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Brief paper

Convergence of iterative learning control for SISO nonrepetitive systems subject to iteration-dependent uncertainties ☆

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ABSTRACT

This paper studies the robust convergence properties of iterative learning control (ILC) for single-input, single-output (SISO), nonrepetitive systems subject to iteration-dependent uncertainties that arise in not only initial states and external disturbances but also plant models. Given an extended relative degree condition, it is possible to propose necessary and sufficient (NAS) conditions for robust ILC convergence. The tracking error bound is shown to depend continuously on the bounds of the iteration-dependent uncertainties. When the iteration-dependent uncertainties are bounded, NAS conditions exist to guarantee bounded system trajectories and output tracking error. If the iteration-dependent uncertainties converge, then NAS conditions ensure bounded system trajectories and zero output tracking error. The results are also extended to a class of affine nonlinear systems satisfying a Lipschitz condition. Simulation tests on a representative batch process demonstrate the validity of the obtained robust ILC convergence results.

1. Introduction

Iterative learning control (ILC) is an effective approach for controlling processes that have both a finite time of operation and are operated over-and-over from nominally the same initial condition. Typically, ILC algorithms are based only on input–output data, one of the most distinct features of ILC (see, e.g., the ILC surveys Ahn, Chen, & Moore, 2007; Bristow, Tharayil, & Alleyne, 2006; Shen & Wang, 2014; Xu, 2011 and references therein). Due to this feature, ILC is easy to implement in practice and many application results of ILC have appeared in the areas of both motion control (Bifaretti, Tomei, & Verrelli, 2011; Chien & Tayebi, 2008; Freeman & Tan, 2013) and process control (Liu & Wang, 2012; Shi, Gao, & Wu, 2005, 2006).

For ILC problems, it is most common to assume that the plant does not change from operation-to-operation though it may be subject to model (parameter) uncertainties (see, e.g., Ahn, Moore, & Chen, 2007; Nguyen & Banjerdpongchai, 2011; van de Wijdeven, Donkers, & Bosgra, 2009; Xu & Tan, 2002). We call this a *repetitive system*, meaning that the system to be controlled is the same from operation-to-operation. If a plant does not have such a property, we call it a *nonrepetitive system*. Because uncertainties are inevitable, especially in practical applications, robustness is an important problem in ILC. Two robust ILC problems related to iteration-dependent uncertainties have been studied in depth. One problem relaxes the initial state resetting condition, and studies whether or not the ILC system still works effectively in the presence of iteration-dependent initial state shifts (see, e.g., Chen, Wen, Gong, & Sun, 1999; Heinzinger, Fenwick, Paden, & Miyazaki, 1992; Sun & Wang, 2002; Zhu, Xu, Huang, & Hu, 2015). The second problem takes into account possible iteration-dependent external disturbances, and discusses the ILC tracking performance from a disturbance rejection point of view (see, e.g., Chin, Qin, Lee, & Cho, 2004; Gunnarsson & Norrlöf, 2006; Kim, Zheng, & Sugie, 2007; Saab, 2001). However, these studies on robust ILC apply only to repetitive systems, whereas we will show that robust ILC is still applicable for nonrepetitive systems subject to iteration-dependent plants. Further, we note that in limiting case as nonrepetitive systems become repetitive, our results become

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equivalent to standard robust ILC results for repetitive systems. Thus, in this paper we provide a more general overall theory.

In practice, we may encounter plants operating iteratively that are nonrepetitive. One motivating class of practical nonrepetitive plants that are often operated in an ILC fashion is the batch process (see, e.g., Shi et al., 2005, 2006). Another class of motivating examples is the execution of a cooperative control task performed by a networked multi-agent system that learns to cooperate, which however has a topology of interconnections between agents that changes with training iteration (see, e.g., Meng, Jia, & Du, 2015; Meng & Moore, 2016, 2017a). Since the interactions between agents also create a class of control plants in addition to all agents' dynamics, the iteration-dependent topologies create a nonrepetitive ILC problem. For other motivating examples, see Chen and Moore (2002) for pick-and-place manipulators transporting mass-dependent objects, Yan and Hou (2010) for freeway traffics subject to the day-dependent free speed, and Butcher and Karimi (2010) for linear permanent magnet synchronous motors subject to the position-dependent force ripple.

Of particular note in looking at the ILC literature is the notion that the nonrepetitiveness can be harnessed by making full use of higher-order internal models in both linear (Chen & Moore, 2002; Moore, Ahn, & Chen, 2008; Zhu et al., 2015) and nonlinear (Yin, Xu, & Hou, 2010) cases. The focus in such studies is improvement of ILC system performance in the face of nonrepetitiveness, such as reducing baseline errors and accommodating iteration-dependent disturbances, uncertainties and/or output reference trajectories. However, for most results, the pattern or structure of iteration-dependent uncertainties in ILC should be known in general, and there are usually restrictive assumptions, such as high-order updating laws in iteration (Chen & Moore, 2002; Moore et al., 2008; Yin et al., 2010; Zhu et al., 2015), exponential convergence of disturbances or noises in iteration (Moore et al., 2008), and identical alignment of initialization conditions (Yin et al., 2010). Although there have been recent studies on ILC for nonrepetitive systems (see, e.g., Altin & Barton, 2015; Meng & Moore, 2014, 2017b;), only sufficient conditions have been obtained for robust convergence of the resulting ILC processes (although notably Altin and Barton (2015) does give design guidelines for achieving convergence). Moreover, there is a challenging problem that exists about the relation between the relative degree and robust convergence conditions of nonrepetitive ILC systems. Though this is clear to follow in repetitive ILC systems (Ahn, Chen et al., 2007), it is not easy to develop even for single-input, single-output (SISO), nonrepetitive systems.

Previously, the authors have considered the case of discrete-time ILC for nonrepetitive systems (Meng & Moore, 2014, 2017b). In this paper, we give additional insights into the basic robust convergence analysis problem for the SISO case. Specifically, we look at robust ILC of nonrepetitive, SISO linear discrete-time systems subject to iteration-dependent initial state shifts, external disturbances, and nonrepetitive system matrices. We present necessary and sufficient (NAS) conditions for boundedness of system trajectories and either boundedness or convergence to zero of the output tracking error, in comparison with Meng and Moore (2017b) and Meng and Moore (2014) that can offer only sufficient conditions. To establish basic robust convergence results for SISO nonrepetitive systems, we develop a new convergence analysis approach for ILC by exploiting stability results for discrete parameterized systems. We call it an "extended contraction mapping-based ILC" approach, which leads to new NAS convergence conditions for the boundedness of system trajectories for nonrepetitive ILC systems. Notably, the resulting ILC processes no longer need to satisfy the contraction mapping principle at all iterations. Furthermore, an extended relative degree condition is given, which provides a NAS guarantee that there is a learning gain such that the convergence conditions can be satisfied. This extended relative degree condition does not require a relative degree requirement at each iteration. By contrast, it is hard to provide the convergence conditions in Meng and Moore (2017b) with clear relative degree guarantees. Finally, we have NAS guarantees for the tracking error to be bounded when the iteration-dependent parameters in the ILC systems are bounded and for the perfect tracking when the iteration-dependent parameters converge with increasing iteration.

This paper is organized as follows. In Section 2, we present the problem statement of ILC for nonrepetitive systems. The ILC system dynamics analysis is performed in Section 3, based on which we propose main NAS convergence results for ILC of nonrepetitive systems (note that all proofs are collected in the Appendices A and B). Section 4 illustrates the theoretical results via simulation tests for a nonrepetitive batch process (Shi et al., 2006). Conclusions are given in Section 5.

Notations: $\mathbb{Z} = \{1, 2, 3, \ldots\}$; $\mathbb{Z}_+ = \{0, 1, 2, \ldots\}$; $\mathbb{Z}_N = \{0, 1, \ldots, N\}$ for any $N \in \mathbb{Z}_+$; $|a|$ denotes the absolute value of a scalar a; and $\|A\|$ denotes any matrix (or vector) norm of a matrix (or vector) A. For any matrix function $f_l(k)$ depending on two independent variables k and l, $\Delta : f_l(k) \to \Delta f_l(k) \triangleq f_{l+1}(k) - f_l(k)$ denotes the forward (iteration) difference operator.

2. Problem statement

Consider a linear SISO discrete-time system with evolution along an infinite iteration process, described by $l \in \mathbb{Z}_+$, and over a finite time duration, denoted by $k \in \mathbb{Z}_N$, as follows:

$$\begin{aligned} x_l(k+1) &= A_l(k)x_l(k) + b_l(k)u_l(k) + w_l(k) \\ y_l(k) &= c_l(k)x_l(k) + v_l(k) \end{aligned} \tag{1}$$

where $x_l(k) \in \mathbb{R}^n$, $u_l(k) \in \mathbb{R}$, and $y_l(k) \in \mathbb{R}$ are the state, control input, and output, respectively; $w_l(k) \in \mathbb{R}^n$ and $v_l(k) \in \mathbb{R}$ are the external load and measurement disturbances, respectively; and $A_l(k) \in \mathbb{R}^{n\times n}$, $b_l(k) \in \mathbb{R}^n$, and $c_l(k) \in \mathbb{R}^{1\times n}$ are both time-dependent and iteration-dependent matrices. From the formulation (1), we can see that the controlled system is not only subject to nonrepetitive external disturbances but also nonrepetitive itself due to the iteration-dependent plant model matrices (Meng & Moore, 2017b). Note that in ILC of repetitive systems, there is a basic assumption on the system relative degree determined by the first nonzero Markov parameter. Here, we similarly address this issue for the nonrepetitive system (1) by considering the parameter $c_l(k+1)b_l(k)$, which has not been solved for ILC of nonrepetitive systems in Meng and Moore (2014, 2017b).

In this paper, we say that the system (1) achieves the perfect tracking of any desired reference trajectory $r(k) \in \mathbb{R}$ over $k \in \mathbb{Z}_N$ if

$$\lim_{l\to\infty} y_l(k) = r(k), \quad \forall k = 1, 2, \ldots, N. \tag{2}$$

It is worth noting that the tracking objective (2) may not be achieved in the presence of iteration-dependent system uncertainties. In this case, we desire the tracking error $e_l(k) = r(k) - y_l(k)$ for $k \in \mathbb{Z}_N$ and $l \in \mathbb{Z}_+$ to be bounded such that

$$\sup_{l\ge 0} |e_l(k)| \le \beta_e \quad \text{and} \quad \limsup_{l\to\infty} |e_l(k)| \le \beta_{e_{\sup}}, \quad \forall k = 1, 2, \ldots, N \tag{3}$$

where $\beta_e > \beta_{e_{\sup}} \ge 0$ are finite bounds to be determined. At the same time as the tracking objective (2) or (3), we desire the input and state of the system (1) to be bounded such that

$$\begin{aligned} &\sup_{l\ge 0} \max_{0\le k\le N-1} |u_l(k)| \le \beta_u < \infty, \\ &\sup_{l\ge 0} \max_{0\le k\le N} \|x_l(k)\| \le \beta_x < \infty \end{aligned} \tag{4}$$