing the current states of its neighbor agents as well as its own state at every discrete-time, a self-triggered approach is presented to determine the triggering time sequence. Furthermore, certain conditions are given to avoid controller update at every discrete time. Finally, a simulation example is given to illustrate the efficiency of the proposed control strategy.

1. Introduction

Consensus analysis of multi-agent systems has attracted intensive attention due to its wide applications such as in multi-robot systems (Ren, 2008), sensor networks (Pereira et al., 2011), formation control (Lin, Qin, Li, & Ren, 2011) and swarms (Veysel Gazi, 2002). Due to practical control applications involving agents with limited computing capability, limited capabilities of communication and actuation, and limited onboard energy resource, some notable control protocols to reduce the transmission load and controller update have received more and more interest. As an effective method, the event-based control is put forward to mitigate the unnecessary waste of computation and communication resources.

Concerning on multi-agent systems, autonomous agents are often equipped with a small embedded micro-processor which usually has only limited on-board resources, limited computing, communicating and actuating capabilities. These factors motivate researchers to develop an event-based control scheme for multi-agent systems on digital platform. The event-based implementation of the consensus protocol was firstly developed in Dimarogonas and Johansson (2009) and then received great interest in recent years. The distributed event-based control strategy for single-integrators to determine the control updates was presented respectively in Chen, Hao, and Shao (2015), Dimarogonas, Frazzoli, and Johansson (2012) and Fan, Feng, Wang, and Song (2013) using the state-dependent triggering function. Some results based on event-based control for double-integrators were also presented in Hu, Chen, and Li (2011), Li, Liao, Huang, and Zhu (2015), Liu, Guo, Hu, Xu, and Yang (2016), Seyboth, Dimarogonas, and Johansson (2013) and Yan, Shen, Zhang, and Shi (2014). The event-based consensus of multi-agent systems with linear dynamics was studied in Garcia, Cao, and Casbeer (2014), Guo, Ding, and Han (2014), Zhu and Jiang (2015) and Zhu, Jiang, and Feng (2014).

Note that the above investigations mostly focused on the multi-agent systems with continuous-time dynamics. From an implementation point of view, consensus of multi-agent systems with discrete-time dynamics is an important issue. Some sufficient conditions to achieve an average consensus for discrete-time event-triggered and self-triggered protocols were reported in Chen and Hao (2012) and Hamada, Hayashi, and Takai (2014). A new distributed event-triggered scheme was proposed for discrete-time multi-agent systems with communication delays in Li, Ho, and Xu (2014). The decentralized event-triggered consensus problem for discrete-time linear multi-agent systems with directed graph

☆ This work is supported partly by National Natural Science Foundation of China under Grant 61673080, 61503053, 61403314, partly by Training Programme Foundation for the Talents of Higher Education by Chongqing Education Commission, partly by Innovation Team Project of Chongqing Education Committee under Grant CXTDX201601019. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Wei Ren under the direction of Editor Christos G. Cassandras.

E-mail addresses: zhuwei@cqupt.edu.cn (W. Zhu), puhuizhu@sina.com (H. Pu), 1060539492@qq.com (D. Wang), huaqingli@hotmail.com (H. Li).

http://dx.doi.org/10.1016/j.automatica.2017.01.042
0005-1098/

was discussed in Yang, Liu, and Chen (2015) and Yang, Ren, Liu, and Chen (2016). Based on the Lyapunov functional method and the Kronecker product technique, the event-triggered consensus problem for a set of discrete-time heterogeneous multi-agent systems was considered in Yin, Yue, and Hu (2013), a sufficient condition obtained by linear matrix inequality was presented. The leader-following and leaderless synchronization of linear discrete-time dynamical networks were achieved in Chen, Zhang, Su, and Li (2015) by a distributed event-triggering strategy.

In this paper, the event-based consensus of multi-agent systems with second-order discrete-time models is studied. Under the assumption that the interconnection topology among agents contains a spanning tree, some sufficient conditions on consensus are presented. The novelties of this paper lie in the following three aspects:

(I) Based on the combinational measurement error of neighborhood agents, a novel controller is designed. This controller avoids updating at the neighbors' triggering time, which may result in much less amount of controller update.

(II) A self-triggered approach is given to determine the triggering time sequence, which can avoid observing the current states of its neighborhood agents as well as its own state at every discrete-time h.

(III) Certain conditions are given to guarantee that the minimum allowable time interval of triggering time sequence for each agent is bigger than the discrete-time h of the multi-agent systems, which implies that the controller will not update at every discrete-time h.

The organization of the remaining part is given as follows. Section 2 gives some preliminaries about the problem formulation. Main theoretical results and a self-triggered approach for determining the triggering time sequence are obtained in Sections 3 and 4, respectively. A numerical simulation example is presented to show the effectiveness of the theoretical results in Section 5. In Section 6, concluding remarks are drawn.

2. Preliminaries

The interconnection topology among n agents is represented by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$, $\mathcal{V} = \{1, 2, \ldots, n\}$. The set of in-neighbors of vertex i is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}\}$. $A = (a_{ij})_{n\times n}$ is the adjacency matrix of agents, where a_{ij} denotes the weight of edge (j, i). $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$, otherwise. Let $D = \mathrm{diag}\{d_1, d_2, \ldots, d_n\}$ be the degree matrix whose diagonal elements are defined by $d_i = \sum_{j\in\mathcal{N}_i} a_{ij}$. The Laplacian $L = (l_{ij})_{n\times n}$ is defined as $L = D - A$.

Consider the following second-order multi-agent systems with discrete time dynamics

$$\begin{cases} x_i(t+h) = x_i(t) + v_i(t)h \\ v_i(t+h) = v_i(t) + u_i(t)h, \end{cases} \quad i = 1, \ldots, n, \tag{1}$$

where $x_i(t), v_i(t) \in R$ denote the state and $u_i(t) \in R$ denotes the control input of agent i, respectively. The update time instants t will be the form $t = ph, p = 1, 2, \ldots. h \in (0, 1)$ is the discrete time.

Definition 1. The multi-agent systems (1) are said to achieve consensus under some designed protocol $u_i(t)$ if for any initial values, the states of all agents satisfy

$$\begin{matrix} \lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0 \\ \lim_{t\to\infty} \|v_i(t) - v_j(t)\| = 0, \end{matrix} \quad \forall i, j = 1, 2, \ldots, n.$$

In order to study the event-based consensus of (1), the triggering time instant t_k^i for each agent i is defined iteratively by

$$t_{k+1}^i = \inf\{t : t > t_k^i \text{ and } f_i(t) \ge 0\}, \tag{2}$$

where

$$f_i(t) = |e_i(t)| - \beta_1 |q_i(t_k^i)| - \beta_2 \mu^t \tag{3}$$

is said to be the triggering function for some $\beta_1 > 0$, $\beta_2 > 0$, $\mu \in (0, 1)$, and $e_i(t) = q_i(t_k^i) - q_i(t)$, $t \in [t_k^i, t_{k+1}^i)$ is the measurement error, and $q_i(t) = \sum_{j\in\mathcal{N}_i} a_{ij}(x_i(t) - x_j(t) + v_i(t) - v_j(t))$ is the combining measurement information. It is clear that $\{t_k^i\}$ is the subsequence of $\{ph\}_{p=1}^{\infty}$. $e_i(t)$ is reset to 0 at $t = t_k^i$. Without loss of generality, we assume that $t_0^i = 0$ for any agent i.

Based on the triggering time sequence $\{t_k^i\}$, the following controller for multi-agent systems (1) will be used:

$$u_i(t) = -\kappa q_i(t_k^i), \quad t \in [t_k^i, t_{k+1}^i), i = 1, 2, \ldots, n, \tag{4}$$

where $\kappa > 0$ is a control parameter to be designed later.

Let $x(t) = (x_1(t), \ldots, x_n(t))^T$, $v(t) = (v_1(t), \ldots, v_n(t))^T$, $e(t) = (e_1(t), \ldots, e_n(t))^T$. Then, (1) can be rewritten as the following compact form

$$\begin{cases} x(t+h) = x(t) + hv(t) \\ v(t+h) = -\kappa hLx(t) + (I_n - \kappa hL)v(t) - \kappa he(t), \end{cases} \tag{5}$$

where I_n denotes the identity matrix with dimension n.

Let $y(t) = (x_1(t) - x_n(t), \ldots, x_{n-1}(t) - x_n(t))^T$, $z(t) = (v_1(t) - v_n(t), \ldots, v_{n-1}(t) - v_n(t))^T$. Then, (5) can be decoupled as following:

$$\begin{cases} y(t+h) = y(t) + hz(t) \\ z(t+h) = -\kappa h\hat{L}y(t) + (I_{n-1} - \kappa h\hat{L})z(t) \\ \qquad\qquad -\kappa h(\bar{e}(t) - \mathbf{1}_{n-1}e_n(t)), \end{cases} \tag{6}$$

where $\bar{e}(t) = (e_1(t), \ldots, e_{n-1}(t))^T$, $\hat{L} = (\hat{l}_{ij})_{(n-1)\times(n-1)}$, $\hat{l}_{ij} = l_{ij} - l_{nj}$, $i, j = 1, \ldots, n-1$ and $\mathbf{1}_{n-1} = (\underbrace{1, \ldots, 1}_{n-1})^T$.

Let $\varepsilon(t) = (y^T(t), z^T(t))^T$, from (6), we can get that

$$\varepsilon(t+h) = H\varepsilon(t) + P\tilde{e}(t), t \ge 0, \tag{7}$$

where

$$H = \begin{pmatrix} I_{n-1} & hI_{n-1} \\ -\kappa h\hat{L} & I_{n-1} - \kappa h\hat{L} \end{pmatrix},$$

$$P = \begin{pmatrix} \mathbf{0}_{(n-1)\times n} & \mathbf{0}_{(n-1)\times n} \\ \mathbf{0}_{(n-1)\times n} & -\kappa hQ_{(n-1)\times n} \end{pmatrix},$$

$\tilde{e}(t) = (\underbrace{0, \ldots, 0}_{n}, e_1(t), \ldots, e_n(t))^T$, $Q_{(n-1)\times n}$ is the first $n-1$ rows of matrix $Q = \begin{pmatrix} I_{n-1} & -\mathbf{1}_{n-1} \\ \mathbf{0}_{n-1}^T & 1 \end{pmatrix}$, $\mathbf{0}_{n-1} = (\underbrace{0, \ldots, 0}_{n-1})^T$.

3. Main results

Lemma 1. *All the eigenvalues of matrix $\hat{L}$ have positive real parts if and only if the interconnection graph of all the agents has a spanning tree.*

Lemma 2. *Assume that the interconnection graph of all the agents has a spanning tree and $h \in (0, 1 - \frac{|b|}{\sqrt{a^2+b^2}})$, $\kappa \in \Omega$ defined by (∗∗) (see Box I). Then, we have $\rho(H) < 1$, where $\rho(H)$ represents the spectral radius of matrix H, a and b denote the real part and image part of the eigenvalues of $\hat{L}$, respectively.*